# The riddle of the $f_0(980)$: is it the quark–antiquark state?

**L.G. Dakhno**

**There exist at least two points of view on the nature of $f_0(980)$:**

**1. $f_0(980)$ is a standard quark–antiquark state;**

**2. $f_0(980)$ is an exotic state — $qq\bar{q}\bar{q}$ or $K\bar{K}$ molecule.**

**Should $f_0(980)$ be included into the lowest scalar nonet?**

**Or, being exotic particle, does it play a special role being beyond the nonet classification?**

## Arguments favouring the quark–antiquark nature of $f_0(980)$:

**1. Results of the $K$-matrix analysis ($\pi\pi, K\bar{K}, \eta\eta, \eta\eta'$, $\pi\pi\pi\pi$ spectra).**

**2. Systematics of mesons on the $(n, M^2)$ and $(J, M^2)$ planes.**

**3. Hadronic decay of the $D_s^+$-meson: $D_s^+ \to \pi^+\pi^+\pi^-$.**

**4. Radiative decays involving $f_0(980)$: $f_0(980) \to \gamma\gamma$, $\phi(1020) \to \gamma f_0(980)$.**

**5. Production of $f_0(980)$ in $p\bar{p}$ central production and large-$t$ $\pi p$ collisions, hadronic $Z^0$-decay.**

## 1. Results of the $K$-matrix analysis (V.V. Anisovich, A.V. Sarantsev, Eur. Phys. J. A 16, 229, 2003).

**$K$-matrix analysis covers the invariant mass range 280–1900 MeV. The following data set has been included into K-matrix analysis:**

**(1) GAMS data on the $S$-wave two-meson production in the reactions $\pi p \to \pi^0\pi^0 n$, $\eta\eta n$ and $\eta\eta' n$ at small, $|t| < 0.2$ (GeV/$c$)$^2$, and large, $0.30 < |t| < 1.0$ (GeV/$c$)$^2$, nucleon momenta transferred, ;**

**(3)BNL data on $\pi^- p \to K\bar{K}n$;**

**(4)CERN-Münich data on $\pi^+\pi^- \to \pi^+\pi^-$**

**(5)Crystal Barrel data on $p\bar{p}$(at rest, from liquid $H_2$)$\to \pi^0\pi^0\pi^0$, $\pi^0\pi^0\eta$, $\pi^0\eta\eta$.**

**(6) Crystal Barrel data on proton-antiproton annihilation in gas: $p\bar{p}$(at rest, from gaseous $H_2$)$\to \pi^0\pi^0\pi^0$, $\pi^0\pi^0\eta$,**

**(7) Crystal Barrel data on proton-antiproton annihilation in liquid: $p\bar{p}$(at rest, from liquid $H_2$)$\to \pi^+\pi^-\pi^0$, $K^+K^-\pi^0$, $K_SK_S\pi^0$, $K^+K_S\pi^-$;**

**(8) Crystal Barrel data on neutron-antiproton annihilation in liquid deuterium: $n\bar{p}$(at rest, from liquid $D_2$)$\to \pi^0\pi^0\pi^-$, $\pi^-\pi^-\pi^+$, $K_SK^-\pi^0$, $K_SK_S\pi^-$;**

**(9) E852 Collaboration data on the $\pi\pi$ $S$-wave production in the reaction $\pi^- p \to \pi^0\pi^0 n$ at the nucleon momentum transfers squared $0 < |t| < 1.5\ (\mathrm{GeV/c})^2$.**

**The $K$-matrix technique is used for the description of the two-meson coupled channels $\pi\pi, K\bar{K}, \eta\eta, \eta\eta'$, $\pi\pi\pi\pi$ ($\rho\rho$ or $\sigma\sigma$).**

**$K$-matrix amplitude:** $\hat{A} = \hat{K}(\hat{I} - i\hat{\rho}\hat{K})^{-1}$**, where $\hat{K}$ is $n \times n$ matrix.**

**$S$-wave scattering amplitude in the scalar-isoscalar sector:**

$$K_{ab}^{00}(s) = \left( \sum_{\alpha} \frac{g_a^{(\alpha)} g_b^{(\alpha)}}{M_\alpha^2 - s} + f_{ab}(s) \right)$$

**where $K_{ab}^{IJ}$ is a 5×5 matrix ($a, b$ = 1,2,3,4,5): 1 = $\pi\pi$, 2 = $K\bar{K}$, 3 = $\eta\eta$, 4 = $\eta\eta'$ and 5 = multimeson states (four-pion state mainly at $\sqrt{s} < 1.6$ GeV). The $g_a^{(\alpha)}$ is the coupling constant of the bare state $\alpha$ to meson channel and $M_\alpha$ is mass of the bare state; $f_{ab}(s)$ is smooth (non-pole) term.**

**For the two-particle states, $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$, the phase space matrix elements are ($m_{1a}$ and $m_{2a}$ are the masses of pseudoscalars):**

$$\rho_a(s) = \sqrt{\frac{s - (m_{1a} + m_{2a})^2}{s}} \quad , \qquad a = 1, 2, 3, 4 \, .$$

**The vertices $q\bar{q} \to \pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$ together with analogous couplings for the transition of gluonium $gg \to \pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$ serve us to obtain the following coupling constants squared for the decays $f_0 \to \pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$:**

$$g^2_{\pi\pi} = \frac{3}{2}\left(\frac{g}{\sqrt{2}}\cos\varphi + \frac{G}{\sqrt{2+\lambda}}\right)^2 ,$$

$$g^2_{K\bar{K}} = 2\left(\frac{g}{2}(\sin\varphi + \sqrt{\frac{\lambda}{2}}\cos\varphi) + G\sqrt{\frac{\lambda}{2+\lambda}}\,\right)^2 ,$$

$$g^2_{\eta\eta} = \frac{1}{2}\left(g\,(\frac{\cos^2\Theta}{\sqrt{2}}\cos\varphi + \sqrt{\lambda}\ \sin\varphi\ \sin^2\Theta\ ) + \frac{G}{\sqrt{2+\lambda}}\ (\cos^2\Theta + \lambda\sin^2\Theta\ )\right)^2 ,$$

$$g^2_{\eta\eta'} = \sin^2\Theta\ \cos^2\Theta\ \left(g\,(\frac{1}{\sqrt{2}}\cos\varphi - \sqrt{\lambda}\ \sin\varphi) + G\ \frac{1-\lambda}{\sqrt{2+\lambda}}\right)^2 .$$

**Here $g = g_0\cos\alpha$ and $G = G_0\sin\alpha$, where $g_0$ is a universal constant for all nonet members and $G_0$ is universal decay constant for the gluonium state.**

$$f_0(980) \Longleftrightarrow f_0^{bare}(700\pm100), \qquad f_0(1300) \Longleftrightarrow f_0^{bare}(1220\pm40),$$
$$f_0(1500) \Longleftrightarrow f_0^{bare}(1250\pm40)), \qquad f_0(1750) \Longleftrightarrow f_0^{bare}(1800\pm40),$$

**1. $f_0(980)$: This resonance is dominantly the $q\bar{q}$ state, $q\bar{q}=n\bar{n}\cos\varphi+s\bar{s}\sin\varphi$, with a large $s\bar{s}$ component ($K\bar{K}\le 20\%$):**
$-90^\circ\le\varphi\le-43^\circ$ **at** $W_{gg}\le 0.3$ **; if** $W_{gg}=0$, $\quad\varphi=-67^\circ\pm10^\circ$.
$f_0^{bare}(700\pm100)$**: nearly flavour octet,** $\varphi_{f_0^{bare}(700\pm100)}=-68^{\circ+3}_{-15}$ **(**$\varphi_{octet}=-54.7^\circ$**).**

**2. $f_0(1300)$: The resonance $f_0(1300)$ is formed due to a strong mixing with the primary gluonium and neighbouring $q\bar{q}$ states. The quark component:**
$q\bar{q}=n\bar{n}\cos\varphi+s\bar{s}\sin\varphi$ **with** $-42^\circ\le\varphi\le10^\circ$ **at** $W_{gg}\le0.3$**;**
**if** $W_{gg}=0$, $\quad\varphi=-7^\circ\pm10^\circ$.
**The $f_0(1300)$ is the descendant of the bare $q\bar{q}$ state which is close to the flavour singlet,**
$f_0^{bare}(1220\pm30)$, $\quad\varphi_{f_0^{bare}(1220\pm30)}=14^{\circ+3}_{-15}$.

**For bare states, $f_0^{bare}$, of the same nonet:**

$$\varphi_{f_0^{bare}(1220\pm30)}-\varphi_{f_0^{bare}(700\pm100)}\simeq 90^\circ.$$

**3.** $f_0(1500)$**: Radial excitation** $n=2$**,** $2^3P_0$**. The resonance is the descendant of the bare state with a large** $n\bar{n}$ **component,** $f_0^{bare}(1230\pm 30)$**: nearly flavour singlet,** $\varphi_{f_0^{bare}(1230\pm 30)}=43^\circ\pm 10^\circ$**.**
**Being similar to** $f_0(1300)$**, the resonance** $f_0(1500)$ **is formed by mixing with the gluonium as well as with neighbouring** $q\bar{q}$ **states.**
**Quark component:** $q\bar{q}=n\bar{n}\cos\varphi+s\bar{s}\sin\varphi$ **with** $-18^\circ\le\varphi\le 23^\circ$ **at** $W_{gg}\le 0.3$**.**

**4.** $f_0(1750)$**: Radial excitation** $n=2$**,** $2^3P_0$**. Flavour wave function has noticeable** $s\bar{s}$ **component,** $-46^\circ\le\varphi\le 7^\circ$ **at** $W_{gg}\le 0.3$**;**
**The bare state** $f_0^{bare}(1830\pm 30)$ **is flavour octet,** $\varphi_{f_0^{bare}(1830\pm 30)}=-55^\circ\pm 10^\circ$**.**

**For bare states,** $f_0^{bare}$**, of the same nonet (**$2^3P_0$**):**

$$\varphi_{f_0^{bare}(1230\pm 30)}-\varphi_{f_0^{bare}(1830\pm 30)}\simeq 90^\circ.$$

**5.** $f_0(1200-1600)$**: The broad state is the descendant of a primary glueball,** $f_0^{bare}(1580\pm 40)$**.**

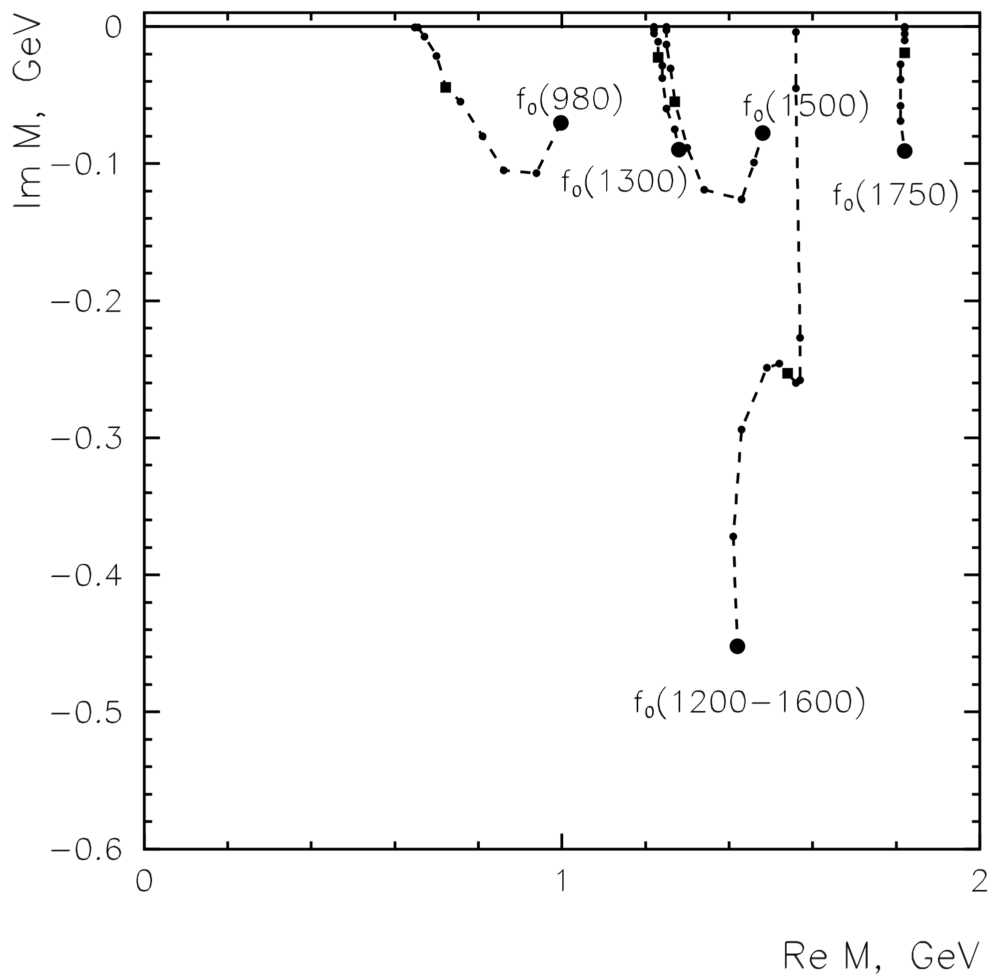



**Figure 1: Complex-$M$ plane: trajectories of poles corresponding to the states $f_0(980)$, $f_0(1300)$, $f_0(1500)$, $f_0(1750)$, $f_0(1200-1600)$ within a uniform onset of the decay channels.**

## 2. The systematics of meson quark–antiquark states on the $(n, M^2)$- and $(J, M^2)$-planes

V.V. Anisovich, V.V. Anisovich, A.V. Sarantsev, Phys. Rev. D 62, 051502, 2000.

**The fact is that $q\bar{q}$ states can be put, with rather good accuracy, on linear trajectories on the $(n, M^2)$- and $(J, M^2)$-planes.** *Vice versa*: **the states which do not belong to linear trajectories should be considered as exotics. In terms of $q\bar{q}$-states, the mesons form the nonets $n^{2S+1}L_J$:**

$$
\begin{aligned}
{}^1S_0 &\rightarrow \pi(10^{-+}),\ \eta(00^{-+})\,;\\
{}^3S_1 &\rightarrow \rho(11^{--}),\ \omega(01^{--})/\phi(01^{--})\,;\\
{}^1P_1 &\rightarrow b_1(11^{+-}),\ h_1(01^{+-})\,;\\
{}^3P_J &\rightarrow a_J(1J^{++}),\ f_J(0J^{++}),\ J=0,1,2\,;\\
{}^1D_2 &\rightarrow \pi_2(12^{-+}),\ \eta_2(02^{-+})\,;\\
{}^3D_J &\rightarrow \rho_J(1J^{--}),\ \omega_J(0J^{--})/\phi_J(0J^{--}),\ J=1,2,3\,;\\
{}^1F_3 &\rightarrow b_3(13^{+-}),\ h_3(03^{+-})\,;\\
{}^3F_J &\rightarrow a_J(1J^{++}),\ f_J(0J^{++}),\ J=2,3,4\,.
\end{aligned}
$$

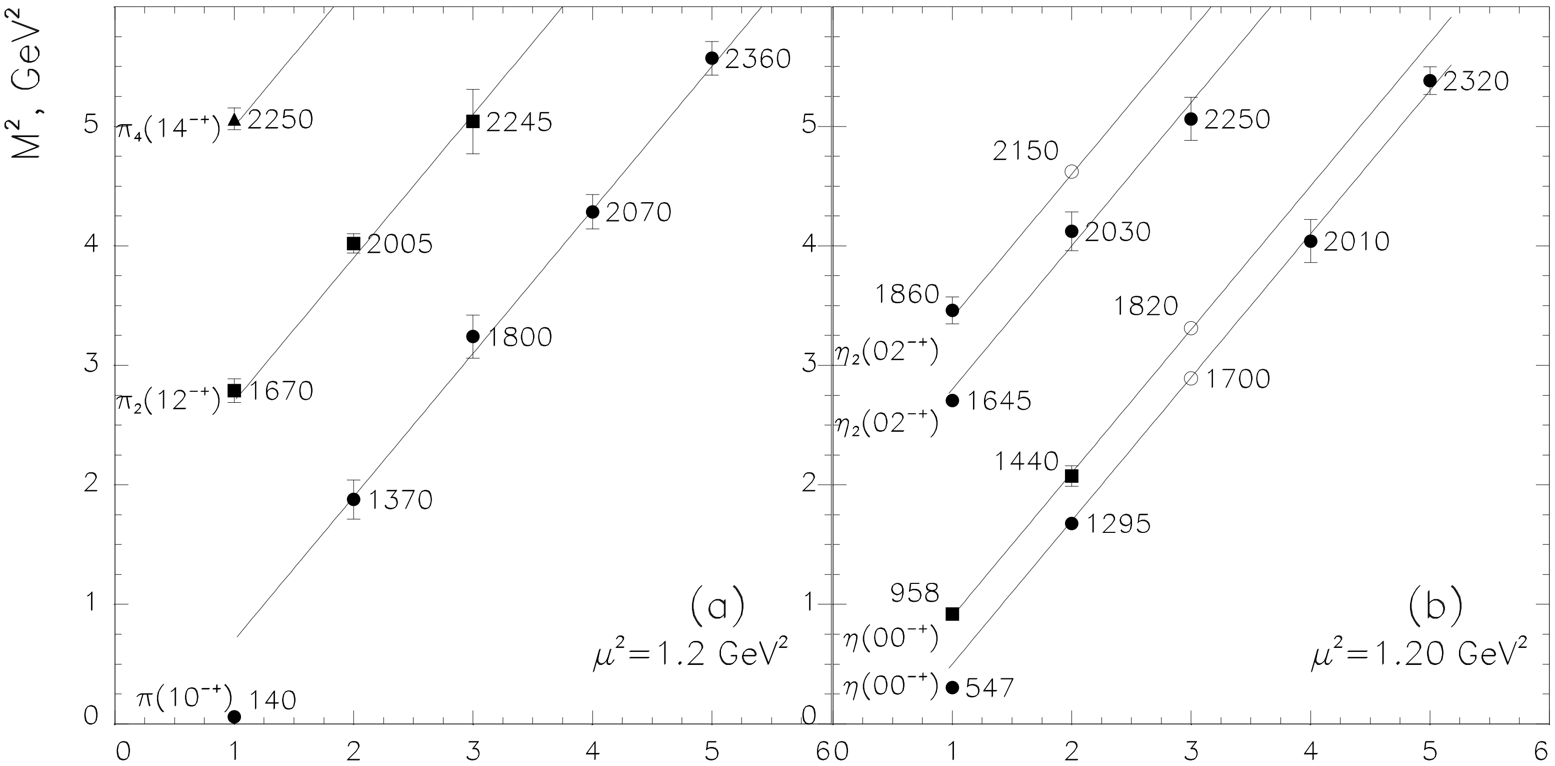



**Figure 2: Trajectories on the $(n, M^2)$ planes for the states with $(C = +)$: $\pi, \eta$-trajectories. Open circles stand for the predicted states.**

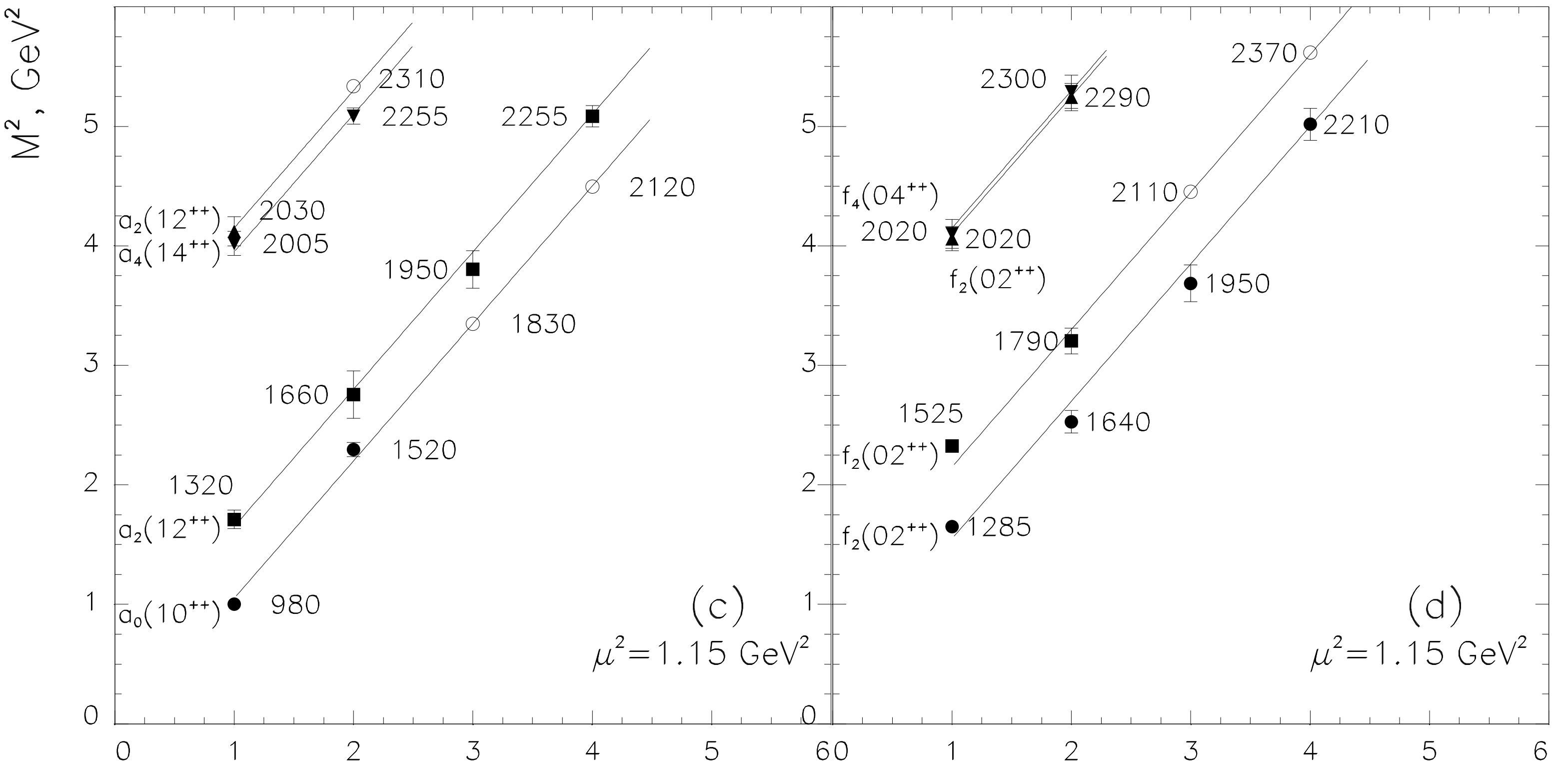



**Figure 3: Trajectories on the $(n, M^2)$ planes for the states with $(C = +)$: $a_0, a_2, f_2$-trajectories. Open circles stand for the predicted states.**

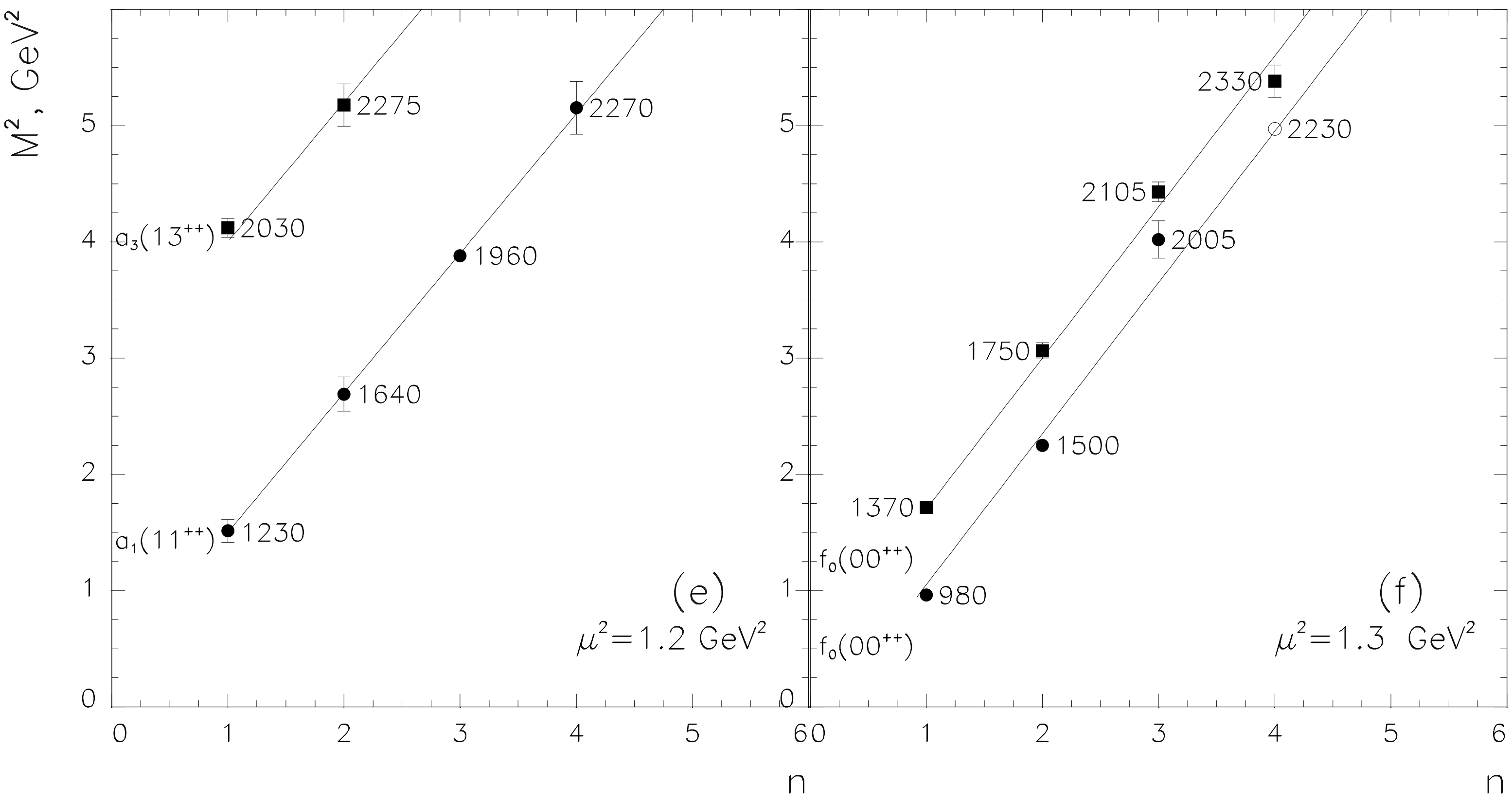



**Figure 4:** **Trajectories on the $(n, M^2)$ planes for the states with $(C = +)$. Open circles stand for the predicted states.**

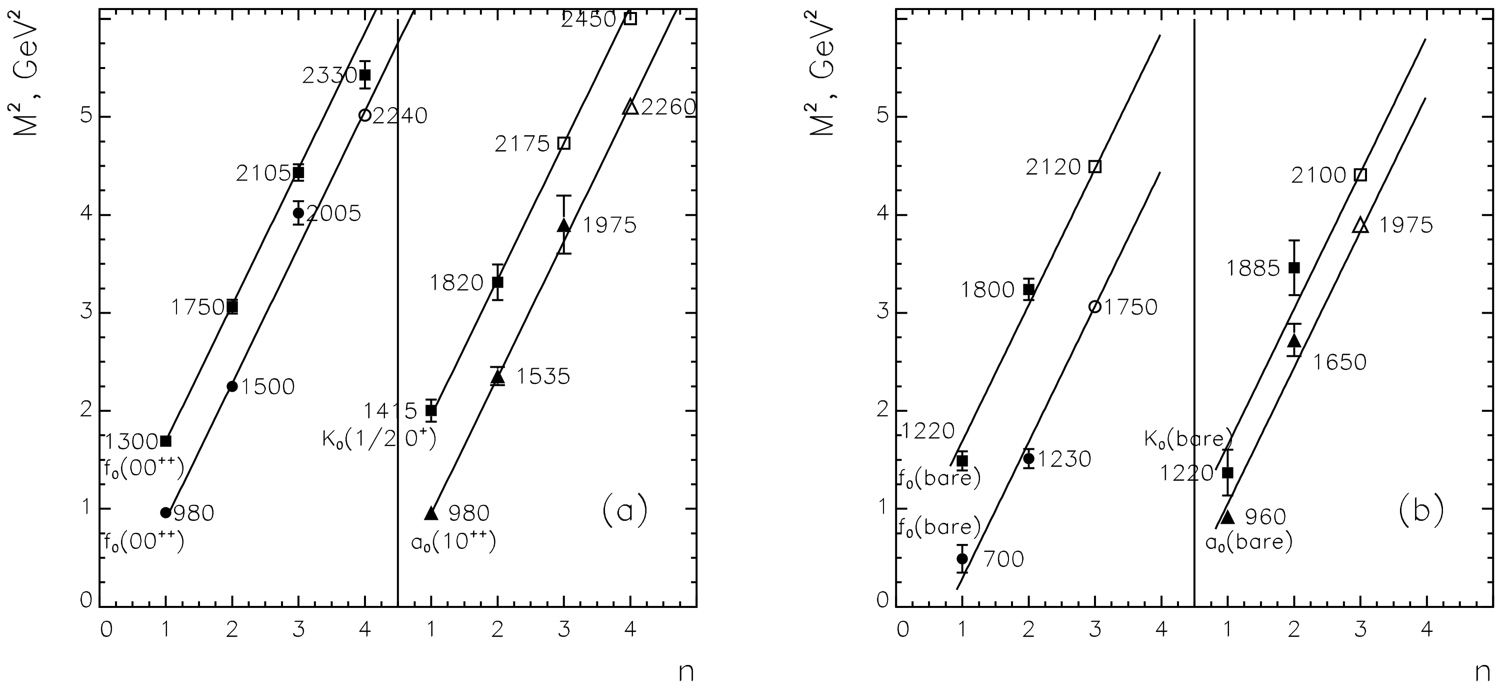


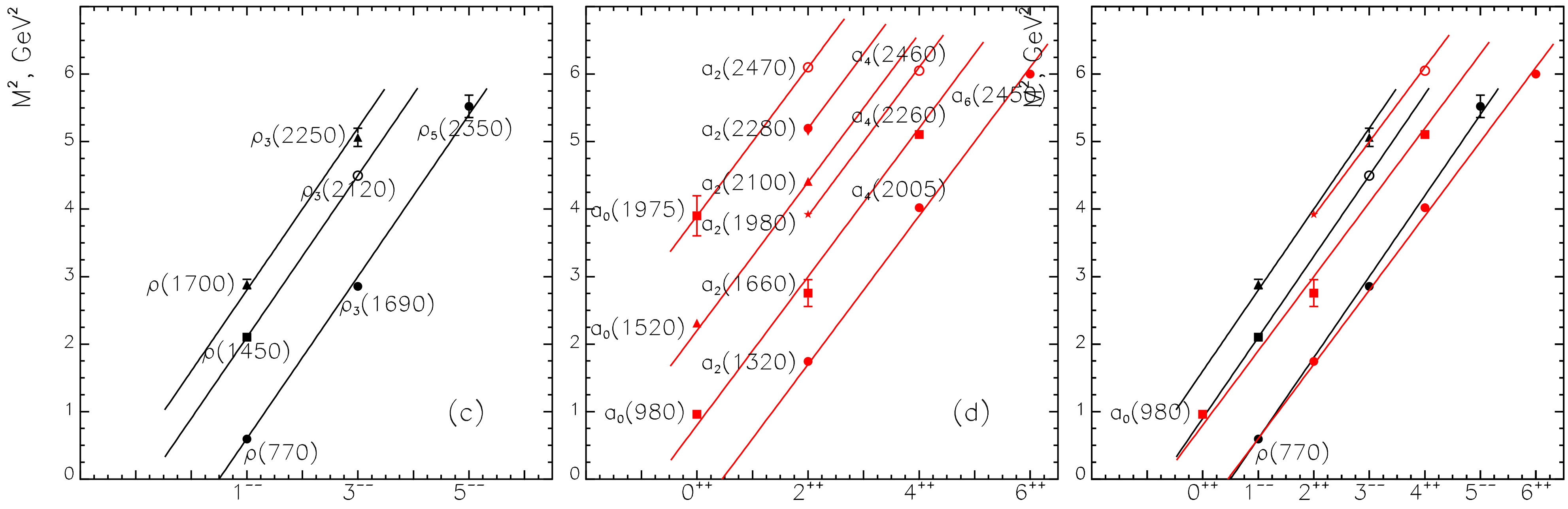

**Figure 5:** **Linear trajectorieson the $(n, M^2)$ plane for scalar resonances (a) and bare scalar states (b). Open labels correspond to the predicted states. The broad state $f_0(1200 - 1600)$ together with its bare-state counterpart, $f_0^{bare}(1580 \pm 40)$, cannot be put on linear $(n, M^2)$ trajectory, so they are not shown in Fig. 5. These states are superfluous for linear trajectories, and the $K$-matrix analysis revealed the gluonium nature of the bare state $f_0^{bare}(1580 \pm 40)$.**

**$f_0(980)$ and its bare-state counterpart $f_0(700 \pm 100)$ lay quite comfortably on the linear $(n, M^2)$ trajectory.**

# Trajectories on the $(J, M^2)$ plane:

**$\rho$-meson and daugther trajectories,**

**$a_2$-meson and daugther trajectories.**




**Combined presentation:**

**$\rho$ and $a_2$ trajectories are degenerate. $a_0(980)$ is on the daughter trajectory.**

**If $a_0(980)$ is not a $q\bar{q}$ state, there should be another $a_0$ in the mass region $\sim$ 1000 MeV.**

## 3. The $D_s^+ \to \pi^+\pi^+\pi^-$ decay and formation of $f_0(980)$

V.V. Anisovich, L.G. Dakhno, V.A. Nikonov, hep-ph/0302137, to be published in Phys. Atom. Nucl. (Yad. Fiz.)

**Now let us discuss the decay $D_s^+ \to \pi^+\pi^+\pi^-$. Main process:**

$$D_s^+ \to \pi^+ s\bar{s} \to \pi^+(f_0(980) \to \pi^+\pi^-) \to \pi^+\pi^+\pi^- \ .$$

**This reaction may be a tool for the estimate of $s\bar{s}$ components in the $f_J$ mesons.**

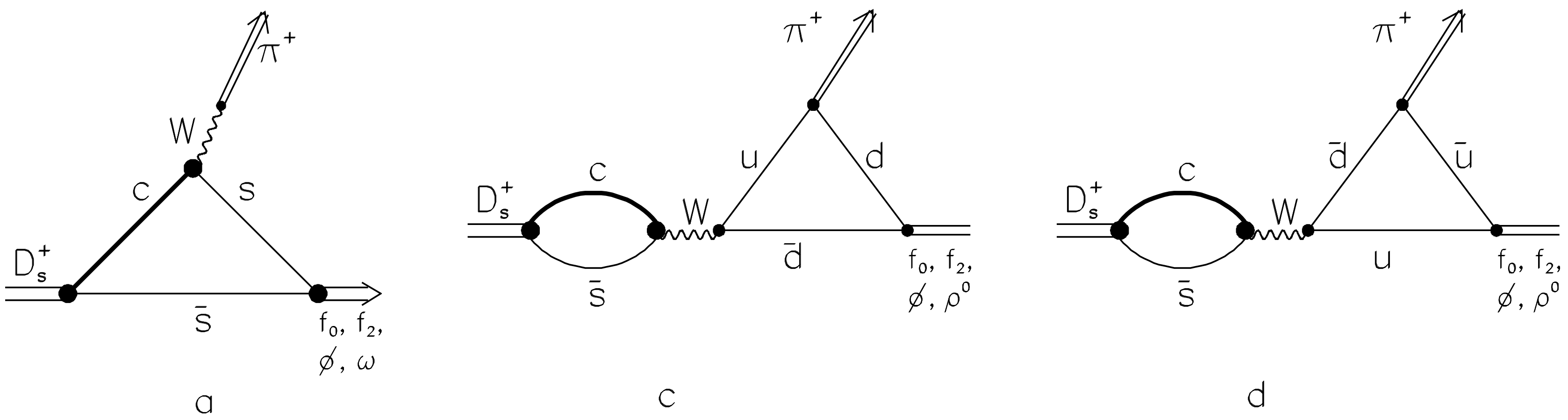



**Figure 6:** **Diagrams determining the decay $D_s^+ \to \pi^+\pi^+\pi^-$: a) diagram for the spectator mechanism; c,d) diagrams for the $W$-annihilation mechanism $D_s^+ \to u\bar{d}$, with subsequent production of $u\bar{u}$ and $d\bar{d}$ pairs.**

**Experimental data:** **E791 Collaboration, E.M. Aitala, et al. Phys. Rev. Lett. 86, 765 (2001).**

**In the spectra measured from the reaction $D_s^+ \to \pi^+\pi^+\pi^-$ the relative weight of channels $\pi^+ f_0(980)$ and $\pi^+\rho^0(770)$ is evaluated,**

$$BR\left(\pi^+ f_0(980)\right) = 57\% \pm 9\%, \qquad BR\left(\pi^+\rho^0(770)\right) = 6\% \pm 6\%,$$

$$BR\left(\pi^+(f_0(1430)\right) = 26\%{\pm}11\% \to BR\left(\pi^+(f_0(1300) + f_0(1500) + f_0(1200-1600)\right).$$

**and the ratio of yields is measured:**

$$\Gamma(D_s^+ \to \pi^+\pi^+\pi^-)/\Gamma(D_s^+ \to \pi^+\phi(1020)) = 0.245 \pm 0.028^{+0.019}_{-0.012}.$$

**Taking into account the branching ratio $BR\left(f_0(980) \to \pi^+\pi^-\right) \simeq 53\%$ and the ratio of yields, we have:**

$$\frac{\Gamma\left(D_s^+ \to \pi^+ f_0(980)\right)}{\Gamma\left(D_s^+ \to \pi^+\phi(1020)\right)} = 0.275(1 \pm 0.25).$$

**These data are basic for the determination of relative weight of the $s\bar{s}$ component in the $f_0(980)$. The production of $\rho$ is a measure of the $W$-annihilation - it is small.**

**For the spectator and $W$-annihilation processes the transition amplitude is:**

$$A(D_s^+ \to \pi^+ M) = \xi_M^{(spectator)} F_M^{(spectator)}(0) + \xi_M^{(W)} F_M^{(W)}(0),$$

**where the factors $\xi^{(spectator)}$ and $\xi^{(W)}$ are determined by flavour content of isoscalar mesons. In terms of the quarkonium states $s\bar{s}$ and $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$, we define flavour wave functions of isoscalar mesons as**

$$\phi(1020): \quad \sim s\bar{s}, \qquad f_0(980): \quad n\bar{n}\cos\varphi + s\bar{s}\sin\varphi,$$

**$D_s^+$-meson wave functions and $s\bar{s}$ component in the meson $M$ are parametrized as follows:**

$$\psi_{D_s}(s) = C_{D_s}\exp(-b_{D_s}s), \qquad s = \frac{m_c^2 + k_\perp^2}{1-x} + \frac{m_s^2 + k_\perp^2}{x}$$

$$\psi_M(s') = C_M\exp(-b_M s'), \qquad s' = \frac{m_s^2 + k_\perp^2}{x(1-x)}$$

**where $C_{D_s}$ and $C_M$ are the normalization constants for the wave functions and $b_{D_s}$ and $b_M$ characterize the mean radii squared of the $c\bar{s}$ and $s\bar{s}$ systems, $R^2_{D_s}$ and $R^2_M$.**

**In the used approximation, the mean radii squared are the only parameters for meson wave functions: $R^2_M$ for $f_0(980)$ and $\phi(1020)$ to be of the order of the pion radius squared, $R^2_M \sim R^2_\pi = 10$ GeV$^{-2}$,**

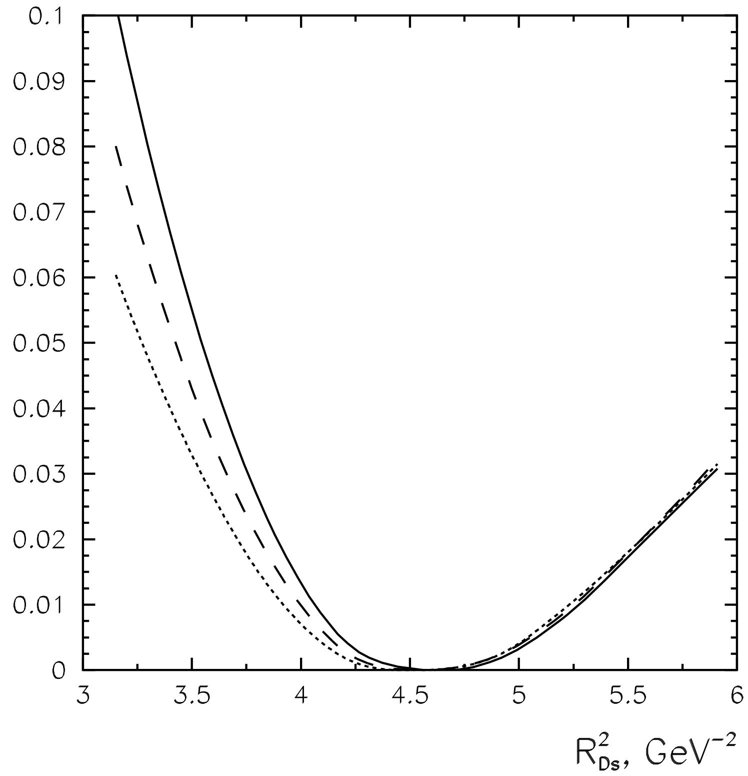
**For the $D_s^+$-meson, the charge radius squared $R^2_{D_s}$ is of the order of $3.5 \div 5.5$ GeV$^{-2}$.**

**In calculations for $f_0(980)$: $-90^\circ \leq \varphi \leq -43^\circ$**

**1.The $W$-annihilation is comparatively small. For $D_s^+ \to \pi^+ f_0$**

$$\frac{\sqrt{2}F^{(W)}_{f_0}(0)}{F^{(spectator)}_{f_0}(0)} = (0.28 + i0.75) \cdot 10^{-3}.$$

**2. In the decay $D_s^+ \to \pi^+\pi^+\pi^-$ the radial excitation states $2^3P_0$ are produced with a small probability as compared to the production of basic $f_0$ mesons.**

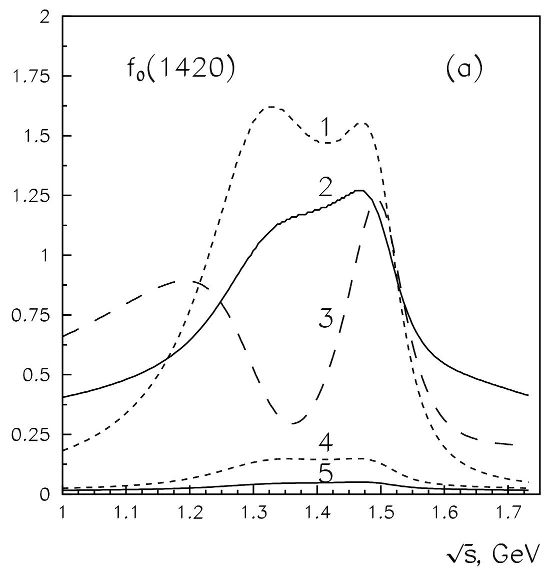



**The ratio $\Gamma\left(D_s^+ \to \pi^+ f_0^{rad.\,excit.}\right)/\Gamma\left(D_s^+ \to \pi^+ f_0^{basic}\right)$ as a function of radius square of the $D_s^+$ meson. Calculations have been carried out at fixed $R^2[f_0^{basic}]$=10 GeV$^{-2}$ for several values of $R^2(f_0^{rad.\,excit.})$: 10 GeV$^{-2}$ (solid line), 13 GeV$^{-2}$ (dashed line) and 16 GeV$^{-2}$ (dotted line).**

$$\psi_{M(rad.\,excit.)}(s) = C_{rad.\,excit.}(d_{rad.\,excit.}s - 1)\exp(-b_{rad.\,excit.}s)$$

**Bump near $1430$ MeV: $D_s^+ \to \pi^+ \left(f_0(1300) + f_0(1500) + f_0(1200-1600)\right)$ The bump was observed in the wave $(IJ^{PC} = 00^{++})$ at $1434 \pm 18$ MeV, with the width $173 \pm 32$ MeV and claimed to be a new meson. Relative weight of this bump is:**

$$BR\left(\pi^+(f_0(1300) + f_0(1500) + f_0(1200-1600))\right) = 26\% \pm 11\% \ .$$




**The $\pi^+\pi^-$ mass spectrum in the vicinity of 1420 MeV: calculated curves respond to the production of $f_0(1300) + f_0(1500)$ and the broad state $f_0(1200-1600)$, with $R^2_{D_s} = 4.15$ GeV$^{-2}$.**

$(1)\ \varphi[f_0(1300)] = -7^\circ,\ \varphi[f_0(1500)] = 7^\circ,\ \varphi[f_0(1200-1600)] = 37^\circ,$

$(2)\ \varphi[f_0(1300)] = -25^\circ,\ \varphi[f_0(1500)] = 17^\circ,\ \varphi[f_0(1200-1600)] = 37^\circ.$

**The variants (1) and (2) reproduce well the bump observed in the $D_s^+ \to \pi^+\pi^+\pi^-$ decay: the relative weight of the bump for variants (1), (2) is of the order of $\sim 15\% \div 20\%$ that agrees with measured peak.**

$$BR\left(\pi^+ f_0(980)\right) = 57\% \pm 9\%,$$

**2/3 $s\bar{s}$ in $f_0(980)$**

$$BR\left(\pi^+ (f_0(1300) + f_0(1500) + f_0(1200-1600))\right) = 26\% \pm 11\% \ .$$

**1/3 $s\bar{s}$ is dispersed over the resonances $f_0(1300)$, $f_0(1500)$, $f_0(1200-1600)$.**

**So the reaction $D_s^+ \to \pi^+ + f_0$ is a measure of the $1^3P_0 s\bar{s}$ component in the $f_0$ mesons.**

**The transform of bare states into real resonances can be illustrated by Fig. 7 for the levels in the potential well: bare states are the levels in a well with impenetrable wall (Fig. 7a); at the onset of the decay channels (under-barrier transitions, Fig. 7b) the stable levels transform into real resonance.**

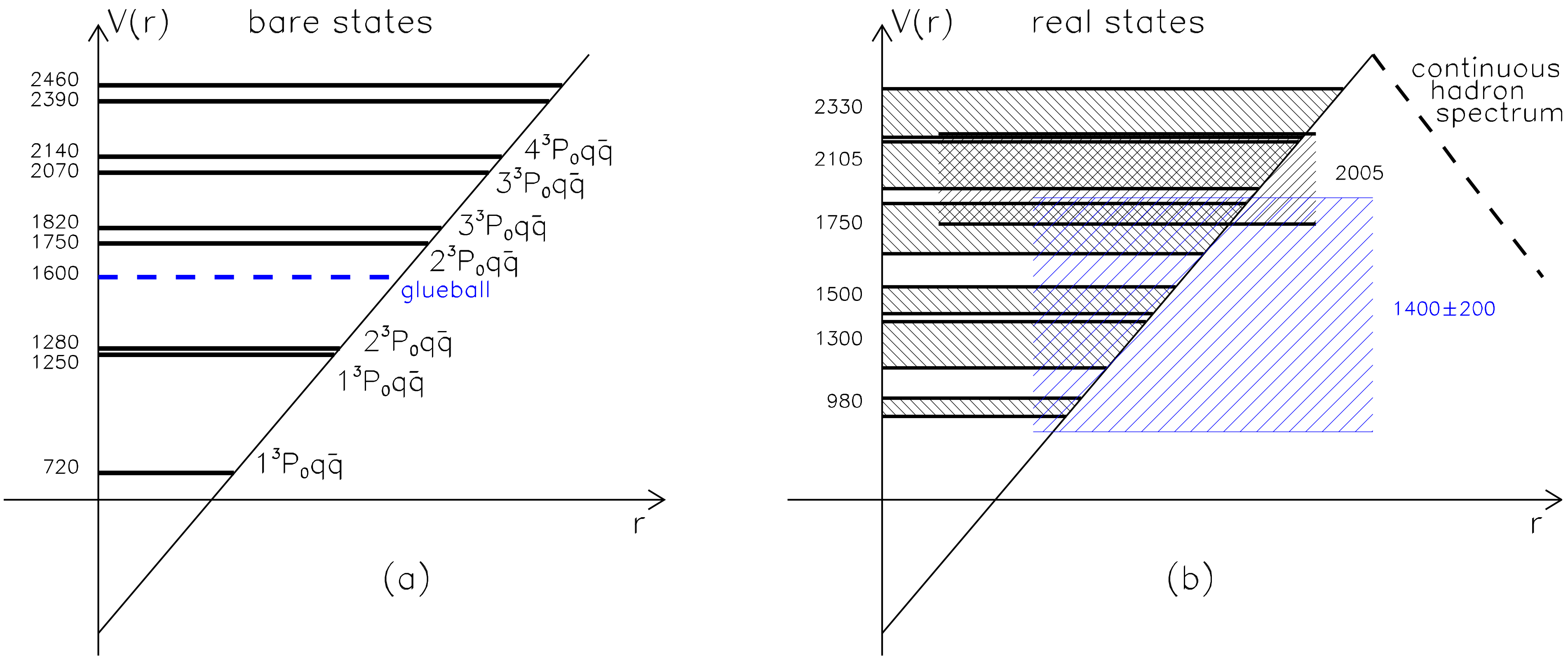



**Figure 7: The $f_0$-levels in the potential well depending on the onset of the decay channels: bare states (a) and real resonances (b).**

## 4. Radiative decays $f_0(980) \to \gamma\gamma$, $\phi \to \gamma f_0(980)$

**$D_s^+ \to \pi^+ f_0(980)$, $f_0(980) \to \gamma\gamma$, $\phi \to \gamma f_0(980)$ were calculated within the same technique: double spectral-integral representation developed for the quark triangle diagram; calculation of double discontinuity of the triangle diagram (cuttings I and II in figure) which correspond to real processes, and the double discontinuity is the integrand of double dispersion representation. The double dispersion integrals may be rewritten in terms of the light-cone variables, by introducing the light-cone wave functions of mesons.**

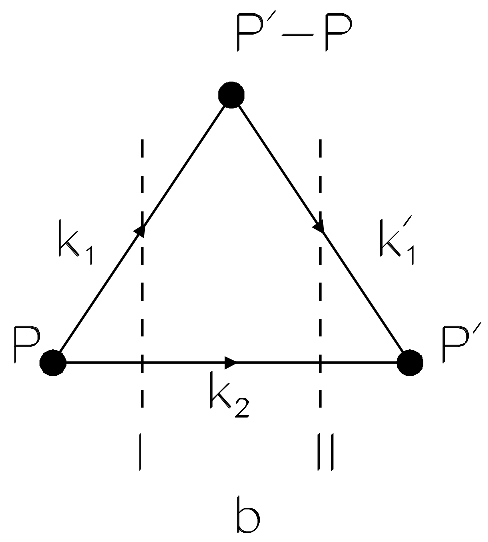
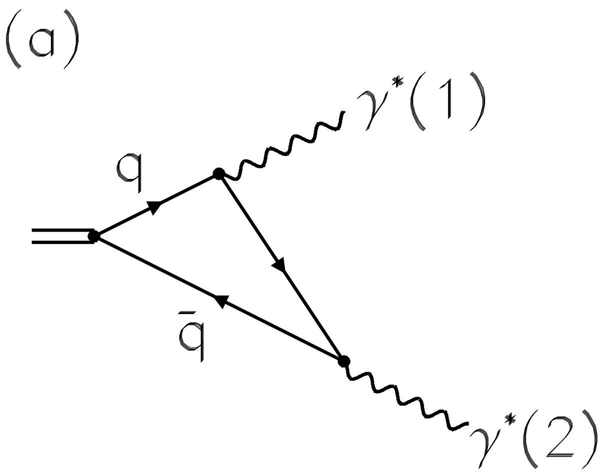



**Figure 8:** **Cuts of triangle diagram in the spectral integral representation.**

**Experimental data:** $\Gamma_{f_0(980)\to\gamma\gamma} = 0.28^{+0.09}_{-0.13}$ **keV**

**M. Boglione, M.R. Pennington, Eur. Phys. Journal C 9, 11 (1999).**



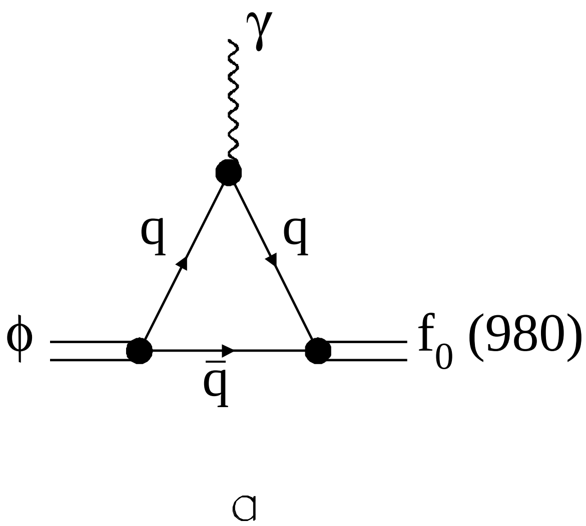




**Figure 9: Diagrammatic representation of the processes $f_0(980) \to \gamma\gamma$ and $\phi(1020) \to f_0(980)\gamma$**

**A.V. Anisovich, et al. EPJA 12, 103 (2001); Phys. Atom. Nucl. 65,497 (2002); hep-ph/0403123.**

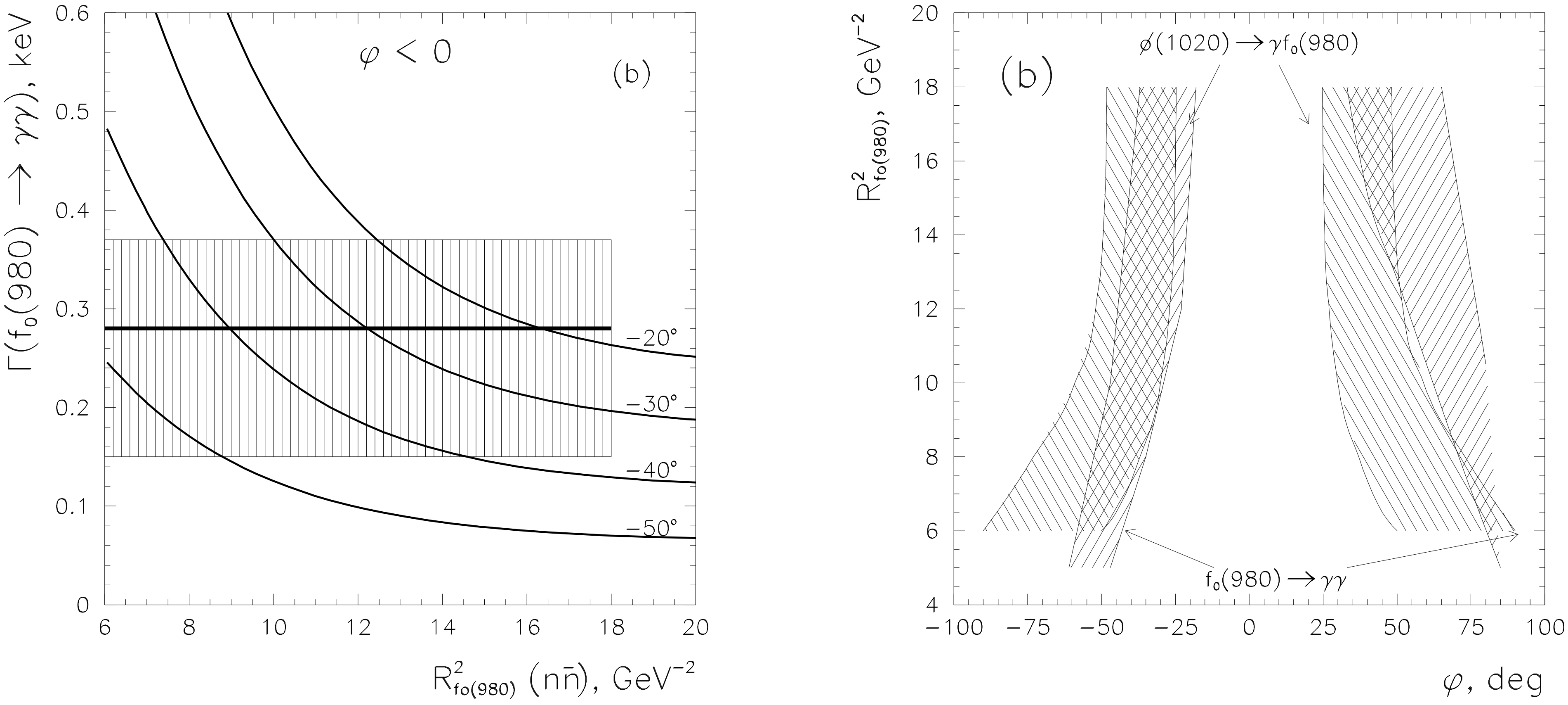



**Figure 10:** **Partial width $\Gamma(f_0(980) \to \gamma\gamma$ calculated under the assumption that $f_0(980)$ is quark–antiquark system, $q\bar{q} = n\bar{n}\cos\varphi + s\bar{s}\sin\varphi$. Shaded area corresponds to values allowed by the experiment. Combined representation of areas of mixing angle $\varphi$ allowed by the data on $f_0(980) \to \gamma\gamma$ and $\phi \to \gamma f_0(980)$**

## Additive quark model approach *vs* dipole emission formula

**Two alternative representations for the $V \to \gamma S$ decay amplitude:**

**(i) the standard additive quark model formula and**

**formula of a photon dipole emission, in the latter the factor $\omega = (m_V - m_S)$ is written in the explicit form.**

**The comparison of these two representations helps us to formulate the problem of application of the threshold theorem (Siegert theorem) to the reaction $V \to \gamma S$.**

$$F^{V\to\gamma S}_{\mu\alpha} = \int d^3r\, \mathbf{Sp}_2 \left[\Psi^+_S(r) r_\alpha \Psi_{V\mu}(r)\right] 2im(\varepsilon_V - \varepsilon_S).$$

**Quantum mechanics consideration of the reaction $\phi \to \gamma f_0$ (dipole emission formula) should be done for $\phi$ and $f_0$ being stable particles:**

$f_0(980) \to f_0^{bare} = 700 \pm 100$ **MeV.**

$$A^{(calc)}_{\phi(1020)\to\gamma f_0(980)}(\text{dipole}) \simeq (0.58 \pm 0.04) \cdot \sqrt{W_{q\bar q}[f_0^{bare}(700)]}\, Z^{(s\bar s)}_{\phi\to\gamma f_0} \times$$

$$\times \frac{2^{7/2}}{\sqrt{3}} \frac{b_\phi^{7/4} b_{f_0}^{5/4}}{(b_\phi + b_{f_0})^{5/2}} m_s \left[m_\phi - (0.7 \pm 0.1)\text{GeV}\right] .$$

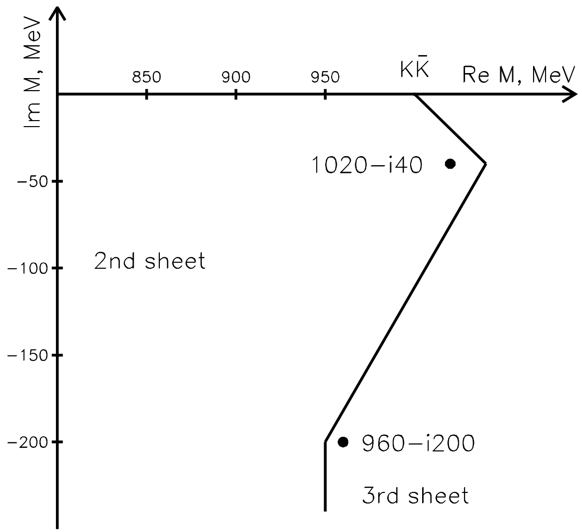



**Figure 11:** **Complex-$M$ plane and location of the poles corresponding to $f_0(980)$; the cut related to the $K\bar{K}$ threshold is shown as broken line.**

$$\frac{g_\pi + i\rho_{K\bar{K}} g_K f}{M_0^2 - M_{\pi\pi}^2 - i g_\pi^2 \rho_{\pi\pi} - i\rho_{K\bar{K}}[g_K^2 + i\rho_{\pi\pi}(2g_\pi g_K f + f^2(m_0^2 - s))]},$$

**with the parameters**

$$g_\pi = 0.386 \text{ GeV}, g_K = 0.447 \text{ GeV}, M_0 = 0.975 \text{ GeV}, f = 0.516 .$$

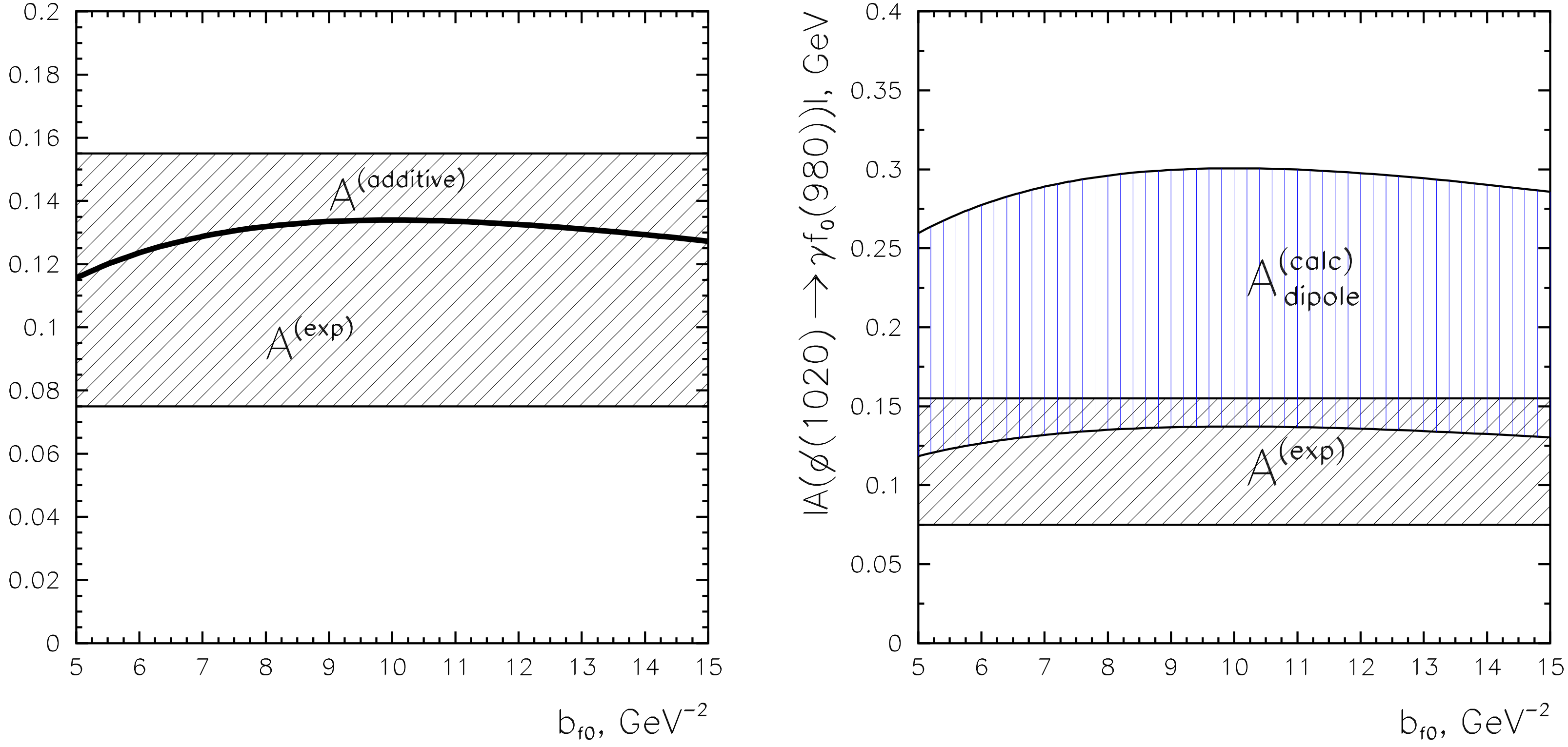



**Figure 12:** **Amplitudes $\phi(1020) \to \gamma f_0(980)$ calculated in the framework of the additive quark model (a) and dipole formula (b) versus $A^{(exp)}$ determined by data for $\Gamma_{\phi(1020)\to\gamma f_0(980)}$**

$$A^{(exp)}_{\phi(1020)\to\gamma f_0(980)} = 0.115 \pm 0.040\,\mathrm{GeV}\,.$$