# Proton Form Factors: Theoretical Aspects

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4. Light Cone Sum Rules for $F_1$ and $F_2$
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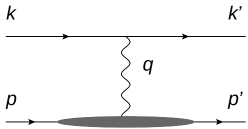
# Electromagnetic Form Factors

- one photon exchange




▷ two form factors

$$F_1(Q^2),\ F_2(Q^2) \quad \text{or} \quad G_E(Q^2),\ G_M(Q^2)$$

this talk

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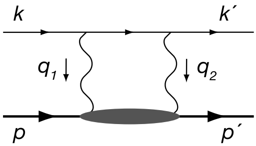
- two photon exchange




▷ three complex function

$$\tilde{G}_M(Q^2,t),\ \tilde{F}_2(Q^2,t),\ \tilde{F}_3(Q^2,t)$$

Kivel,

Kuraev,

Tomasi-Gustavson

# Some Definitions and Properties I

- Dirac- and Pauli-form factor

$$\langle N(P-q)| j_\mu^{em} |N(P)\rangle = \bar{N}(P-q)\left[\gamma_\mu F_1(Q^2) - i\frac{\sigma_{\mu\nu}q^\nu}{2\,m_N} F_2(Q^2)\right] N(P)$$

- Electric- and Magnetic form factor

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2), \qquad G_E(Q^2) = F_1(Q^2) + \frac{q^2}{4m_N^2} F_2(Q^2)$$

- for zero momentum transfer squared

$$\begin{array}{llll} G_M^p(0) = \mu_p, & G_E^p(0) = 1, & G_M^n(0) = \mu_n, & G_E^n(0) = 0 \\ F_1^p(0) = 1, & F_2^p(0) = \kappa_p = \mu_p - 1, & F_1^n(0) = 0, & F_2^n(0) = \mu_n \end{array}$$

# Some Definitions and Properties II

- cross section

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{E_e}{E_{beam}} \left\{ F_1^2(Q^2) + \tau \left[ F_2^2(Q^2) + 2\left(F_1(Q^2) + F_2(Q^2)\right)^2 \tan^2 \frac{\Theta_e}{2} \right] \right\}$$

$$= \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left( \frac{G_E^2(Q^2) + G_M^2(Q^2)}{1+\tau} + 2\tau\, G_M^2(Q^2) \tan^2 \frac{\Theta_e}{2} \right)$$

- experimentally $G_E$, $G_M$ preferred
  - separable in Rosenbluth cross section
  - approximated by dipole form factor

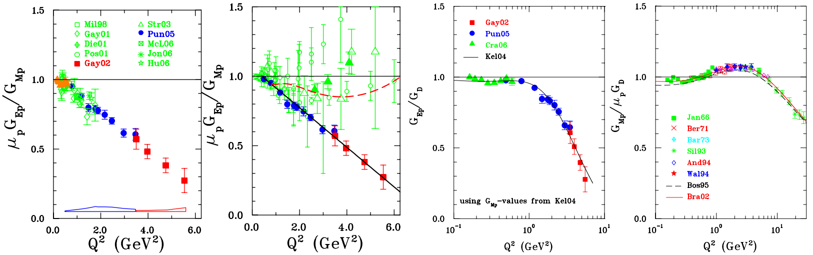
clear interpretation only in Breit-frame

$$G_D = \frac{1}{\left(1 + \frac{Q^2}{0.71\,\text{GeV}^2}\right)^2}, \quad \text{with} \quad G_E^P = G_D, \quad G_M^P = \mu_P\, G_D \quad \text{and} \quad G_M^n = \mu_n\, G_D$$

- theoretically $F_1$, $F_2$
  - helicity conserving and helicity flip form factor
- scaling for large $Q^2$

$$F_1(Q^2) \sim \frac{1}{Q^4}, \quad F_2(Q^2) \sim \frac{1}{Q^6}$$

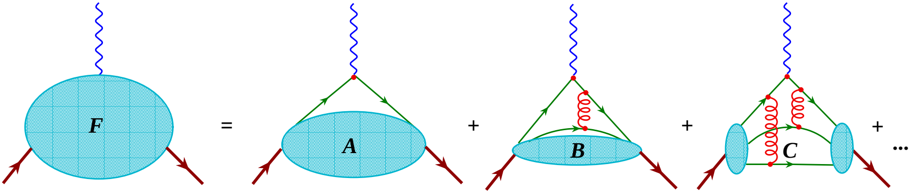
# Deviation from Expectation




Perdrisat, Punjabi, Vanderhaeghen

- scaling violation clearly visible in polarisation experiments
- deviation from dipole form factor for $Q^2 \geq 1\,\text{GeV}^2$

# The General Picture



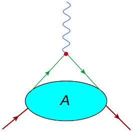

$$F_1(Q^2) \sim A(Q^2) + \frac{\alpha_s}{4\pi}\frac{B(Q^2)}{Q^2} + \left(\frac{\alpha_s}{4\pi}\right)^2 \frac{C}{Q^4}$$

- $A(Q^2)$, $B(Q^2)$ soft overlap functions $\leq \frac{1}{Q^6}$, $\leq \frac{1}{Q^4}$ ?
- $C$ hard (soft) scattering part determined by nucleon distribution amplitudes
- $F_2$ scales like $\sim \frac{1}{Q^6}$

higher twist effect

no factorisation for hard scattering part
overlap of B and C

# Soft Overlap Region




- form factors can be written as integrals over generalized parton distributions (GPDs)

$$F_1 \sim \int_0^1 dx (2H_v^u(x,t,\mu^2) - H_v^d(x,t,\mu^2))$$

- GPDs defined as matrix element of Fourier-transformed em-current

$$H_v^q \sim \int dx^- \, e^{ix^- px} \langle P' | \bar{q}(-x^-) \gamma_\mu q(x^-) | P \rangle$$

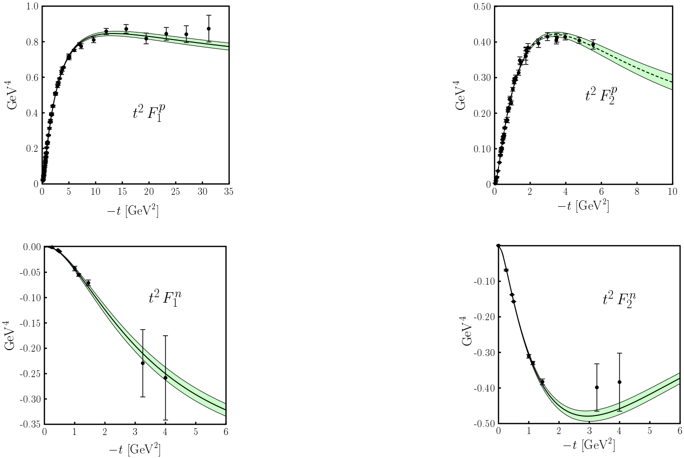
- can be interpreted as overlap of light cone wave functions (LCWFs)

$$H_v \sim \int d^2 k_\perp \Psi(x_i, k_\perp) \Psi^\dagger(x_f, k_\perp + \bar{x} r_\perp)$$

- LCWFs defined as probability to find nucleon in certain Fock-state
- Power counting and GPD models imply
  - scaling for ultrasoft region $\sim \frac{1}{Q^8}$
  - Feynman-mechanism scales like $\sim \frac{1}{Q^4}$

more later

# Some Fits

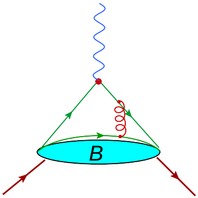



Diehl, Feldmann, Jakob, Kroll

- fits to data no prediction!

calculations depend strongly on model assumptions for GPDs

# $B$-function

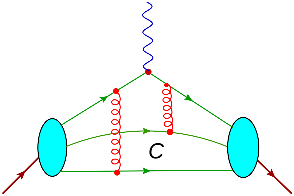



- not much to say...
- for $F_2$ overlap of $C$ and $B$ — both scale as $\frac{1}{Q^6}$
  - end-point divergence in convolution of hard scattering kernel and distribution amplitudes
- results from NLO light cone sum rules for $F_2$ give large $B \sim \frac{1}{Q^4}$ contribution

  Kumericki, Peters
- for $F_1$ still work in progress

  Anikin, Braun, Offen

# $C$-Region I




- for $F_1$ leading in $\frac{1}{Q^2}$
- factorisation for hard part

$$F_1^{(h)} = \psi \otimes H \otimes \psi$$

Brodsky, Lepage

$H$: hard scattering kernel, $\psi$ distribution amplitude, $\otimes$: convolution

- distribution amplitude ($x_1$, $x_2$, $x_3$ coordinates on the light cone)

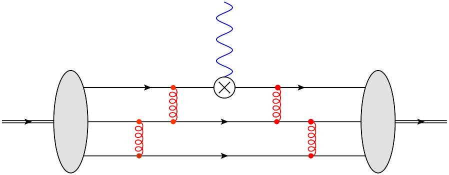
$$\langle 0 \,|\, q_\alpha(x_1)\, q_\beta(x_2)\, q_\gamma(x_3) \,|\, P \,\rangle \sim (\not{p}C)_{\alpha\beta}\gamma_5 N_\gamma^+ V_1 + \cdots$$

23 more structures up to twist 6

renormalisation: Efremov, Radyushkin; Brodsky, Lepage; Bukhvostov, Frolov, Lipatov, Kuraev; Braun, Manashov, Rohrwild

- but...

# $C$-Region II



- gives subleading large log $\frac{Q^2}{m^2}$ at leading order in $\frac{1}{Q^2}$ not reproduced by factorisation

non-renorm group log
no such contribution for meson form factors

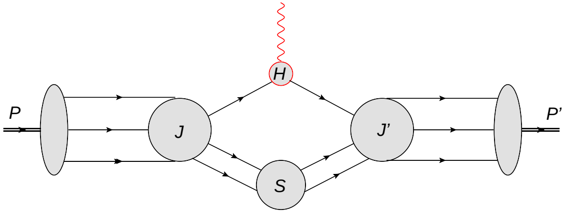
Duncan, Mueller; Fadin, Milshtein; Kivel

- at three loop order ($\alpha_s^5$) appears Sudakov like double log in front of this term

Fadin,Milshtein

- Sudakov suppression would imply leading hard term

# Factorisation of Hard and Soft Contributions I




Kivel, Vanderhaeghen

- tentative factorisation for hard and soft rescattering proposed using SCET-framework

$$
\begin{aligned}
F_1 &= F_1^{(h)} + F_1^{(s)} \\
F_1^{(h)} &= \psi \otimes H \otimes \psi \\
F_1^{(s)} &\sim \int Dy_i \psi(y_i) \int d\omega_1 d\omega_2 J'(y_i, \omega_i Q) \int Dx_i \psi(x_i) \int d\nu_1 d\nu_2 J(x_i, \nu_i Q) S(\omega_i, \nu_i)
\end{aligned}
$$

- but soft and hard contributions overlap... Kivel

# Factorisation of Hard and Soft Contributions II

- ... and soft convolution integrals over $S$ diverge
- giving rise to aforementioned large $\log \frac{Q^2}{m^2}$ shown for perturbative $S$, $J$ and $\psi$
- divergence in soft convolution integral implies end point divergence in hard part as well
- but convolutions with DA that vanishes at end point finite
- does DA not vanish at end points at low renormalisation scale?
- regularisation to separate hard- and soft part for non perturbative $S$, $J$ and $\psi$ not settled

# First Summary

- very contrived picture
- $A$- and $B$-function for $F_1$ not accessible via perturbative QCD
  - only model calculations available
  - dominant contribution at available experimental energies
- factorisation of formally leading $C$ not completely solved
  - separation of hard and soft contributions not settled
  - end-point behaviour of nucleon DA ?
  - Sudakov suppression of non-renormalisation group logarithm sufficient to justify leading hard part?
- $F_2$ even worse
  - end point divergence in leading hard part, overlap of $C$ and $B$
  - only model estimates available

except for lattice

Systematic calculation of $F_1$ and $F_2$ possible?

lattice calculations $\longrightarrow$ see next talk

# Light Cone Sum Rules (LCSRs)

- ▷ use analyticity, operator product expansion and quark-hadron duality
- ▷ calculate soft $A$-, $B$- and hard $C$ functions in terms of same nucleon DAs
- ▷ no double counting
- ▷ systematic improvement possible

though limited accuracy $\sim$ 10-20 %

correlation function

$$T_\mu(P,q) = i \int d^4x\, e^{iqx} \langle 0 | T\{\eta(0) j_\mu(x)\} | P \rangle$$

hadronic dispersion relation

$$n^\mu T_\mu^{(had)}(P^2,Q^2) = \frac{f_N}{m_N^2 - P'^2} P'\cdot n \left\{ \left[ 2F_1(Q^2) P'\cdot n - F_2(Q^2) q\cdot n \right] \not{n} \right\} + \cdots$$

operator-product-expansion

$$T^{(OPE)}(P^2,Q^2) = \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\, T_1^{(OPE)}(Q^2,s)}{s - P'^2}(P'\cdot n)^2 + \cdots$$

quark-hadron-duality

$$\int_{s_0}^\infty ds \frac{\rho_1(s)}{s - P'^2} \approx \frac{1}{\pi}\int_{s_0}^\infty ds \frac{\mathrm{Im}\, T_1^{(OPE)}(Q^2,s)}{s - P'^2}$$

$$\frac{2 f_N F_1(Q^2)}{(m_N^2 - P'^2)} = \frac{1}{\pi}\int_0^{s_0} ds \frac{\mathrm{Im}\, T_1^{(OPE)}(Q^2,s)}{s - P'^2}$$

Borel-transformation

$$F_1(Q^2) = \frac{1}{2\pi f_N}\int_0^{s_0} ds\, \mathrm{Im}\, T_1^{(OPE)}(Q^2,s)\, e^{(m_N^2 - s)/M^2}$$

**main uncertainties**

- ▷ universal input

correlation function

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operator-product-expansion

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**main uncertainties**

- ▷ universal input
- ▷ truncated OPE

correlation function

$$T_\mu(P,q) = i \int d^4x\, e^{iqx} \langle 0 | T\{\eta(0) j_\mu(x)\} | P \rangle$$

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operator-product-expansion

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quark-hadron-duality

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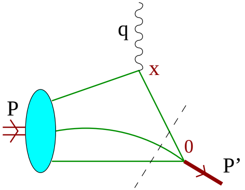
**main uncertainties**

- ▷ universal input
- ▷ truncated OPE
- ▷ quark-hadron duality

# Leading Order Example I




- starting point: correlation function

$$T_\mu(P,q) = i\int d^4x e^{iq\cdot x}\langle 0|\, T\{\eta(0) j_\mu(x)\}|P\rangle$$

$\eta$ nucleon interpolating current

- hadronic sum (only nucleon intermediate state)

$$n^\mu T_\mu = \frac{f_N}{m_N^2 - P'^2} P'\cdot n \left[F_1(Q^2)(P'\cdot n) - F_2(Q^2)(q\cdot n)\right] \not{n} N(P) + \cdots$$

- OPE at leading order and leading twist:

$$n^\mu T_\mu = -2\left[e_d \int Dx \frac{x_3 V_1(x_i)}{(q - x_3 P)^2} + 2e_u \int Dx \frac{x_2 V_1(x_i)}{(q - x_2 P)^2}\right] (P\cdot n)^2 \not{n} N(P)$$

$V_1(x_i)$: distribution amplitude

# Leading Order Example II

- written as dispersion integral

$$n^\mu T_\mu = \int_0^{s_0} ds \frac{\rho(s, Q^2)}{s - P'^2} 2(P \cdot n)^2 \not{n} N(P) + \cdots$$

$s = \frac{\bar{x}_i}{x_i} Q^2$, $Q^2 = -q^2$, $s_0$: duality threshold

- Borel-Transformation

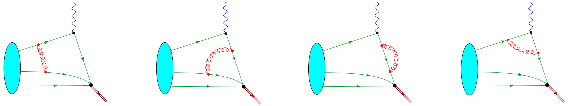
$$\frac{1}{s - P'^2} \longrightarrow \exp\left\{-\frac{s}{M_B^2}\right\}$$

$M_B^2$: Borel-Parameter

- sum rule at leading twist

$$\begin{aligned} F_1^{tw-3}(Q^2) &= \frac{1}{f_N}\left[e_d \int Dx\, V_1(x_i) \exp\left(-\frac{\bar{x}_3 Q^2 - x_3^2 m_N^2}{x_3 M_B^2}\right) \Theta\left(x_3 - \frac{Q^2}{Q^2 + s_0}\right)\right. \\ &+ \left. 2e_u \int Dx\, V_1(x_i) \exp\left(-\frac{\bar{x}_2 Q^2 - x_2^2 m_N^2}{x_2 M_B^2}\right) \Theta\left(x_2 - \frac{Q^2}{Q^2 + s_0}\right)\right] \end{aligned}$$

# NLO LCSRs

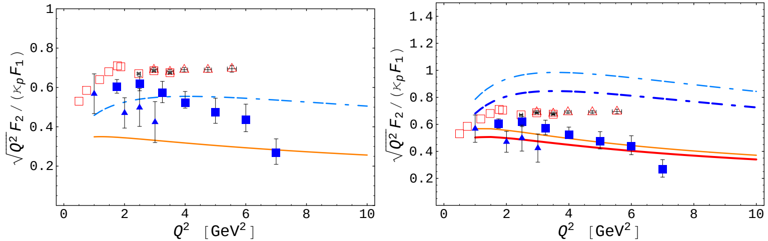


- First NLO calculation with $m_N = 0$ gives $\sim 60$ % of leading term corrections
- due to structure of correlation function only corrections to $F_2$ Kumericki, Peters
- result for $A$ scales as $\frac{1}{Q^6}$
- result for $B$ scales as $\frac{1}{Q^4}$
- going beyond $m_N = 0$:
  - matrix element with non light-like coordinates needed

$$\langle 0 |\, q_\alpha(x_1)\, q_\beta(x_2)\, q_\gamma(x_3) |\, P\,\rangle \longrightarrow (\not{P}C)_{\alpha\beta}\gamma_5 N_\gamma\, V_1 + m_N(\not{P}C)_{\alpha\beta} \sum_i \not{x}_i \gamma_5\, N_\gamma \mathbb{V}_2^{(i)}$$

  - required new relations for distribution amplitudes

# Some Results



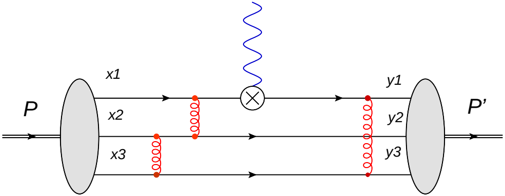

LO(left) and NLO(right) results for asymptotic (solid line) and Braun-Lenz-Wittmann (BLW, dashed line). NLO results for variation of twist 4 normalisation

$$\lambda_1 = ((-3.4) - (-2.7)) \times 10^{-2}\,\mathrm{GeV}^2$$

Kumericki, Peters

- NLO corrections to $F_1$ will lower results again
- moments of DAs from lattice QCD needed to improve accuracy

# Hard Scattering at $\alpha_s^3$




$$\int \frac{d^D k}{(2\pi)^D} \frac{\mathrm{Tr}\,\{\gamma_\alpha \not{k} \gamma_\mu (y_1 \not{P}' + \not{k} - \not{q}) \gamma_\beta \not{P} \gamma_\delta (\not{k} - y_3 \not{P}') \gamma_\beta \not{P}'\}}{N_1\, N_2\, N_3\, N_4\, N_5\, N_6\, N_7} \bar{N}(P)\, \gamma^\alpha \not{k} \gamma^\delta\, N(P')$$

- non-renorm group logs not seen at this level
- challenging calculation
  - large number of diagrams $\sim 4000$ — though most of them redundant...
  - up to 7-point integrals
  - up to tensor rank 4
- logarithmic corrections to $C \sim \frac{1}{Q^4}$

Knoedlseder, Offen, work in progress

- not much changed since summary one
- despite recent advances in understanding factorisation of $F_1$ using SCET, problem not completely solved
  - numerical importance of soft rescattering part? $\mathcal{O}(\alpha_s^4)$
- most GPD calculations of $F_1$ and $F_2$ depend strongly on model assumptions
- light cone sum rules give possibility to approach both $F_1$ and $F_2$ but input from lattice QCD on moments of distribution amplitudes is dearly needed
- first NLO-calculations in LCSRs with $m_N \neq 0$ on the way
- calculation of perturbative part of $F_1$ at $\mathcal{O}(\alpha_s^3)$ in progress
- new experiments will surely give us an interesting time...