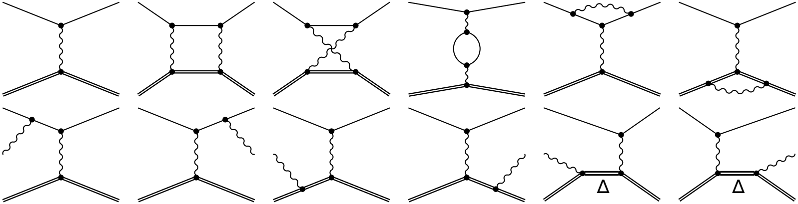
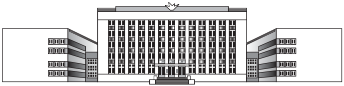
Radiative corrections in the Novosibirsk two-photon exchange experiment

Alexander Gramolin
Budker Institute of Nuclear Physics, Novosibirsk, Russia

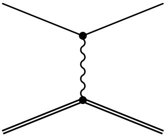


Symposium "Experimental and theoretical aspects of the proton form factors"

OLYMPUS



Differential cross section for elastic ep-scattering is given by the Rosenbluth formula:



$$\frac{d\sigma_{\text{Ros}}}{d\Omega_\ell} = \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\,\mathrm{tg}^2\frac{\theta_\ell}{2}\right]\frac{d\sigma_{\text{Mott}}}{d\Omega_\ell},$$

$$\frac{d\sigma_{\text{Mott}}}{d\Omega_\ell} = \frac{\alpha^2}{4E_\ell^2}\,\frac{\cos^2(\theta_\ell/2)}{\sin^4\theta_\ell/2}\,\frac{E'_\ell}{E_\ell},$$

where $\tau = Q^2/(4M^2)$, $Q^2 = 2M(E_\ell - E'_\ell)$, $d\sigma_{\text{Mott}}/d\Omega_\ell$ — Mott cross section, $G_E(Q^2)$ and $G_M(Q^2)$ — electric and magnetic form factors of the proton.

G_E and G_M are functions of the 4-momentum transfer squared (Q^2) only and describe the distributions of charge and magnetic moment inside the proton.

Introducing the variable ε (virtual photon polarization)

$$\varepsilon = \left[1 + 2(1+\tau)\,\mathrm{tg}^2\frac{\theta_\ell}{2}\right]^{-1},$$

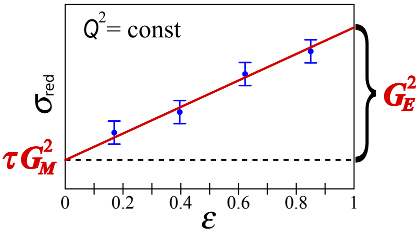
the Rosenbluth formula can be written as follows:

$$\frac{d\sigma_{\text{Ros}}}{d\Omega_\ell} = \frac{1}{\varepsilon(1+\tau)}\left[\varepsilon G_E^2(Q^2) + \tau G_M^2(Q^2)\right] = \frac{\sigma_{\text{red}}}{\varepsilon(1+\tau)},$$

where σ_{red} (reduced cross section) is a linear function of ε if $Q^2 = \text{const}$.

The proton's form factors, two methods of measuring

$$\sigma_{\text{red}} = \varepsilon(1+\tau)\frac{d\sigma}{d\Omega_\ell} = \varepsilon G_E^2 + \tau G_M^2$$



Rosenbluth method

It consists in measuring of $d\sigma/d\Omega_\ell$ for fixed Q^2, but with different E_ℓ, θ_ℓ.
$\Rightarrow$ Dipole formula for G_E and G_M:

$$G_E(Q^2) \approx \left(1 + \frac{Q^2}{0.71\ \text{GeV}^2}\right)^{-2},$$

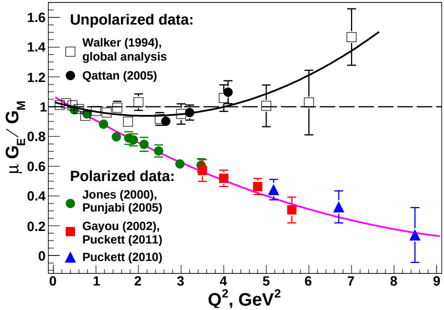
$$G_M(Q^2) \approx \mu G_E(Q^2).$$

Polarization transfer method

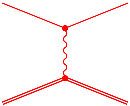
(Akhiezer and Rekalo, 1968)

The ratio G_E/G_M is proportional to the ratio of transverse P_T and longitudinal P_L polarization components of the recoil proton in reaction $\vec{e}p \to e'\vec{p}'$:

$$\frac{G_E}{G_M} = -\frac{P_T}{P_L}\frac{E_\ell + E'_\ell}{2M}\,\text{tg}\,\frac{\theta}{2}.$$



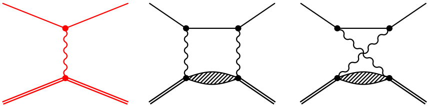
Born cross section and radiative corrections of order α^3



$\mathcal{M}_{\text{Born}}$

$$\sigma(e^{\pm}p) = |\mathcal{M}_{\text{Born}}|^2$$

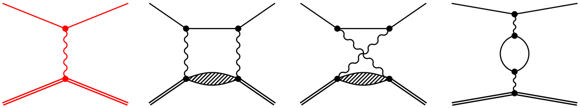
Born cross section and radiative corrections of order α^3



$\mathcal{M}_{\text{Born}}$ $\mathcal{M}_{2\gamma}$

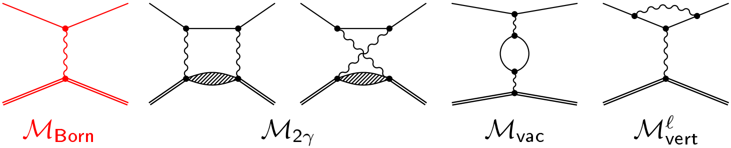
$$\sigma(e^{\pm}p) = |\mathcal{M}_{\text{Born}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right)$$

Born cross section and radiative corrections of order α^3



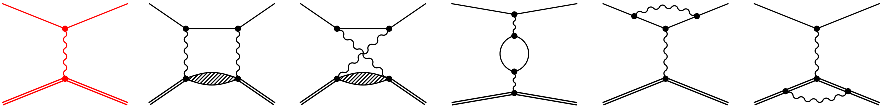
$$\sigma(e^{\pm}p) = |\mathcal{M}_{\text{Born}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) + $$
$$+ 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right)$$

Born cross section and radiative corrections of order α^3



$$\sigma(e^{\pm}p) = |\mathcal{M}_{\text{Born}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) + $$
$$+\, 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{\ell}_{\text{vert}}\right)$$

Born cross section and radiative corrections of order α^3

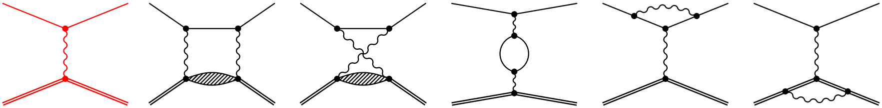


$\mathcal{M}_{\text{Born}}$ $\mathcal{M}_{2\gamma}$ $\mathcal{M}_{\text{vac}}$ $\mathcal{M}^{\ell}_{\text{vert}}$ $\mathcal{M}^{p}_{\text{vert}}$

$$\sigma(e^{\pm}p) = |\mathcal{M}_{\text{Born}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) + $$
$$+ 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{\ell}_{\text{vert}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{p}_{\text{vert}}\right)$$

Born cross section and radiative corrections of order α^3

"Elastic" scattering ($e^{\pm}p \to e^{\pm}p$):



$\mathcal{M}_{\text{Born}}$ $\mathcal{M}_{2\gamma}$ $\mathcal{M}_{\text{vac}}$ $\mathcal{M}^{\ell}_{\text{vert}}$ $\mathcal{M}^{p}_{\text{vert}}$

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Born cross section and radiative corrections of order α^3

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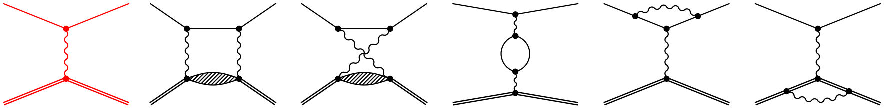
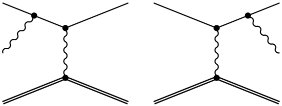


$\mathcal{M}_{\text{Born}}$ $\mathcal{M}_{2\gamma}$ $\mathcal{M}_{\text{vac}}$ $\mathcal{M}^{\ell}_{\text{vert}}$ $\mathcal{M}^{p}_{\text{vert}}$

Bremsstrahlung ($e^{\pm}p \to e^{\pm}p\,\gamma$):



$\mathcal{M}^{\ell}_{\text{brem}}$

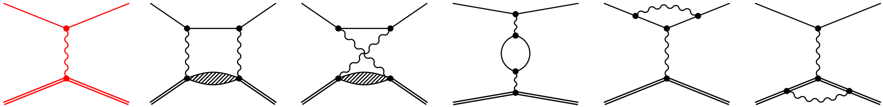
$$\sigma(e^{\pm}p) = |\mathcal{M}_{\text{Born}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) +$$

$$+\,2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{\ell}_{\text{vert}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{p}_{\text{vert}}\right) +$$

$$+\,|\mathcal{M}^{\ell}_{\text{brem}}|^2$$

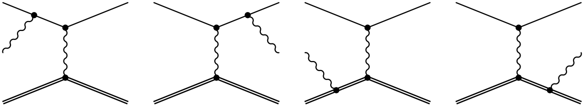
Born cross section and radiative corrections of order α^3

"Elastic" scattering ($e^{\pm}p \to e^{\pm}p$):



$\mathcal{M}_{\text{Born}}$ $\mathcal{M}_{2\gamma}$ $\mathcal{M}_{\text{vac}}$ $\mathcal{M}^{\ell}_{\text{vert}}$ $\mathcal{M}^{p}_{\text{vert}}$

Bremsstrahlung ($e^{\pm}p \to e^{\pm}p\,\gamma$):



$\mathcal{M}^{\ell}_{\text{brem}}$ $\mathcal{M}^{p}_{\text{brem}}$

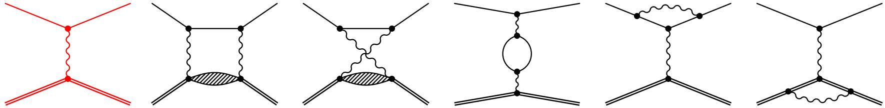
$$\sigma(e^{\pm}p) = |\mathcal{M}_{\text{Born}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) +$$

$$+\, 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{\ell}_{\text{vert}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{p}_{\text{vert}}\right) +$$

$$+\, |\mathcal{M}^{\ell}_{\text{brem}}|^2 + |\mathcal{M}^{p}_{\text{brem}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\ell\dagger}_{\text{brem}}\mathcal{M}^{p}_{\text{brem}}\right)$$

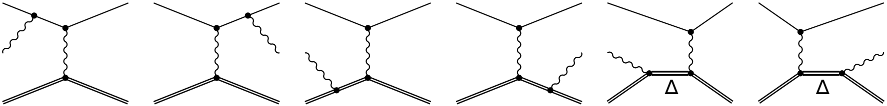
Born cross section and radiative corrections of order α^3

"Elastic" scattering ($e^{\pm}p \to e^{\pm}p$):



$\mathcal{M}_{\text{Born}}$ $\mathcal{M}_{2\gamma}$ $\mathcal{M}_{\text{vac}}$ $\mathcal{M}^{\ell}_{\text{vert}}$ $\mathcal{M}^{p}_{\text{vert}}$

Bremsstrahlung ($e^{\pm}p \to e^{\pm}p\,\gamma$):



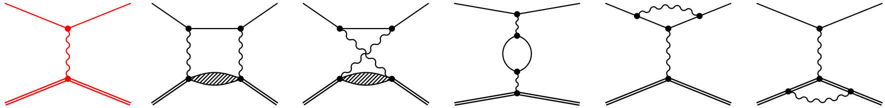
$\mathcal{M}^{\ell}_{\text{brem}}$ $\mathcal{M}^{p}_{\text{brem}}$ $\mathcal{M}_{\Delta}$

$$\sigma(e^{\pm}p) = |\mathcal{M}_{\text{Born}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) +$$

$$+\,2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{\ell}_{\text{vert}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{p}_{\text{vert}}\right) +$$

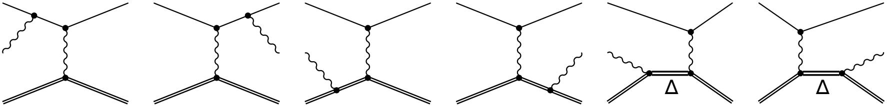
$$+\,|\mathcal{M}^{\ell}_{\text{brem}}|^2 + |\mathcal{M}^{p}_{\text{brem}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\ell\dagger}_{\text{brem}}\mathcal{M}^{p}_{\text{brem}}\right) \pm 2\,\text{Re}\left(\mathcal{M}^{\ell\dagger}_{\text{brem}}\mathcal{M}_{\Delta}\right) + \ldots$$

"Elastic" scattering ($e^{\pm}p \to e^{\pm}p$):



$\mathcal{M}_{\text{Born}}$ $\mathcal{M}_{2\gamma}$ $\mathcal{M}_{\text{vac}}$ $\mathcal{M}^{\ell}_{\text{vert}}$ $\mathcal{M}^{p}_{\text{vert}}$

Bremsstrahlung ($e^{\pm}p \to e^{\pm}p\,\gamma$):



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$$+\,|\mathcal{M}^{\ell}_{\text{brem}}|^2 + |\mathcal{M}^{p}_{\text{brem}}|^2 \pm 2\,\text{Re}\left(\mathcal{M}^{\ell\dagger}_{\text{brem}}\mathcal{M}^{p}_{\text{brem}}\right) \pm 2\,\text{Re}\left(\mathcal{M}^{\ell\dagger}_{\text{brem}}\mathcal{M}_{\Delta}\right) + \ldots$$

✓ Cancellation of infrared divergences (corresponding terms are marked in color)

✓ Some of the terms are of different signs ("±") for $e^{+}p$ and $e^{-}p$ scattering

Asymmetry A and ratio R for the cross sections

$$A = \frac{\sigma(e^+p) - \sigma(e^-p)}{\sigma(e^+p) + \sigma(e^-p)}$$

$$R = \frac{\sigma(e^+p)}{\sigma(e^-p)}$$

How are they related?

$$A = \frac{R-1}{R+1} \approx \frac{R-1}{2}$$

$$R = \frac{1+A}{1-A} \approx 1 + 2A$$

After taking into account the radiative corrections:

$$A \approx 2\frac{\mathrm{Re}\left(\mathcal{M}^\dagger_{\mathrm{Born}}\mathcal{M}_{2\gamma}\right)}{|\mathcal{M}_{\mathrm{Born}}|^2}$$

$$R \approx 1 + 4\frac{\mathrm{Re}\left(\mathcal{M}^\dagger_{\mathrm{Born}}\mathcal{M}_{2\gamma}\right)}{|\mathcal{M}_{\mathrm{Born}}|^2}$$

How to take into account the radiative corrections?

$$A = A_{\mathrm{exp}} - A_{\mathrm{MC}}$$

(exp = experimental,

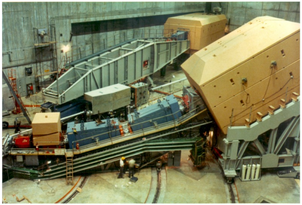
MC = Monte Carlo)

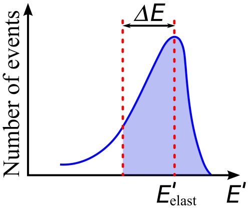
$$R = \frac{R_{\mathrm{exp}}R_{\mathrm{MC}} + 3R_{\mathrm{exp}} - R_{\mathrm{MC}} + 1}{R_{\mathrm{exp}}R_{\mathrm{MC}} - R_{\mathrm{exp}} + 3R_{\mathrm{MC}} + 1}$$

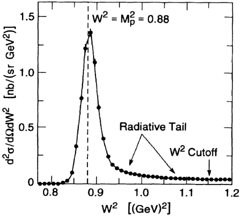
$$R \approx R_{\mathrm{exp}} - R_{\mathrm{MC}} + 1$$

The asymmetry is more natural, but the ratio is used more often.

Radiative corrections in single arm experiments







To select elastic events the following condition is commonly used:

$$E' > E'_{\text{elast}}(E,\theta_\ell) - \Delta E, \text{ where } E'_{\text{elast}}(E,\theta_\ell) = \frac{ME}{M + E(1-\cos\theta_\ell)}.$$

Or another condition:

$$W^2 < W^2_{\text{cut}}, \text{ where } W^2 = M^2 + 2M(E-E') - 4EE'\sin^2\frac{\theta_\ell}{2},$$

and W^2 is the missing mass squared. Typically used value is $W^2_{\text{cut}} = 1.1 \div 1.15$ GeV^2 (and $W^2 = M^2 = 0.88$ GeV^2 in the case of purely elastic scattering). It is easy to express ΔE through W^2_{cut}:

$$\Delta E = \frac{W^2_{\text{cut}} - M^2}{2M + 4E\sin^2(\theta_\ell/2)}.$$

Radiative corrections in single arm experiments

When only electron is detected (so we are measuring its energy E' and scattering angle θ_ℓ) the procedure of elastic scattering event selection can be described with a single parameter ΔE. Then the following simple formula is used for taking into account the radiative corrections:

$$\frac{d\sigma_{\text{exp}}}{d\Omega_\ell} = \left[1 + \delta_{\text{virt}} + \delta_{\text{brem}}(\Delta E)\right] \frac{d\sigma_{\text{Ros}}}{d\Omega_\ell}.$$

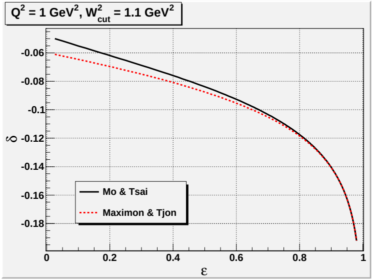
In this case, integration over kinematic parameters of recoil proton and bremsstrahlung photon can be done analytically (using some approximations). Then theorists write in their papers simple formulas for experimentalists to calculate δ_{virt} and δ_{brem} from E, E', θ_ℓ and ΔE.

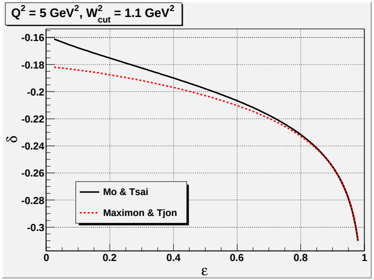
The main theoretical works about radiative corrections for such experiments:

- *Yung-Su Tsai,* Phys. Rev. **122** (1961) 1898.
- *L. W. Mo, Y. S. Tsai,* Rev. Mod. Phys. **41** (1969) 205.
- *L. C. Maximon, J. A. Tjon,* Phys. Rev. C **62** (2000) 054320.

Formulas of Mo & Tsai was a standart recipe for taking into account RC during a few decades (and are still in use)!

Comparison between Mo–Tsai and Maximon–Tjon





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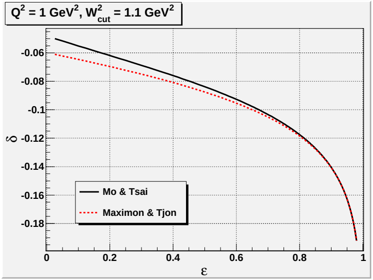

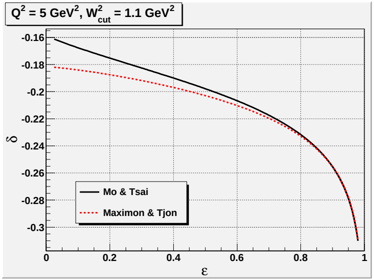

$$\sigma_{\text{red}} \frac{1+\delta_{\text{MTs}}}{1+\delta_{\text{MTj}}}:$$

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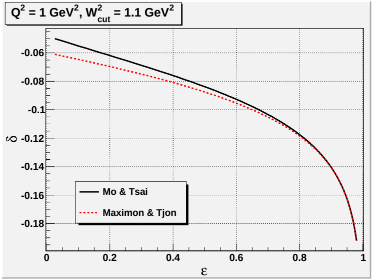
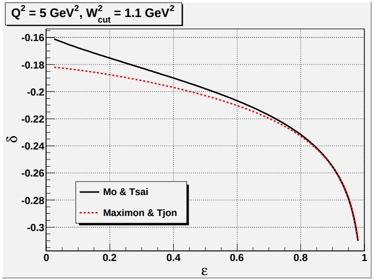
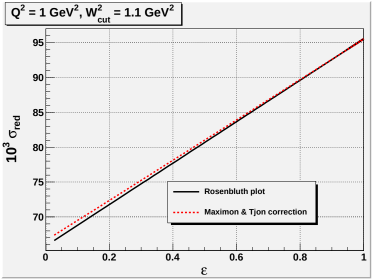
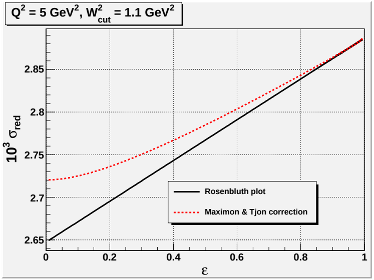




$$\sigma_{\text{red}}\frac{1+\delta_{\text{MTs}}}{1+\delta_{\text{MTj}}}\ :$$





Comparison between Mo–Tsai and Maximon–Tjon

$$\delta' = \delta_{\mathrm{IR}}(\mathrm{MTj}) - \delta_{\mathrm{IR}}(\mathrm{MoT}) =$$

$$= -\frac{\alpha}{\pi}\left[\ln\left(\frac{E}{E'}\right)\ln\left(\frac{Q^4}{4M^2EE'}\right) + 2\Phi\left(1 - \frac{M}{2E}\right) - 2\Phi\left(1 - \frac{M}{2E'}\right)\right],$$

$$\Phi(x) = -\int\limits_0^x \frac{\ln|1-x|}{x}dx.$$

Reference: *J. Arrington, et al.* Prog. Part. Nucl. Phys. **66** (2011) 782.

Comparison between Mo–Tsai and Maximon–Tjon

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$$\sigma_{\mathrm{red}}\frac{1+\delta_{\mathrm{MoT}}}{1+\delta_{\mathrm{MTj}}-\delta'}:$$

Comparison between Mo–Tsai and Maximon–Tjon

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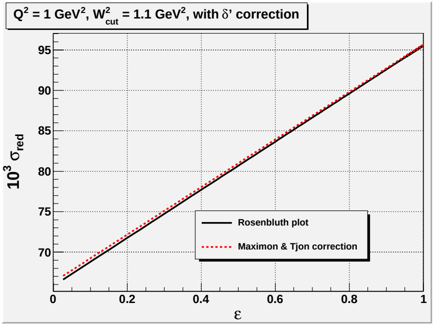
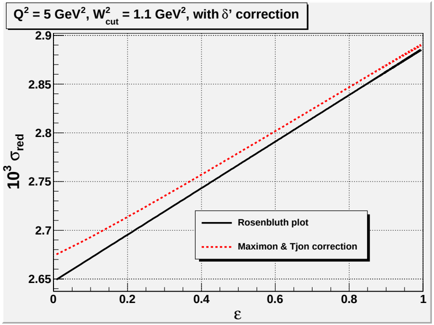
$$\sigma_{\mathrm{red}}\frac{1+\delta_{\mathrm{MoT}}}{1+\delta_{\mathrm{MTj}}-\delta'}:$$





Radiative corrections in coincidence experiments

✓ In general, magnitude of radiative corrections depends on the type of detector (magnetic or not), detector geometry, its spatial and energy resolutions and the kinematic cuts used in event selection. It is impossible to consider all this using only single parameter ΔE. So we need to have an event generator and to conduct Monte Carlo simulation of the detector.

✓ It is important also, that this approach allows us for bremsstrahlung replace analytical integration over kinematic variables of electron, proton and photon on numerical. And numerical integration allows us avoid the use of soft photon approximati and other simplifications. We need an analytical integration only in the kinematic region where photons are very soft (this is necessary to separate infrared divergent terms).

✓ Another advantage is an opportunity to take into account such complex processes as bremsstrahlung with the delta-isobar $\Delta(1232)$ excitation. We need to know only the square of the amplitude of the process. Analytical integration is not required. Modern computer algebra packages can be used for the calculation of amplitudes (for example, `Mathematica` + `FeynCalc`).

The ESEPP event generator

A new Monte Carlo event generator ($\ell p \to \ell' p'$ Pë $\ell p \to \ell' p' \gamma$), called ESEPP (Elastic Scattering of Electrons and Positrons by Protons), has been developed. We used the following formulas from the paper of Ent et al., which is the development of Mo & Tsai approach applied to coincidence experiments:

$$\left.\frac{d\sigma_{\text{elast}}}{d\Omega_\ell} + \frac{d\sigma_{\text{brem}}}{d\Omega_\ell}\right|_{E_\gamma < E_\gamma^{\text{cut}}} = \left(1 + \delta_{\text{virt}} + \delta_{\text{brem}}\right) \frac{d\sigma_{\text{Ros}}}{d\Omega_\ell},$$

$$\delta_{\text{virt}} = \delta_{\text{vac}}^{\ell} + \delta_{\text{vert}}, \quad \delta_{\text{brem}} = \delta_{\text{brem}}^{\ell\ell} \pm \delta_{\text{brem}}^{\ell p} + \delta_{\text{brem}}^{pp},$$

$$\delta_{\text{vac}}^{\ell} = \frac{2\alpha}{\pi}\left(-\frac{5}{9} + \frac{1}{3}\ln\frac{Q^2}{m^2}\right),$$

$$\delta_{\text{vert}} = \frac{\alpha}{\pi}\left(\frac{3}{2}\ln\frac{Q^2}{m^2} - 2\right),$$

$$\delta_{\text{brem}}^{\ell\ell} = -2\alpha\left[\widetilde{B}(\ell, \ell, E_\gamma^{\text{cut}}) - 2\widetilde{B}(\ell, \ell', E_\gamma^{\text{cut}}) + \widetilde{B}(\ell', \ell', E_\gamma^{\text{cut}})\right],$$

$$\delta_{\text{brem}}^{\ell p} = 4\alpha\left[\widetilde{B}(\ell, p, E_\gamma^{\text{cut}}) - \widetilde{B}(\ell, p', E_\gamma^{\text{cut}}) - \widetilde{B}(\ell', p, E_\gamma^{\text{cut}}) + \widetilde{B}(\ell', p', E_\gamma^{\text{cut}})\right],$$

$$\delta_{\text{brem}}^{pp} = -2\alpha\left[\widetilde{B}(p, p, E_\gamma^{\text{cut}}) - 2\widetilde{B}(p, p', E_\gamma^{\text{cut}}) + \widetilde{B}(p', p', E_\gamma^{\text{cut}})\right].$$

Reference: *R. Ent, et al.*, Phys. Rev. C **64** (2001) 054610.

Events of two different types are generated: "elastic" (with $E_\gamma < E_\gamma^{\text{cut}} = 1 \div 10$ MeV) and inelastic (three particles in the final states). To generate inelastic events we use either the formula (soft photon approximation)

$$\frac{d\sigma_{\text{brem}}}{d\Omega_\ell\, d^3k} = \frac{-\alpha}{4\pi^2 E_\gamma}\left[\pm\frac{\ell}{k\cdot\ell} - \pm\frac{\ell'}{k\cdot\ell'} + \frac{p}{k\cdot p} - \frac{p'}{k\cdot p'}\right]^2 \frac{d\sigma_{\text{Ros}}}{d\Omega_\ell},$$

or expression (by V. S. Fadin and A. L. Feldman)

$$\frac{d\sigma_{\text{brem}}}{dE_\gamma\, d\Omega_\gamma\, d\Omega_\ell} = \frac{1}{(2\pi)^5}\,\frac{1}{32I}\,\frac{E_\gamma E'_\ell}{\left|M + E_\ell(1-\cos\theta_\ell) - E_\gamma(1-\cos\psi)\right|}\,|\mathcal{M}_{\text{brem}}|^2,$$

where

$$I = \sqrt{(\ell p)^2 - m^2 M^2} \approx M E_\ell.$$

$$\mathcal{M}^{\ell}_{\text{brem}} = \frac{e^6}{q_1^4}(\mathcal{L}_{1\mu\nu} + \mathcal{L}_{2\mu\nu})\mathcal{P}^{\mu\nu},$$

where

$$\mathcal{L}_{1\mu\nu} = \frac{1}{2}\,\text{tr}\left[(\not{\ell}' + m)\gamma^{\alpha}\frac{\not{\ell}' + \not{k} + m}{2(k\cdot\ell')}\gamma_{\mu}(\not{\ell} + m)\gamma_{\alpha}\frac{\not{\ell} - \not{k} + m}{2(k\cdot\ell)}\gamma_{\nu}\right] -$$

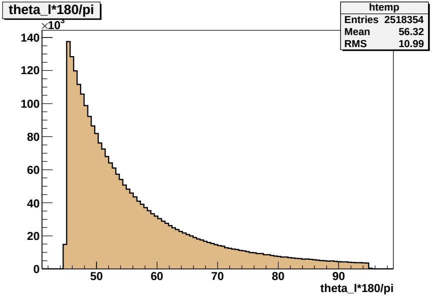
$$-\frac{1}{2}\,\text{tr}\left[(\not{\ell}' + m)\gamma^{\alpha}\frac{\not{\ell}' + \not{k} + m}{2(k\cdot\ell')}\gamma_{\mu}(\not{\ell} + m)\gamma_{\nu}\frac{\not{\ell}' + \not{k} + m}{2(k\cdot\ell')}\gamma_{\alpha}\right],$$

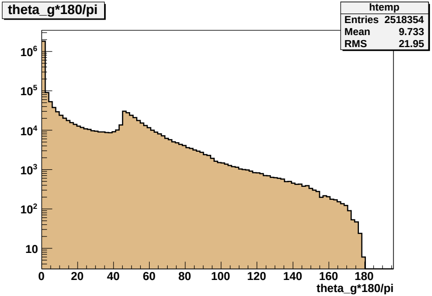
$$\mathcal{P}^{\mu\nu} = \frac{1}{2}\,\text{tr}\left[(\not{p} + M)\left\{(F_1(q_1) + F_2(q_1))\gamma^{\nu} - \frac{F_2(q_1)}{2M}P^{\nu}\right\}\right.$$

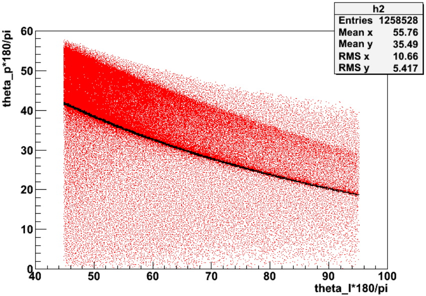
$$\left.(\not{p}' + M)\left\{(F_1(q_1) + F_2(q_1))\gamma^{\mu} - \frac{F_2(q_1)}{2M}P^{\mu}\right\}\right],$$

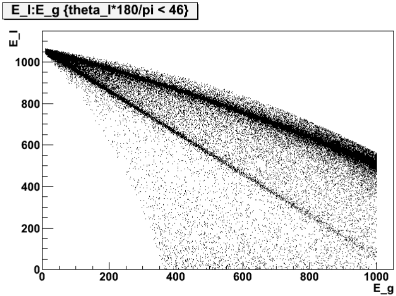
and expression for the tensor $\mathcal{L}_{2\mu\nu}$ is obtained from the expression for $\mathcal{L}_{1\mu\nu}$ after the substitution $\ell \leftrightarrow -\ell'$.

Some examples of events generated



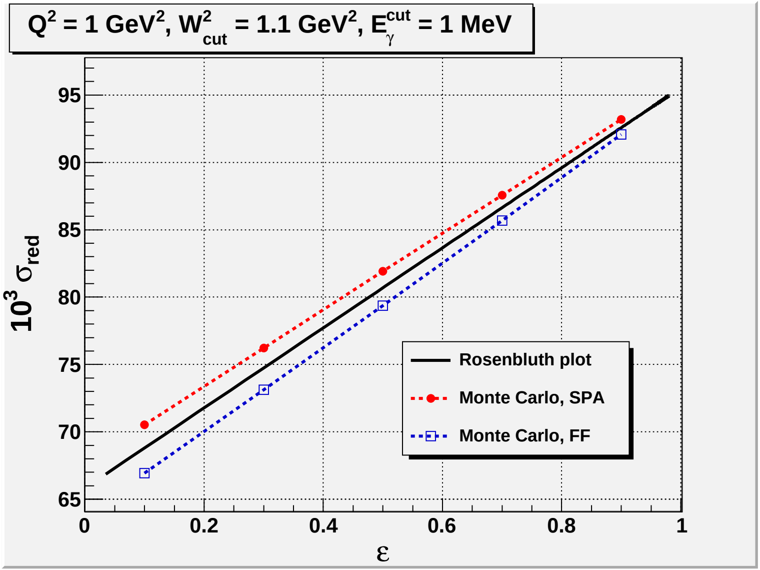






Impact on Rosenbluth separation

$$\sigma_{\text{red}} \frac{1+\delta_{\text{MTs}}}{1+\delta_{\text{MC}}}$$



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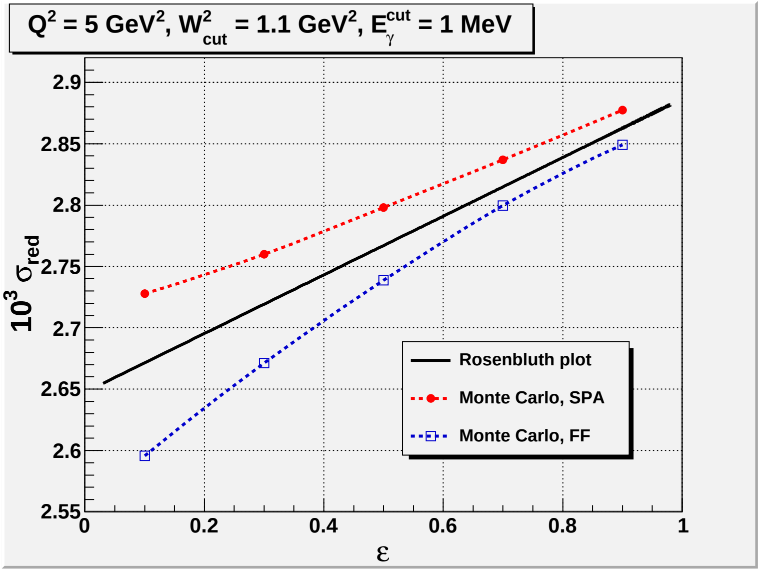
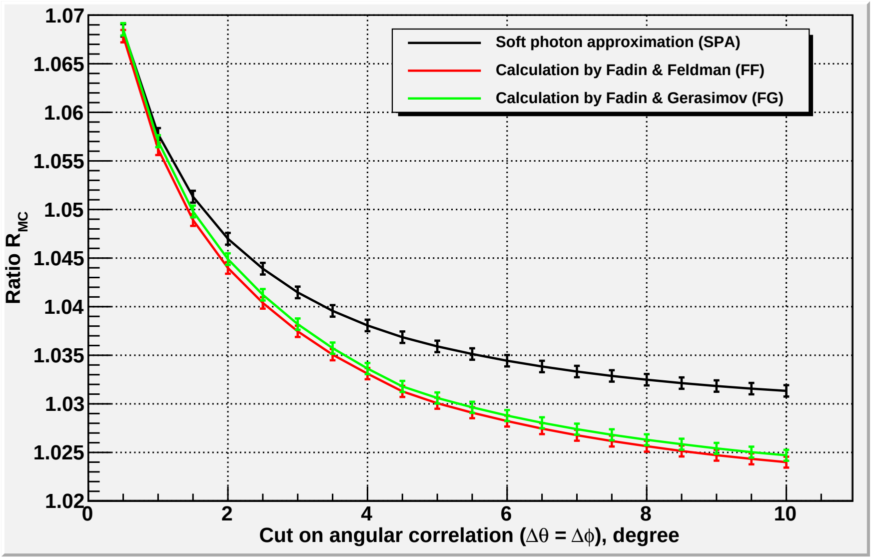


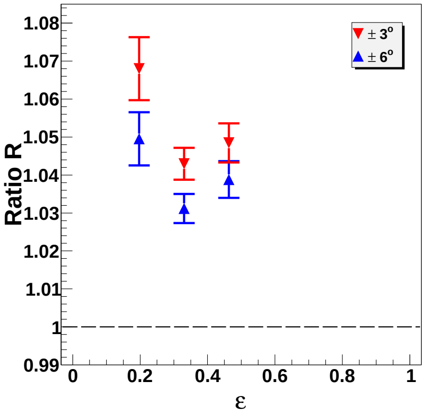
Comparison between the SPA, FF and FG models



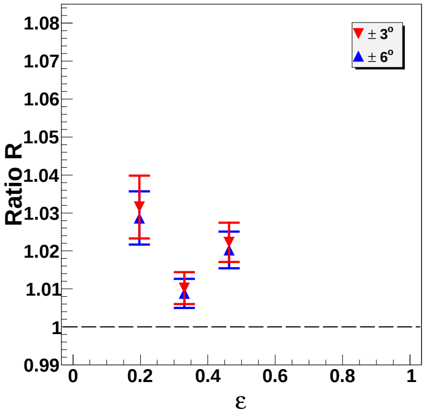
The ratio R_{MC} as a result of geometric MC simulation. Kinematics: LA, run I.

Ratio R and RC depend both on the kinematic cuts used

Raw data for the ratio R:



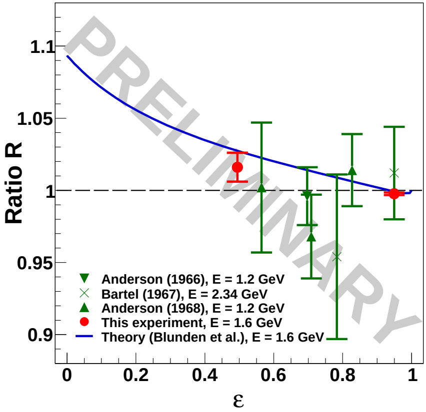
Radiatively corrected ratio R:



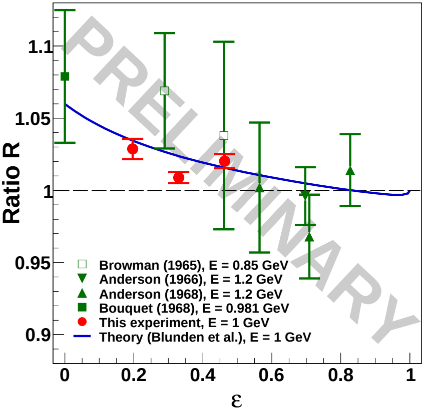
Experimentally measured ratio R is shown before (left figure) and after (right figure) taking into account the radiative corrections (FF model). Red markers correspond to the cut $\Delta\theta = \Delta\phi = 3^\circ$ on the angular correlations, blue markers correspond to the cut $\Delta\theta = \Delta\phi = 6^\circ$ (data for LA range of the run II).

Preliminary results of the Novosibirsk experiment

Run I (2009):
$E_{\text{beam}} = 1.6$ GeV



Run II (2011–2012):
$E_{\text{beam}} = 1$ GeV



Theory: *P. G. Blunden, et al.*, Phys. Rev. C **72** (2005) 034612

Only statistical errors are shown. Systematic errors for both the runs: $\leqslant 0.3\%$

Note that the radiative corrections have been taken into account. Some minor corrections have not yet been made (for example, corrections related to the variation in time of beam energy and position).

Conclusion and acknowledgements

✓The procedure for accounting of radiative corrections in the experiment was designed, the ESEPP event generator was written, Geant4 simulation of the detector was carried out.
✓To account for the first-order bremsstrahlung the soft-photon approximation was used, as well as the more accurate calculation provided by V. S. Fadin and A. L. Feldman.
✓The effect of this clarification on the Rosenbluth separation was studied.
✓The first calculation (by V. S. Fadin and R. E. Gerasimov) of first-order bremsstrahlung with the delta-isobar excitation was used.

- I'm very grateful to R. E. Gerasimov, D. M. Nikolenko, I. A. Rachek, D. K. Toporkov, V. S. Fadin and A. L. Feldman for their help and for many fruitful discussions.
- My participation in the Symposium was supported in part by Russian Foundation for Basic Research under the grant 12–02–16065_mob_z_ros.

Thank you for your attention!

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