On radiative corrections to electron-proton scattering

V.S. Fadin

Budker Institute of Nuclear Physics

Novosibirsk

The results of study of radiative corrections performed with R.E. Gerasimov

Contents

- Introduction
- One-loop corrections
  - Virtual corrections
  - Real emission
- Comparison of different approaches
- Summary

Introduction

There is an evident discrepancy between results of measurements of G_E/G_M by two methods:
Rosenbluth separation

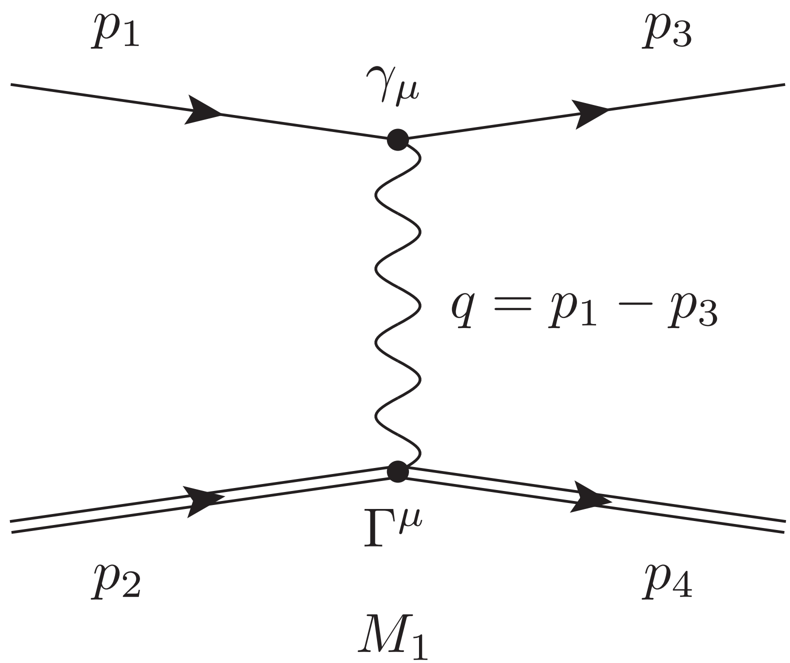
$$\frac{G_E^2}{G_M^2} = \tau \frac{f'(\epsilon)}{f(0)}, \quad \tau = \frac{Q^2}{4M^2}, \quad f(\epsilon) = \frac{1}{\tau+\epsilon} \left(\frac{d\sigma}{d\Omega}\right)^{-1}_{point} \left(\frac{d\sigma^B}{d\Omega}\right),$$

$\epsilon = \left(1 + 2(1+\tau)\tan^2(\theta/2)\right)^{-1}$ – the virtual photon polarization parameter, and polarization transfer at scattering of polarized electron beam on unpolarized target

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\epsilon)}{2\epsilon}} \frac{P_T}{P_L},$$

$P_T(P_L)$ — the polarization of the recoil proton transverse (longitudinal) to the proton momentum in the scattering plane.
Formulas above are obtained in the Born approximation.

Introduction



$$\Gamma^\mu(q) = F_1(Q^2)\,\gamma^\mu + F_2(Q^2)\,\frac{i\sigma^{\mu\nu}q_\nu}{2M},$$

$$Q^2 = -q^2,\;\; G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2),\;\; G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Introduction

The cause of the discrepancy must be understood.

One possible cause is the radiative corrections.

Most dangerous they are for the Rosenbluth method.

At large Q^2 contribution of the term with G_E^2 to the cross section becomes small.

It makes determination of G_E very sensitive to ϵ-dependent corrections.

Account of the radiative corrections in

[1] R. C. Walker et al., Phys. Rev. D 49, 5671 (1994)

[2] J. Arrington, Phys. Rev. C 68, 034325 (2003)

[3] M. E. Christy et al., Phys. Rev. C 70, 015206 (2004)

[4] I. A. Qattan et al., Phys. Rev. Lett. 94, 142301 (2005)

must be analyzed.

Introduction

In these papers, the main theoretical source of the radiative corrections is

Y. -S. Tsai, Phys. Rev. **122**, 1898 (1961)

L. W. Mo and Y. -S. Tsai, Rev. Mod. Phys. **41**, 205 (1969).

In the following we will call it MoT.

The radiative corrections in this source have factored form:

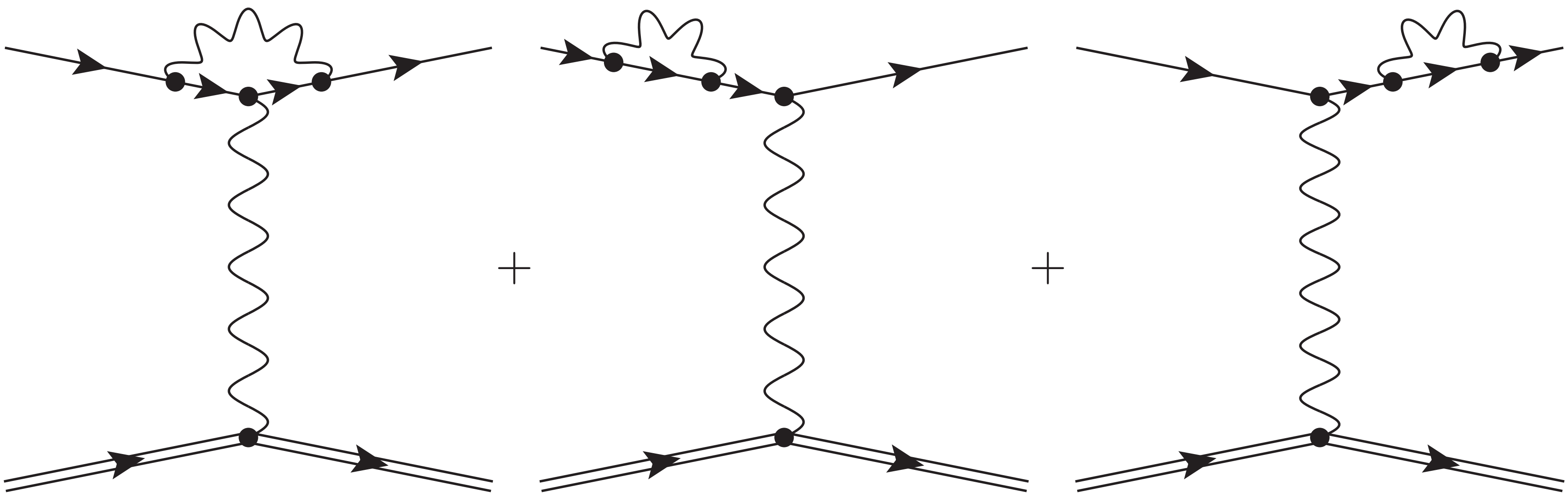
$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^B}{d\Omega}(1+\delta)$$

To distinguish corrections of different type let us denote electron charge e and proton one $-Ze$.

There are virtual (corresponding to elastic process) and real (accounting inelasticity) corrections.

Virtual electron correction

The first order virtual correction proportional to Z^0 come from interference of the Born diagram with the diagrams



M_e

Virtual electron correction

$$\delta^{ve}_{MoT} = \frac{Z^0\alpha}{\pi}\left(-K(p_1,p_3) + K(p_1,p_1) + \frac{3}{2}\ln\left(\frac{-q^2}{m^2}\right) - 2\right),$$

where

$$K(p_i,p_j) = \frac{2(p_i\cdot p_j)}{-i\pi^2}\int\frac{d^4k}{(k^2-\lambda^2+i0)(k^2-2(k\cdot p_i)+i0)(k^2-2(k\cdot p_j)+i0)}$$

$$= (p_i\cdot p_j)\int_0^1\frac{dy}{p_y^2}\ln\left(\frac{p_y^2}{\lambda^2}\right),$$

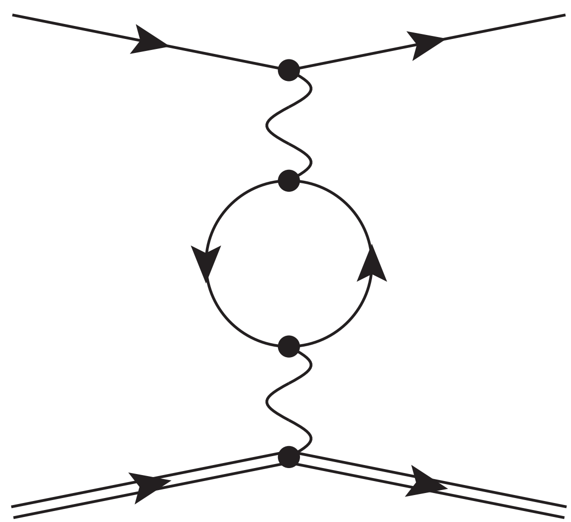
$p_y = y\,p_i + (1-y)\,p_j$. The only approximation in calculation of this correction is smallness of m^2. Explicitly,

$$\delta^{ve}_{MoT} = \frac{Z^0\alpha}{\pi}\left\{-\left(\ln\left[\frac{-q^2}{m^2}\right]-1\right)\ln\left[\frac{m^2}{\lambda^2}\right] - \frac{1}{2}\ln^2\left[\frac{-q^2}{m^2}\right] + \frac{\pi^2}{6} + \right.$$

$$\left. + \frac{3}{2}\ln\left[\frac{-q^2}{m^2}\right] - 2\right\}$$

Vacuum polarization correction

The interference of the Born diagram and the diagram with vacuum polarization gives also correction proportional to Z^0



M_v

Vacuum polarization correction

Here, besides electron, muon and τ-lepton loops, hadron vacuum polarization must be accounted.

For light leptons

$$\delta_{MoT}^{ll} = \frac{Z^0\alpha}{\pi}\left(\frac{2}{3}\ln\left[\frac{-q^2}{m^2}\right] - \frac{10}{9}\right),$$

for heavy

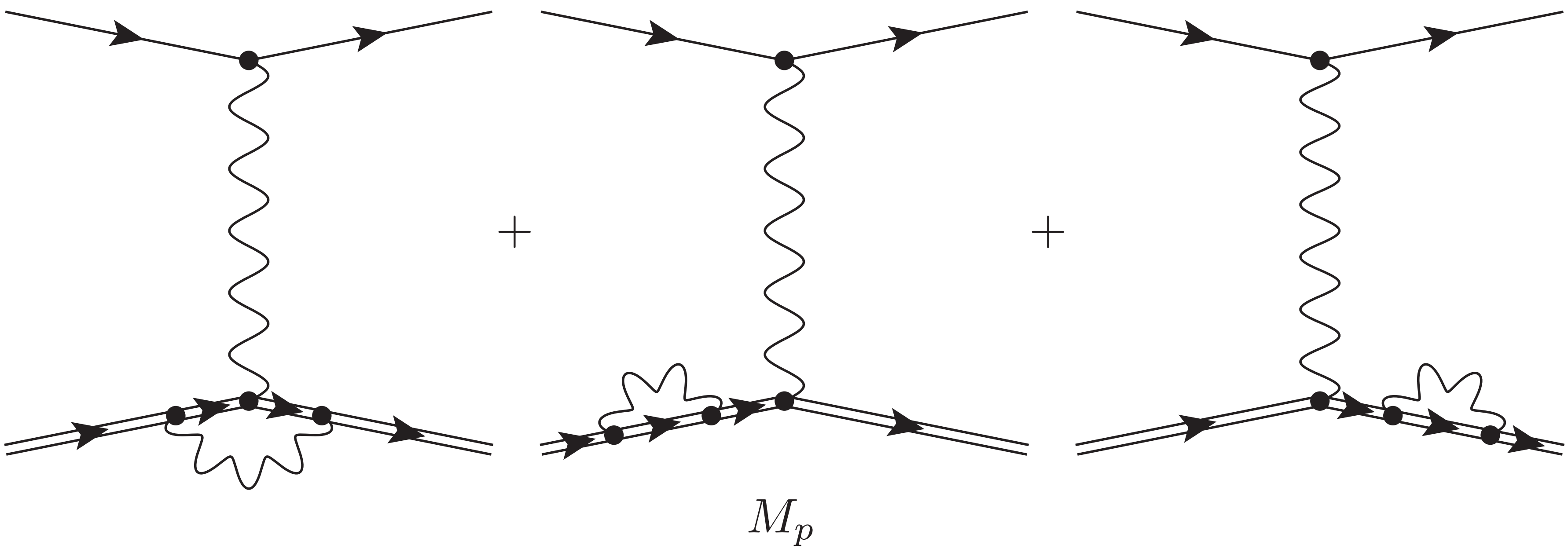
$$\delta_{MoT}^{hl} = \frac{2Z^0\alpha}{\pi}\left[\frac{1}{9} - \frac{2q^2+4m^2}{3t}\left(1 - \sqrt{\frac{4m^2-q^2}{-q^2}}\ln\left(\sqrt{-\frac{q^2}{4m^2}} + \sqrt{1-\frac{q^2}{4m^2}}\right)\right)\right.$$

These expressions also are well known.

The hadron contribution to the vacuum polarization can not be presented in an analogous form, because it includes strong interaction effect. It is calculated using dispersion relations and $e^+e^- \rightarrow hadrons$ experimental data. Now this contribution is well known.

Virtual proton correction

The virtual correction proportional to Z^2 come from interference of the Born diagram with the diagrams



Virtual proton correction

This contribution can not be calculated "from the first principles". They are evaluated by MoT using the soft photon approximation

$$\delta^{vp}_{MoT} = \frac{Z^2\alpha}{\pi}\left(-K(p_2,p_4) + K(p_2,p_2)\right).$$

Other evaluations are possible. In the paper

L. C. Maximon and J. A. Tjon, Phys. Rev. C **62** (2000) 054320

which is called in the following MTj, this correction is calculated using dipole or monopole form factors

$$F_1(Q^2) = F_2(Q^2) = \left(\frac{\Lambda^2}{\Lambda^2 + Q^2}\right)^n, \quad n = 1, 2$$

Therefore

$$\delta^{vp}_{MTj} = \delta^{vp}_{MoT} + \delta^{(1)}_{el},$$

where $\delta^{(1)}_{el}$ is infrared finite.

Virtual proton correction

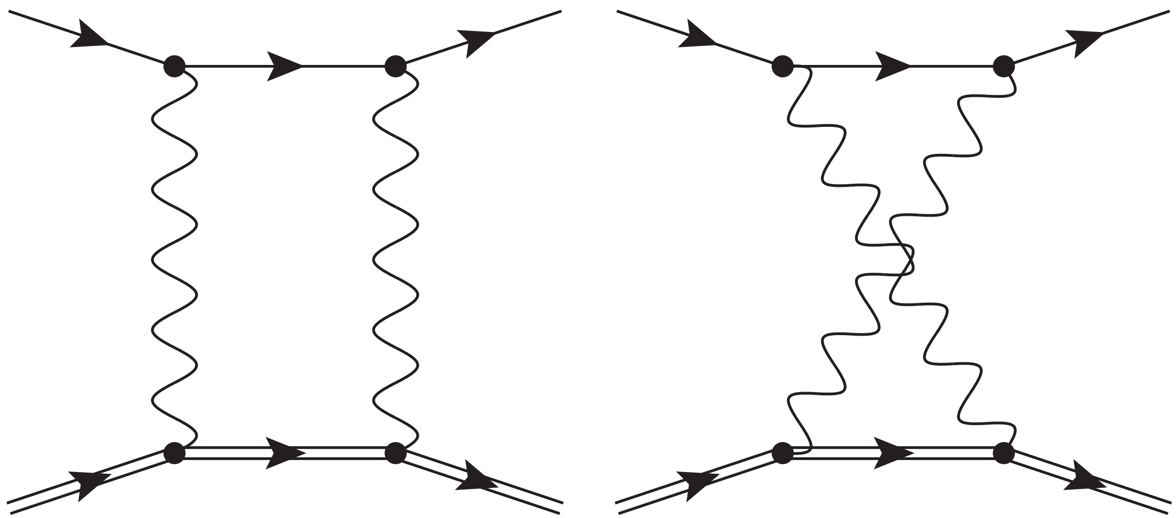
For the dipole parametrization with $\Lambda = 700$ MeV/c

$\delta_{el}^{(1)} = 0.0116$ for $Q^2 = 16$ (GeV/c)^2

In any case, this correction depends only on Q^2.

Virtual electron-proton correction

The most interesting virtual, proportional to Z^1 corrections, come from the interference of the Born diagram with the diagrams



M_{ep}

Virtual electron-proton correction

In their evaluation MoT used the soft photon approximation with additional simplification.

$$\delta^{vep}_{MoT} = \frac{Z^2\alpha}{\pi} \left(-K(p_1, p_2) - K(p_3, p_4 + K(p_1, p_4) + K(p_2, p_3)\right),$$

whereas in the soft photon approximation

$$\delta^{vep}_{sf} = \frac{Z^2\alpha}{\pi} Re\left(-K(p_1, -p_2) - K(p_3, -p_4 + K(p_1, p_4) + K(p_2, p_3)\right).$$

and

$$-Re[K(p_1, -p_2)] - Re[K(p_3, -p_4)] + K(p_1, p_2) + K(p_3, p_4)$$

$$= \pi^2 - 2\int_{1-\frac{M}{2\epsilon_1}}^{1+\frac{M}{2\epsilon_1}} \frac{dx}{x} \ln|1-x|$$

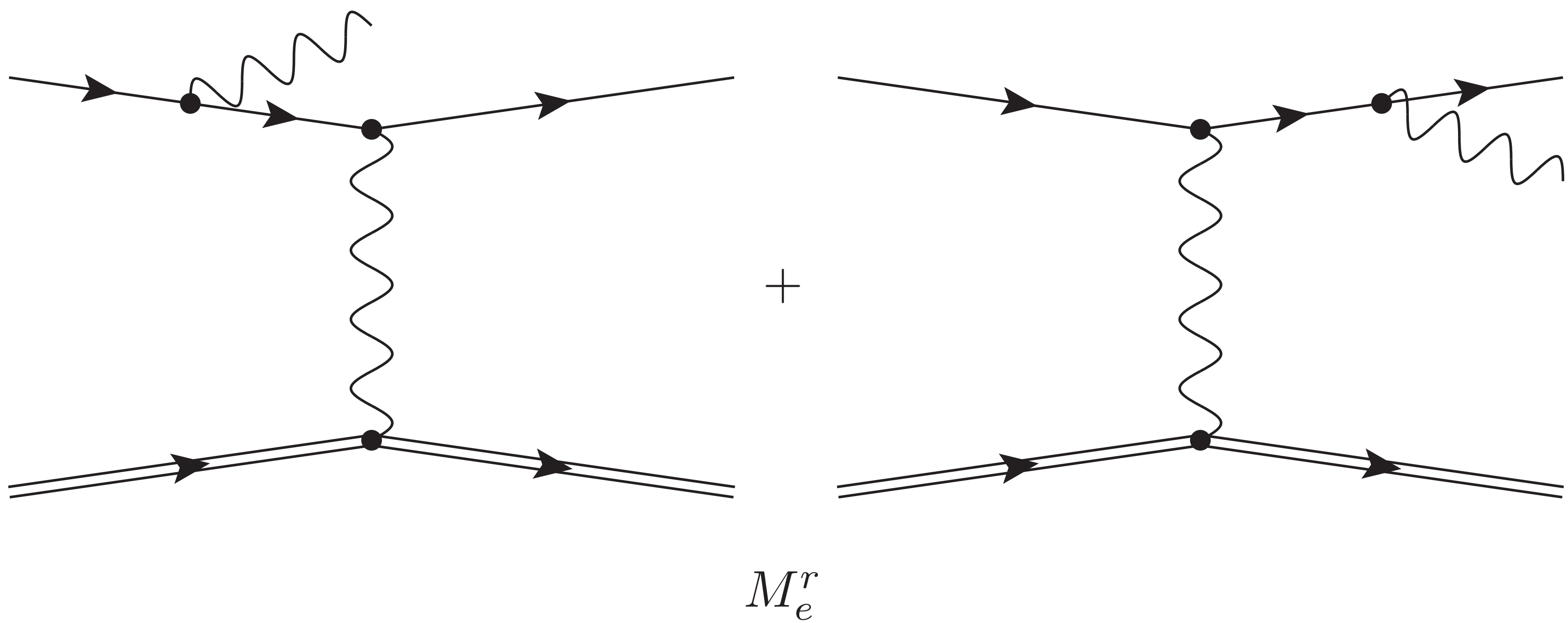
Virtual proton correction

But the soft photon approximation in its standard form is not good here.
More reliable approximation is used by MTj.

$$\delta^{vep}_{MTj} - \delta^{vep}_{MoT} = 2\,Z\,\frac{\alpha}{\pi}\left(-\ln\frac{\epsilon_1}{\epsilon_3}\ln\left[\sin\left[\frac{\theta}{2}\right]^2\right] - Li_2\left[1-\frac{2\epsilon_3}{M}\right] + Li_2\left[1-\frac{2\epsilon_1}{M}\right]\right).$$

Real electron correction

The first order real correction proportional to Z^0 come from the diagrams



Real electron correction

$$\delta^{re}_{MoT} = \frac{Z^0\alpha}{\pi}\left(\left(\ln\left[\frac{-q^2}{m^2}\right]-1\right)\ln\left[\frac{\omega^2 m^2}{\lambda^2\epsilon_1\epsilon_3}\right]+\frac{1}{2}\ln\left[\frac{-q^2}{m^2}\right]^2-\frac{\ln[\eta]^2}{2}-\frac{\pi^2}{6}\right),$$

where

$$\eta = 1+\frac{\epsilon_1}{M}(1-\cos\theta))$$

Here the term

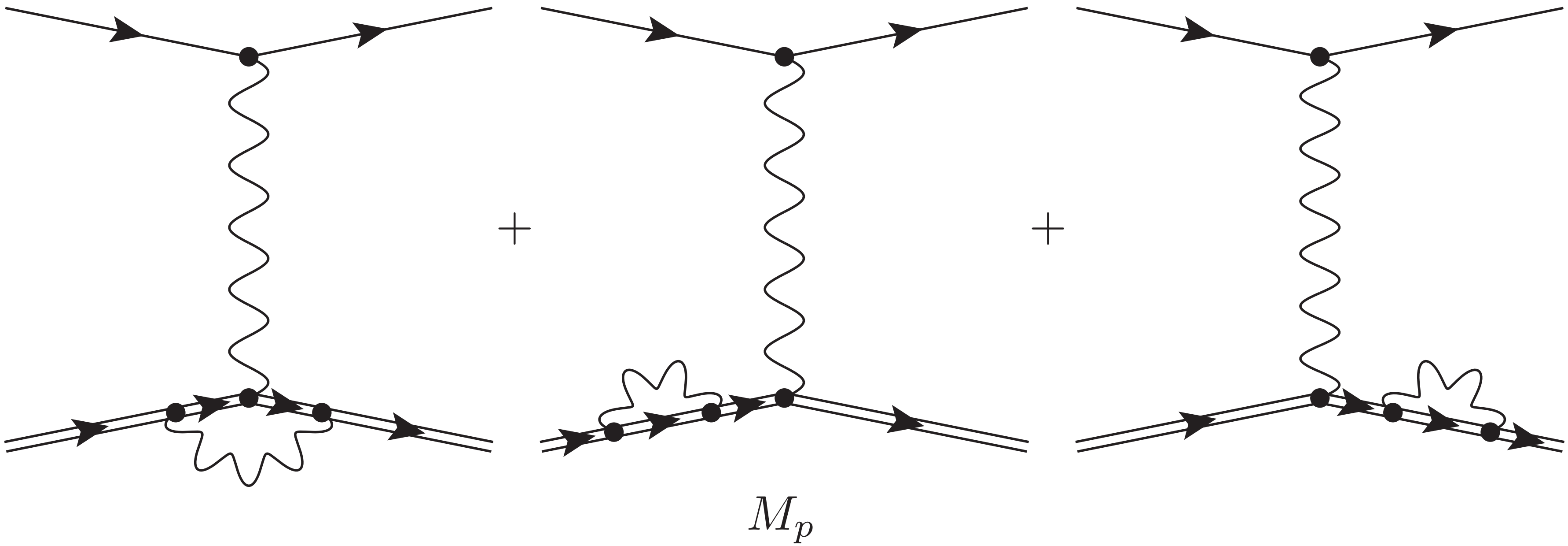
$$-\frac{\alpha}{\pi}\left[\frac{\pi^2}{6}-\Phi[cos^2(\theta/2)]\right]$$

was omitted in papers MoT. Later on was restored in
Y. -S. Tsai, SLAC-PUB-0848, (1971)

and included in experimental papers as δ_{Sch} (Schwinger's correction)).

Virtual proton correction

The real correction proportional to Z^2 come from the diagrams



Real proton correction

This contribution also can not be calculated "from the first principles". They are evaluated by using the soft photon approximation.

$$\delta^{rp}_{MoT} = Z^2 \frac{\alpha}{\pi} \left(\left(\frac{\epsilon_4}{|\mathbf{p}_4|} \ln[x] - 1 \right) \ln\left[\frac{4\omega^2}{\lambda^2}\right] - \right.$$

$$\left. - \frac{\epsilon_4}{|\mathbf{p}_4|} \left(\ln[x]^2 + Li_2\left[-\frac{1}{x^2}\right] + \frac{\pi^2}{12} \right) + \ln\left[\frac{2\epsilon_4}{M}\right] + \ln[2] \right)$$

But in derivation of this result there is the error. Correct result is given by MTj.

$$\delta^{rp}_{MTj} - \delta^{rp}_{MoT} = Z^2 \frac{\alpha}{\pi} \left(-\frac{\epsilon_4}{|\mathbf{p}_4|} \left(Li_2\left[1 - \frac{1}{x^2}\right] - Li_2\left[-\frac{1}{x^2}\right] - \frac{\pi^2}{12} \right) + \right.$$

$$\left. + \frac{\epsilon_4}{|\mathbf{p}_4|} \ln[x] + 1 - \ln\left[\frac{2\epsilon_4}{M}\right] - \ln[2] \right)$$

Real electron-proton correction

The most interesting real correction, proportional to Z^1, comes from the interference diagrams with electron and photon emission.

$$\delta^{rep}_{MoT} = 2Z\frac{\alpha}{\pi}\left(\ln[\eta]\ln\left[\frac{4\omega^2}{\lambda^2}\right] + \frac{1}{2}Li_2\left[1-\eta\frac{2\epsilon_4}{M}\right] - \frac{1}{2}Li_2\left[1-\frac{1}{\eta}\frac{2\epsilon_4}{M}\right]\right)$$

Again there is an error here. Correct result is given by MTj

$$\delta^{rep}_{MTj} - \delta^{rep}_{MoT} = 2\,Z\,\frac{\alpha}{\pi}\left(-\ln[\eta]\ln[x] - Li_2\left[1-\frac{1}{\eta\,x}\right] + Li_2\left[1-\frac{\eta}{x}\right] - \right.$$

$$\left. -\frac{1}{2}\,Li_2\left[1-\eta\frac{x^2+1}{x}\right] + \frac{1}{2}\,Li_2\left[1-\frac{x^2+1}{\eta\,x}\right]\right)$$

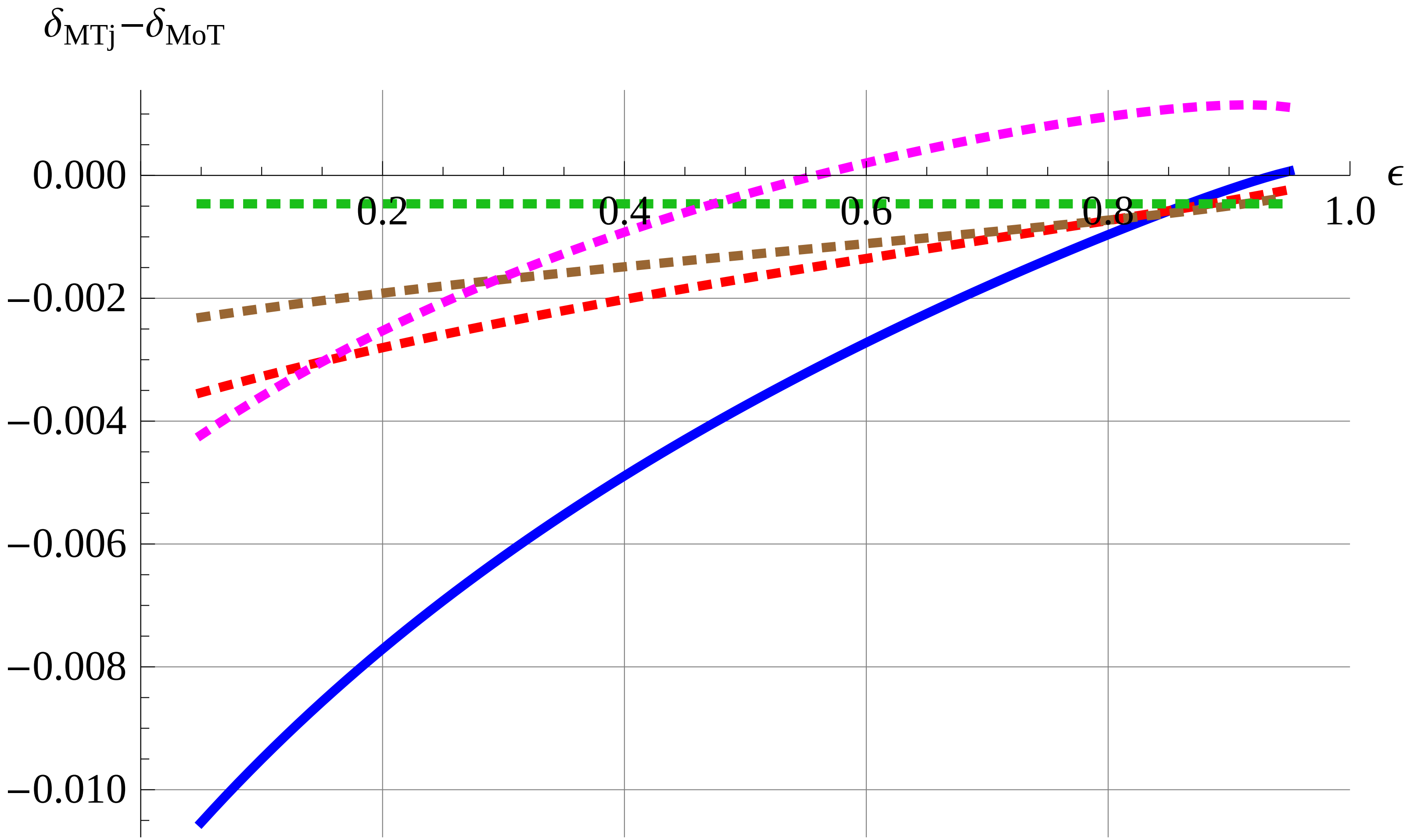


Figure 1: Difference at $Q^2 = 1\ GeV^2$.

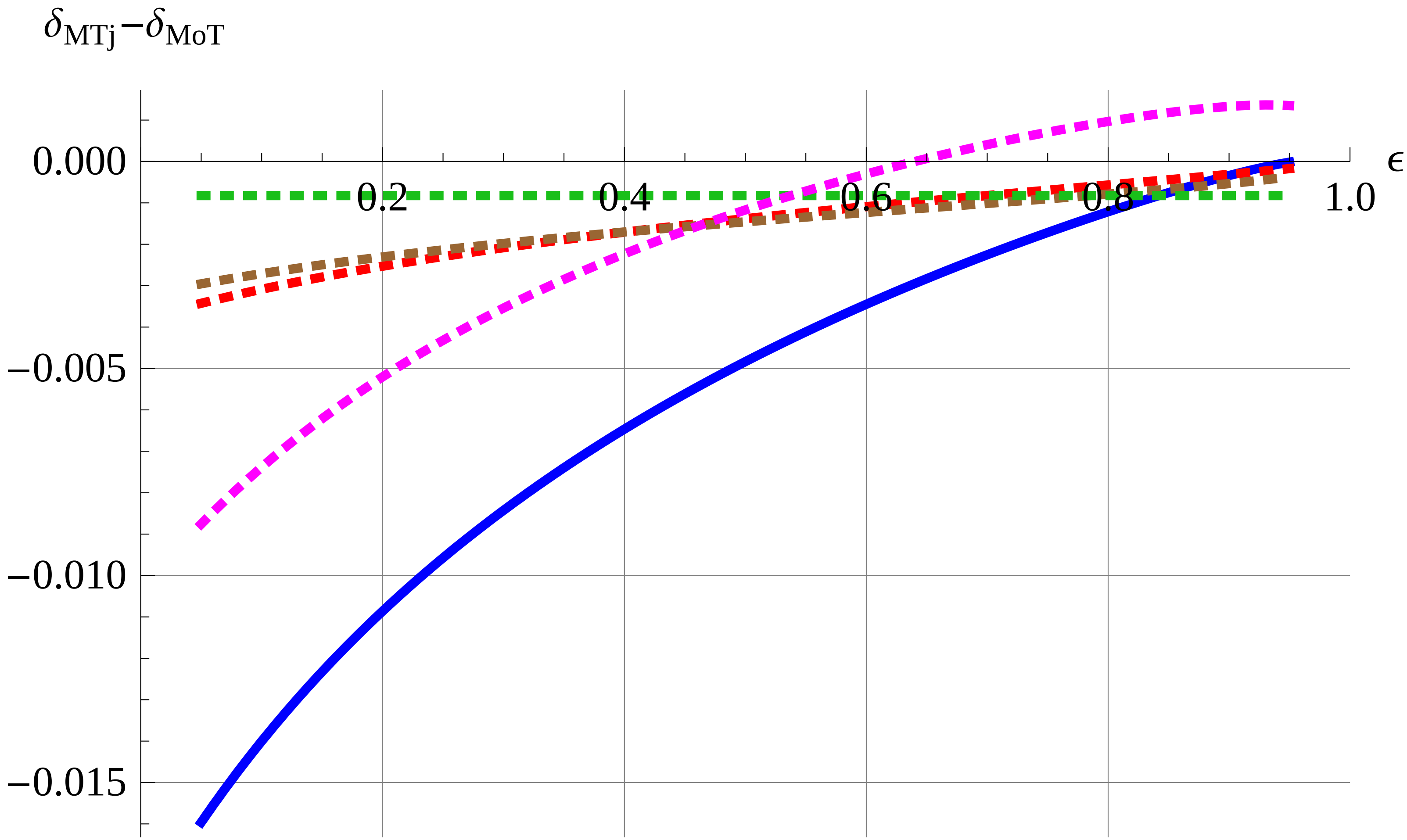


Figure 2: Difference at $Q^2 = 3\ GeV^2$.

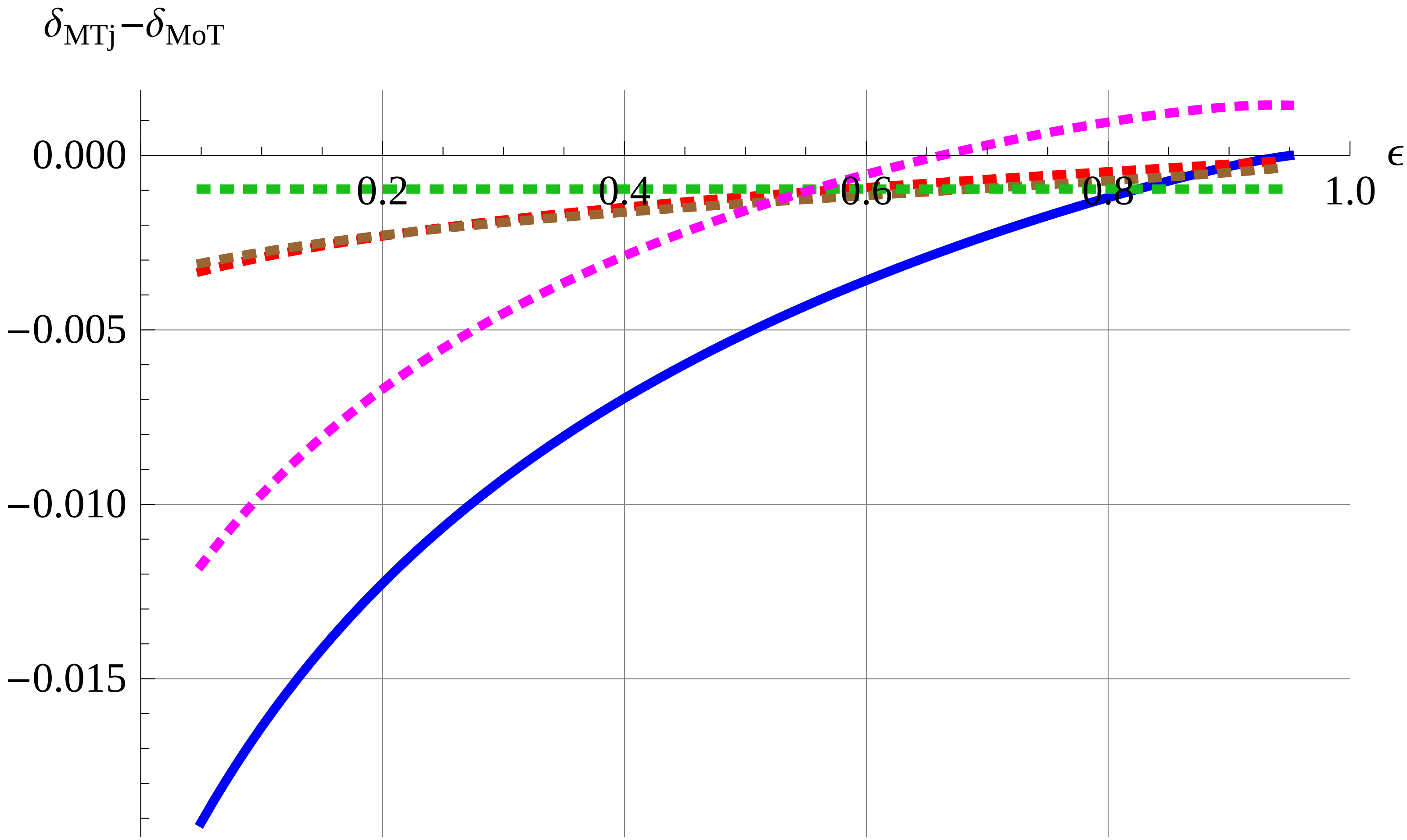


Figure 3: Difference at $Q^2 = 5\ GeV^2$.

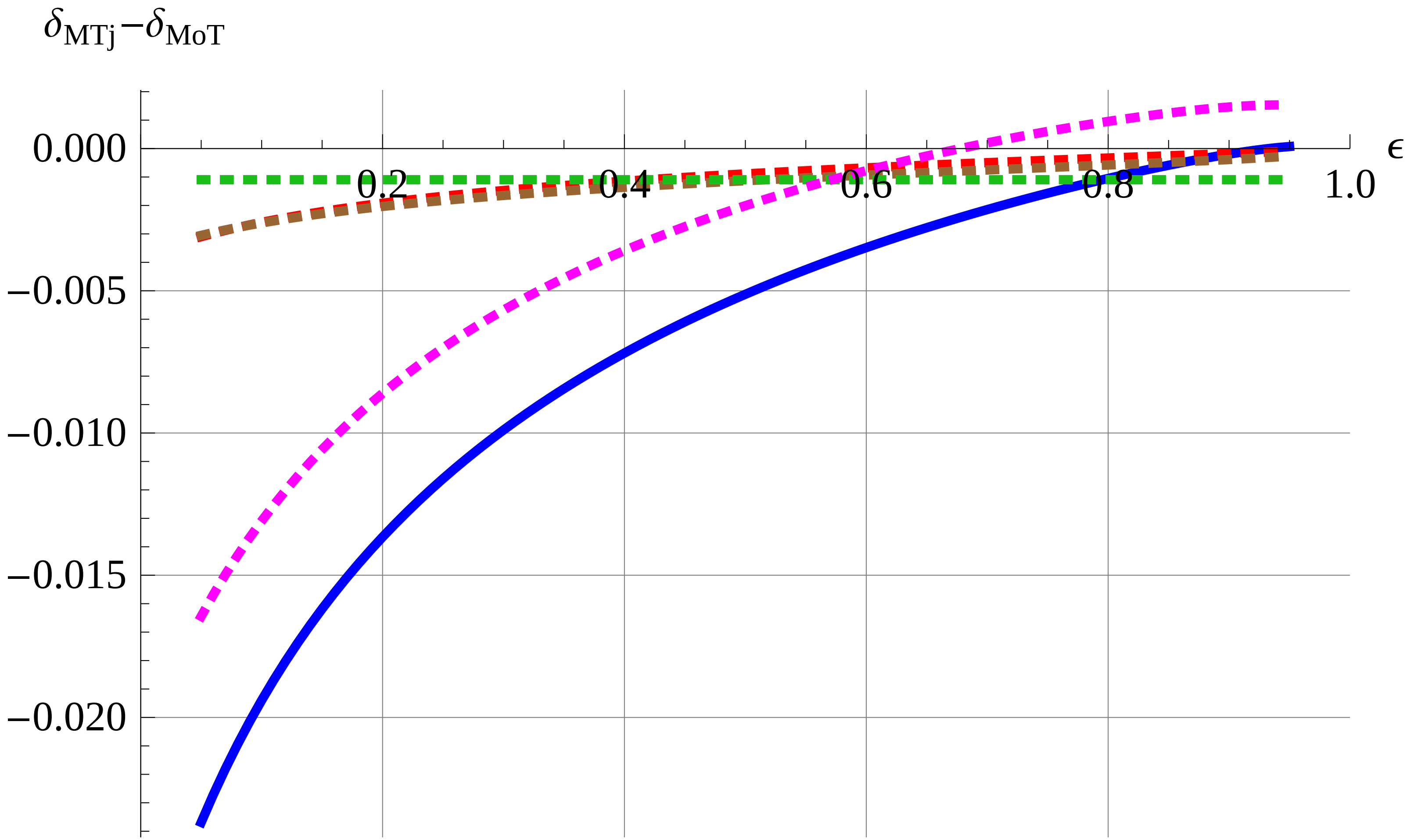


Figure 4: Difference at $Q^2 = 10\ GeV^2$.

Summary

- There is an evident discrepancy between results of measurements of G_E/G_M by two methods.
- There are no strict theoretical justifications for any of these results.
- But the result obtained by Rosenbluth separation method seems less reliable.
- There are evident shortages in account of radiative corrections in this method.
- It is desirable to understand influence of these shortages on the result.
- It is very desirable to perform accurate account of radiative corrections in two-photon exchange experiments in such a way that it will be possible to connect results of different experiments.