

*Model dependent and model **in**dependent considerations on two photon exchange*



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Plan

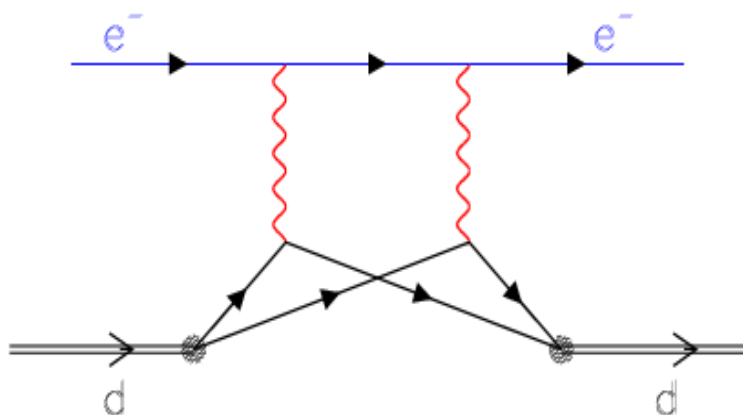
- Introduction
 - Generalities
- Model independent considerations
 - Space-like
 - Time-like
- Model dependent considerations
 - What do data say?
 - Which alternative for GEp problem?
- Conclusions

Two-Photon exchange

- 1γ - 2γ interference is of the order of $\alpha=e^2/4p=1/137$ (in usual calculations of radiative corrections, one photon is 'hard' and one is 'soft')
- "Invent a mechanism" to enhance this contribution
- In the 70's it was shown [J. Gunion and L. Stodolsky, V. Franco, F.M. Lev, V.N. Boitsov, L. Kondratyuk and V.B. Kopeliovich, R. Blankenbecker and J. Gunion] that, at large momentum transfer, due to the sharp decrease of the FFs, if the momentum is shared between the two photons, the 2γ -contribution can become very large.
- The 2γ amplitude is expected to be mostly imaginary.
- In this case, the 1γ - 2γ interference is more important in time-like region, as the Born amplitude is complex.

Qualitative estimation of 2γ exchange

For *ed* elastic scattering:



$$\mathcal{M}_1 = \alpha F_d(t).$$

$$\mathcal{M}_2 = \alpha^2 F_N^2 \left(\frac{t}{4} \right).$$

From quark counting rules: $F_d \sim t^{-5}$ and $F_N \sim t^{-2}$. At $t = 4$ GeV²

$$\frac{\mathcal{M}_2}{\mathcal{M}_1} = \alpha F_N^2 / F_d(t) = 256 \alpha t / m_x^2$$

$$t / m_x^2 \simeq 6$$

$$\frac{\mathcal{M}_2}{\mathcal{M}_1} \simeq 1500 \alpha \rightarrow 10!$$

For d, 3He, 4He, 2γ effect should appear at ~ 1 GeV²,
for protons ~ 10 GeV²

Two-Photon exchange in ed -scatterng

- *Discrepancy between the results from*
 - *Hall A [L.C. Alexa et al., P.R.L. 82, 1374 (1999)]*
 - *Hall C [D. Abbott et al., P.R.L. . 82, 1379 (1999)].*
- *Model-independent parametrization of the 2γ - contribution.*
- *Applied to ed -elastic scattering data.*

M. P. Rekalo, E. T-G and D. Prout, Phys. Rev. C60, 042202 (1999)

Crossing Symmetry

Scattering and annihilation chan

- Described by the same amplitude

$$|\overline{\mathcal{M}}(e^\pm h \rightarrow e^\pm h)|^2 = f(s, t) = |\overline{\mathcal{M}}(e^+ e^- \rightarrow \bar{h} h)|^2,$$

- function of two kinematical va.

$$s = (k_1 + p_1)^2$$

$$t = (k_1 - k_2)^2$$

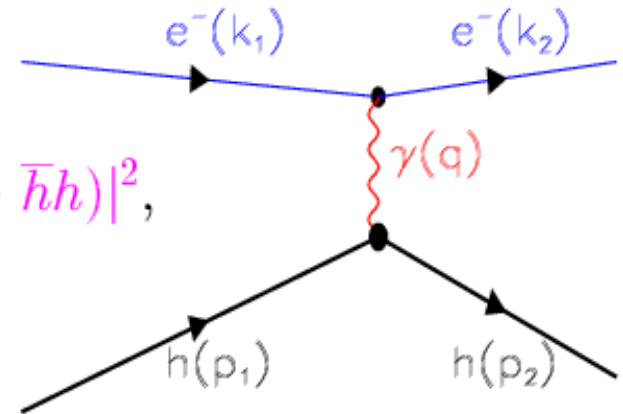
- which scan different kinematical regions

$$k_2 \rightarrow -k_2$$

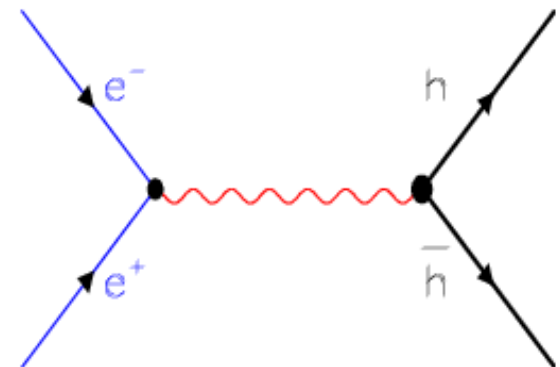
$$p_2 \rightarrow -$$

$$\cos^2 \tilde{\theta} = 1 + \frac{st + (s - M^2)^2}{t(\frac{t}{4} - M^2)} \rightarrow 1 + \frac{ctg^2 \frac{\theta}{2}}{1 + \tau}$$

$$e^- + h \rightarrow e^- + h$$

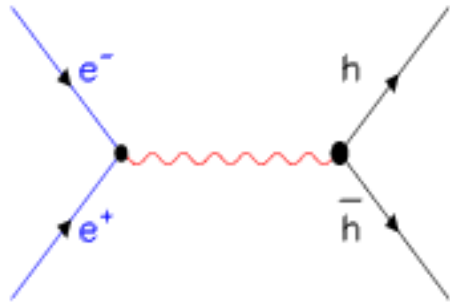


$$e^- + e^+ \rightarrow \bar{h} + h$$

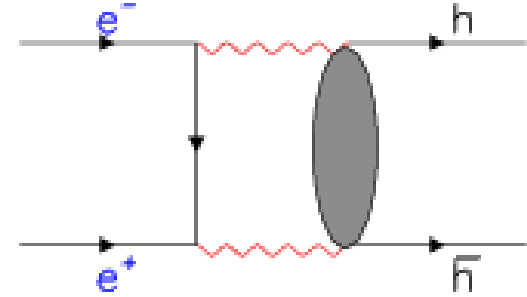


1 γ -2 γ interference

M. P. Rekalov, E. T.-G. and



$$C(\gamma) = -1$$



$$C(2\gamma) = +1$$

$S = 1, \ell = 0$ and $S = 1, \ell = 2$ with $J^P = 1^-$,

$$|\mathcal{M}_1(e^+e^- \rightarrow \bar{h}h)|^2 = a(t) + \cos^2 \tilde{\theta} b(t)$$

$$\text{Re} \mathcal{M}_1 \mathcal{M}_2^* = \cos \tilde{\theta} (a_0 + a_1 \cos^2 \tilde{\theta} + \dots)$$

$$\frac{d\sigma}{d\Omega_e}(e^-h \rightarrow e^-h) = \sigma_0 \left(\underbrace{A \cot^2 \frac{\theta_e}{2}}_{1\gamma} + B + \underbrace{C \cot \frac{\theta_e}{2}}_{2\gamma} + D \cot^3 \frac{\theta_e}{2} + \dots \right)$$

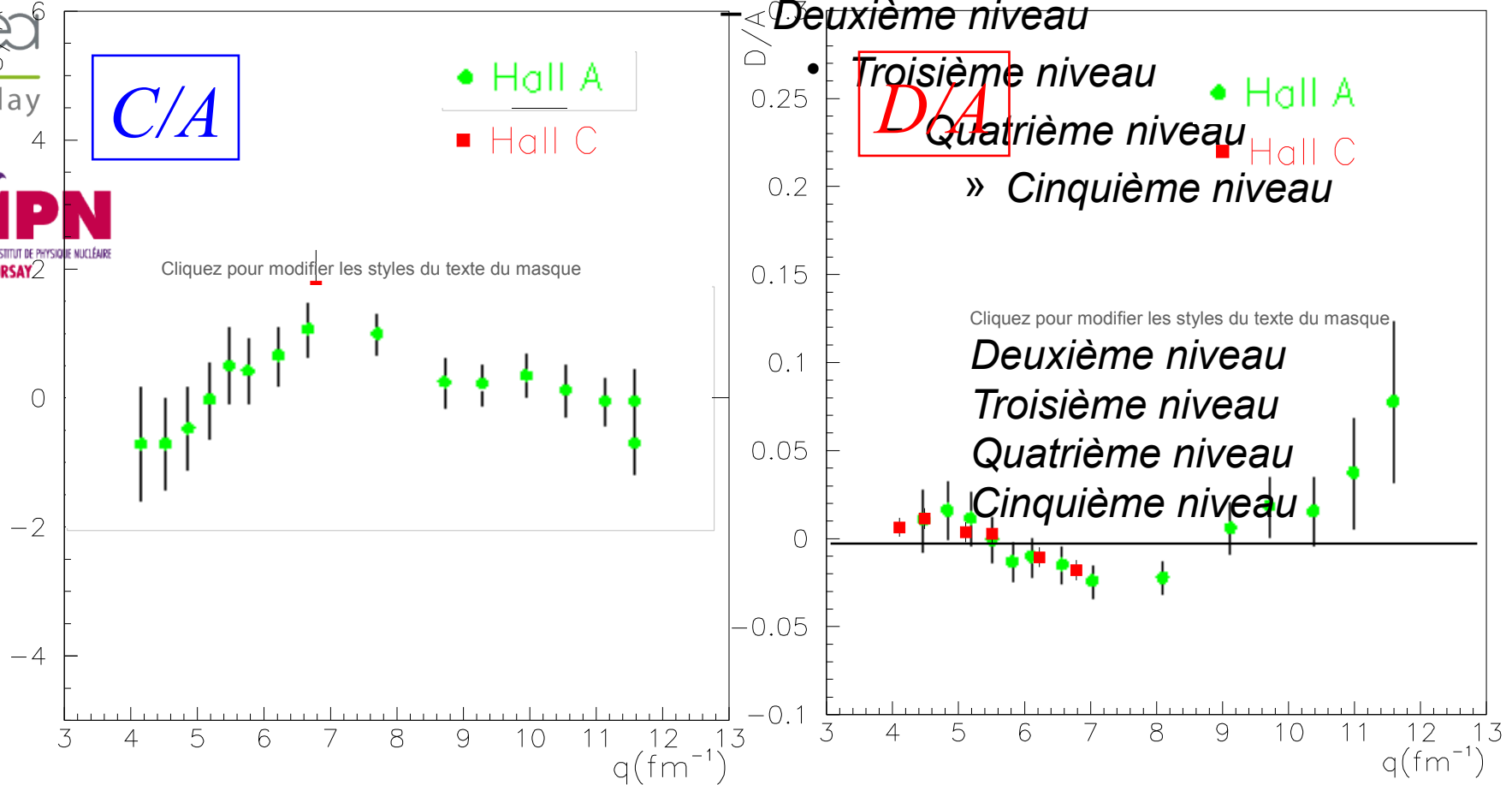
The 1γ - 2γ interference destroys the linearity of the Rosenbluth plot!

What about data?

1 γ -2 γ interference

$$\frac{d\sigma}{d\Omega_e}(e^-h \rightarrow e^-h) = \sigma_0 \left(A \cot^2 \frac{\theta_e}{2} + B + C \cot \frac{\theta_e}{2} + D \cot^3 \frac{\theta_e}{2} + \dots \right)$$

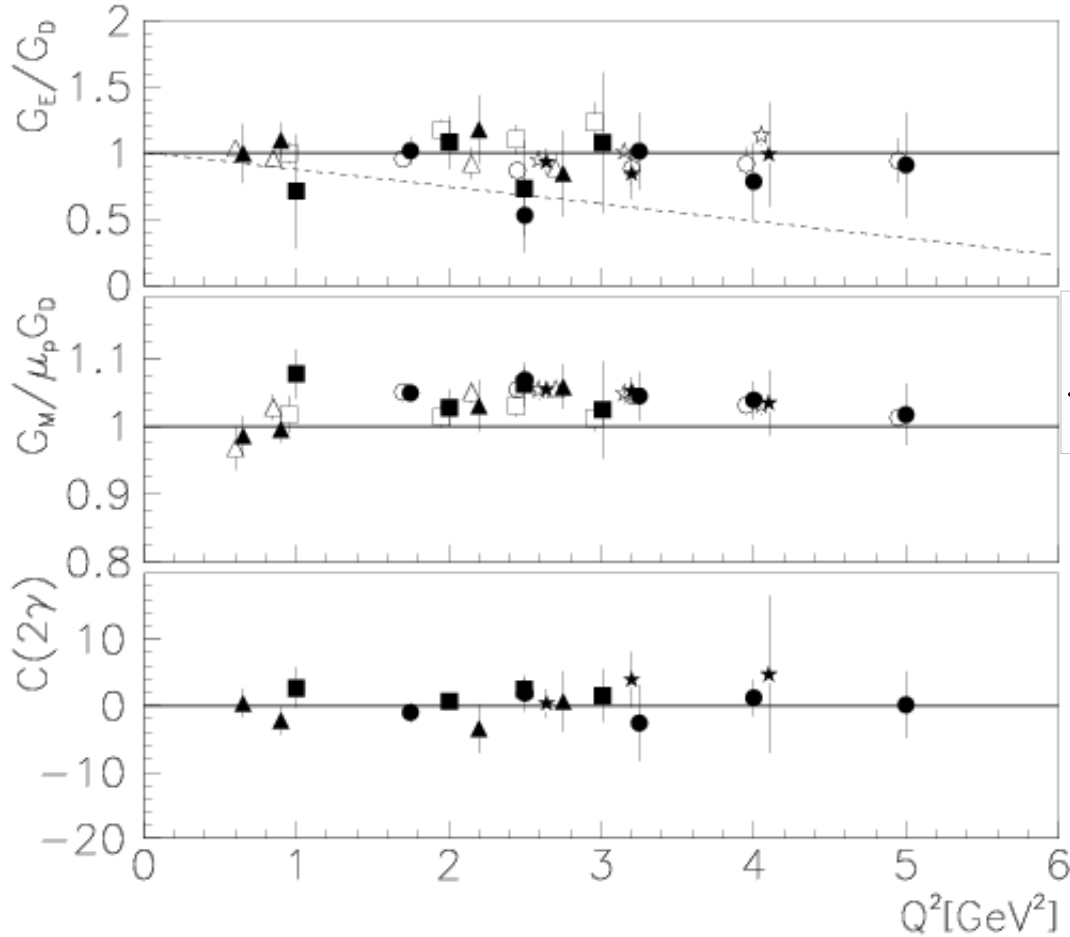
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M. P. Rekaló, E. T-G and D. Prout, Phys. Rev. C60, 042202 (1999)

Parametrization of 2γ -contribution for $e+p$

$\sigma^{red}(Q^2) = C_{2\gamma}(Q^2) G_E(Q^2) G_M(Q^2) F(Q^2, \epsilon)$



$$F(Q^2, \epsilon) \rightarrow \frac{\sqrt{1+\epsilon}}{\sqrt{1-\epsilon}} f^{(a)}(Q^2)$$

$$f^{(a)}(Q^2) = \frac{C_{2\gamma} G_D}{[1 + Q^2 [\text{GeV}^2/m_a^2]^2]}$$

**From the data:
deviation from linearity
 $\ll 1\%$**

E. T.-G., G. Gakh, Phys. Rev. C 72, 015209 (2005)



e+4He scattering

G.I Gakh, and E. T.-G., Nucl.Phys. A838 (2010) 50-60

Spin 0 particle: $F(q^2)$ in Born approximation

$$\frac{d\sigma_{un}^{Born}}{d\Omega} = \frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}} \left[1 + 2\frac{E}{M} \sin^2 \frac{\theta}{2} \right]^{-1} F^2(q^2),$$



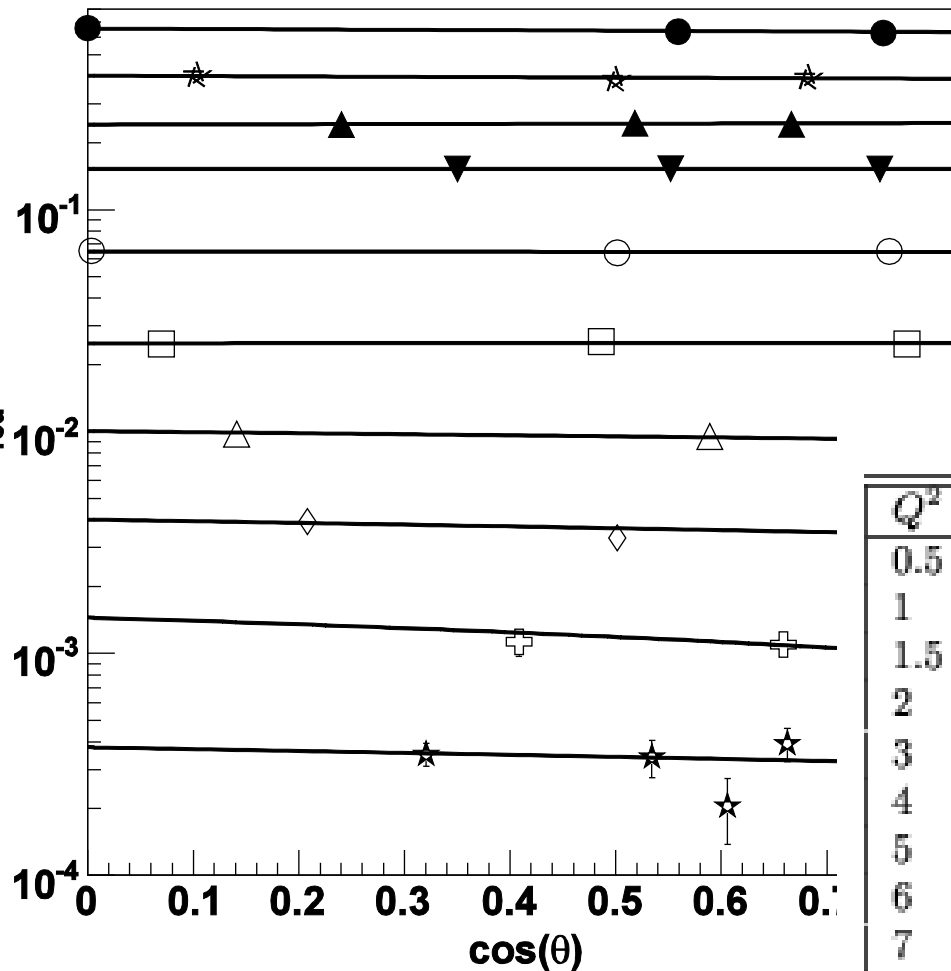
2 γ exchange : $F_1(s, q^2), F_2(s, q^2) = F(q^2) + f(s, q^2)$

$$F(q^2) \sim \alpha^0, F_1(s, q^2) \sim \alpha \quad F_1^{Born}(s, q^2) = 0, \quad F_2^{Born}(s, q^2) = F(q^2),$$

$$\frac{d\sigma_{un}}{d\Omega} = \frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}} \left[1 + 2\frac{E}{M} \sin^2 \frac{\theta}{2} \right]^{-1} \left\{ F^2(q^2) + 2F(q^2) \operatorname{Re} f(s, q^2) + |f(s, q^2)|^2 + \right. \\ \left. + \frac{m^2}{M^2} \left[\frac{M}{E} + \left(1 + \frac{M}{E} \right) \tan^2 \frac{\theta}{2} \right] F(q^2) \operatorname{Re} F_1(s, q^2) \right\}.$$

Linear fit to $e+4\text{He}$ elastic scattering

G.I Gakh, and E. T.-G., Nucl.Phys. A838 (2010) 50-60



$$\sigma_{red}|_{\bar{Q}^2}(\theta) = a + \alpha b \cos \theta.$$

Q^2 [fm^{-2}]	$a \pm \Delta a$	$b \pm \Delta b$	χ^2
0.5	$(66 \pm 4) \text{ E-02}$	-6 ± 9	0.1
1	$(0.40 \pm 3) \text{ E-02}$	-3 ± 8	0.2
1.5	$(0.24 \pm 2) \text{ E-02}$	1.0 ± 0.1	0.1
2	$(15 \pm 2) \text{ E-03}$	0.0 ± 0.1	0.1
3	$(65 \pm 4) \text{ E-03}$	$0. \pm 1$	0.1
4	$(25 \pm 2) \text{ E-03}$	0.0 ± 0.4	0.1
5	$(101 \pm 8) \text{ E-04}$	-0.2 ± 0.2	0.5
6	$(40 \pm 5) \text{ E-04}$	-0.1 ± 0.1	0.6
7	$(15 \pm 3) \text{ E-04}$	-0.09 ± 0.07	1.0
8	$(38 \pm 9) \text{ E-05}$	-0.01 ± 0.03	1.0

Interaction of 4 spin $\frac{1}{2}$ fermions

16 amplitudes in the general case.

- P- and T-invariance of EM interaction,
- helicity conservation,

- One-photon exchange:

- Two form factors (real in SL, complex in TL)
- Functions of one variable (t)

- Two-photon exchange:

- Three (complex) amplitudes
- Functions of two variables (s, t)

*Is it still possible to extract
the « real » FFs in presence
of 2γ exchange?*



Space-like region

Possible but difficult!

Model independent considerations for $e\pm N$ scattering

Determination of EM form factors, in presence of 2γ exchange:

- electron and positron beams
 - longitudinally polarized ,
 - in identical kinematical conditions,

M. P. Rekalov, E. T.-G. , EPJA (2004), Nucl. Phys. A (2003)

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Model independent considerations for $e\pm N$ scattering

Determination of EM form factors, in presence of $2g$ exchange

- electron and positron beams,
- longitudinally polarized ,
- in identical kinematical conditions,

$$\frac{d\sigma^{(-)}}{d\Omega_e} + \frac{d\sigma^{(+)}}{d\Omega_e} = 2\sigma_0 \mathcal{N} = 2\sigma_0 \left[\epsilon G_E^2(Q^2) + \tau G_M^2(Q^2) \right],$$

$$\frac{1}{2} \mathcal{N}(P_x^{(+)} + P_x^{(-)}) = -\lambda_e \sqrt{2\epsilon(1-\epsilon)} \tau G_E(Q^2) G_M(Q^2),$$

$$\frac{1}{2} \mathcal{N}(P_z^{(+)} + P_z^{(-)}) = \lambda_e \tau \sqrt{(1-\epsilon^2)} G_M^2(Q^2),$$

Generalization of the polarization method

(A. Akhiezer and M.P. Rekaló)

M. P. Rekaló and E. T-G Nucl. Phys. A740 (2004) 271,

M. P. Rekaló and E. T-G Nucl. Phys. A742 (2004) 322

If no positron beam...

Either three T-odd polarization observables...

- *A_y*: unpolarized leptons, transversally polarized target or

P_y: outgoing nucleon polarization with unpolarized leptons, unpolarized target

- *Depolarization tensor (D_{ab})*: dependence of the *b*-component of the final nucleon polarization on the *a*-component of the nucleon target with longitudinally polarized leptons

M. P. Rekalov and E. T-G Nucl. Phys. A740 (2004) 271,

M. P. Rekalov and E. T-G Nucl. Phys. A742 (2004) 322

If no positron beam...

Either three T-odd polarization observables...

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$$\begin{aligned}\mathcal{N}P_y = \mathcal{N}A_y &= \sqrt{2\epsilon\tau(1+\epsilon)}\mathcal{I}_3(Q^2, \epsilon) + \sqrt{2\epsilon(1-\epsilon)(1+\tau)}\mathcal{I}_1(Q^2, \epsilon), \\ \mathcal{N}D_{xy}(\lambda_e) = \mathcal{N}D_{yx}(\lambda_e) &= 2\lambda_e\epsilon\sqrt{\tau(1+\tau)}\mathcal{I}_2(Q^2, \epsilon), \\ \mathcal{N}D_{yz}(\lambda_e) = -\mathcal{N}D_{zy}(\lambda_e) &= \lambda_e\sqrt{2\epsilon(1+\tau)(1+\epsilon)}\mathcal{I}_1(Q^2, \epsilon),\end{aligned}$$

$$\mathcal{I}_1(Q^2, \epsilon) = \text{Im}G_E(Q^2)\mathcal{A}(Q^2, \epsilon), \quad \mathcal{I}_2(Q^2, \epsilon) = \text{Im}G_M(Q^2)\mathcal{A}(Q^2, \epsilon),$$

$$\mathcal{I}_3(Q^2, \epsilon) = \text{Im}G_E(Q^2, \epsilon)G_M^*(Q^2, \epsilon).$$

$$\mathcal{R}_{EM}(Q^2) = \frac{\mathcal{I}_1(Q^2, \epsilon)}{\mathcal{I}_2(Q^2, \epsilon)} = \frac{G_E(Q^2)}{G_M(Q^2)}$$

M. P. Rekalo and E. T-G Nucl. Phys. A740 (2004) 271,

M. P. Rekalo and E. T-G Nucl. Phys. A742 (2004) 322

If no positron beam...

This ratio contains the 'TRUE' form factors!

$$\mathcal{R}_{EM}(Q^2) = \frac{\mathcal{I}_1(Q^2, \epsilon)}{\mathcal{I}_2(Q^2, \epsilon)} = \frac{G_E(Q^2)}{G_M(Q^2)}$$

$$\mathcal{R}_{EM}(Q^2) = -\frac{\lambda_e}{D_{xy}(\lambda_e)} \left[P_y + \sqrt{\frac{1+\epsilon}{1-\epsilon}} \frac{D_{zy}(\lambda_e)}{\lambda_e} \right] \sqrt{\frac{1-\epsilon}{2\epsilon}} \tau.$$

Very difficult experiments

Three T-odd polarization observables....

Expected small, of the order of α , triple spin correlations

but... Model independent way



If no positron beam...

Either three T-odd polarization observables....

..or five T-even polarization observables....

among $d\sigma/d\Omega$, $P_x(\lambda_e)$, $P_z(\lambda_e)$, D_{xx} , D_{yy} , D_{zz} ,

D_{xz}


$$\frac{G_E(Q^2)}{G_M(Q^2)} = -\frac{1}{2\epsilon} \sqrt{\tau} \frac{1 - \epsilon^2}{2\epsilon} \frac{\left[\sqrt{1 + \epsilon} \frac{P_x(\lambda_e)}{\lambda_e} - D_{xz} \sqrt{1 - \epsilon} \right]}{1 + \frac{1 - \epsilon}{2\epsilon} D_{xx} - \frac{1 + \epsilon}{2\epsilon} D_{yy}},$$

Again very difficult experiments

Only Model independent ways (without positron beams)

M. P. Rekalo and E. T-G Nucl. Phys. A740 (2004) 271,

M. P. Rekalo and E. T-G Nucl. Phys. A742 (2004) 322

*Is it still possible to extract
the « real » FFs in presence
of 2γ exchange?*



Time-like region

much easier!

Time-like observables: $|G_E|^2$ and $|G_M|^2$

- The cross section for $\bar{p} + p \rightarrow e^+ + e^-$ (1 γ -exchange):

Deuxième niveau
Troisième niveau
Quatrième niveau
Cinquième niveau

$$\frac{d\sigma}{d(\cos \theta)} = \frac{\pi \alpha^2}{8m^2 \sqrt{\tau}} [|G_M|^2 (1 + \cos^2 \theta) + |G_E|^2 \sin^2 \theta]$$

θ : angle between e^- and \bar{p} in cms.

A. Zichichi, S. M. Berman, N. Cabibbo, R. Gatto, Il Nuovo Cimento XXIV, 170 (1962)

B. Bilenkii, C. Giunti, V. Wataghin, Z. Phys. C 59, 475 (1993).

G. Gakh, E.T-G., Nucl. Phys. A761,120 (2005).

As in SL region:

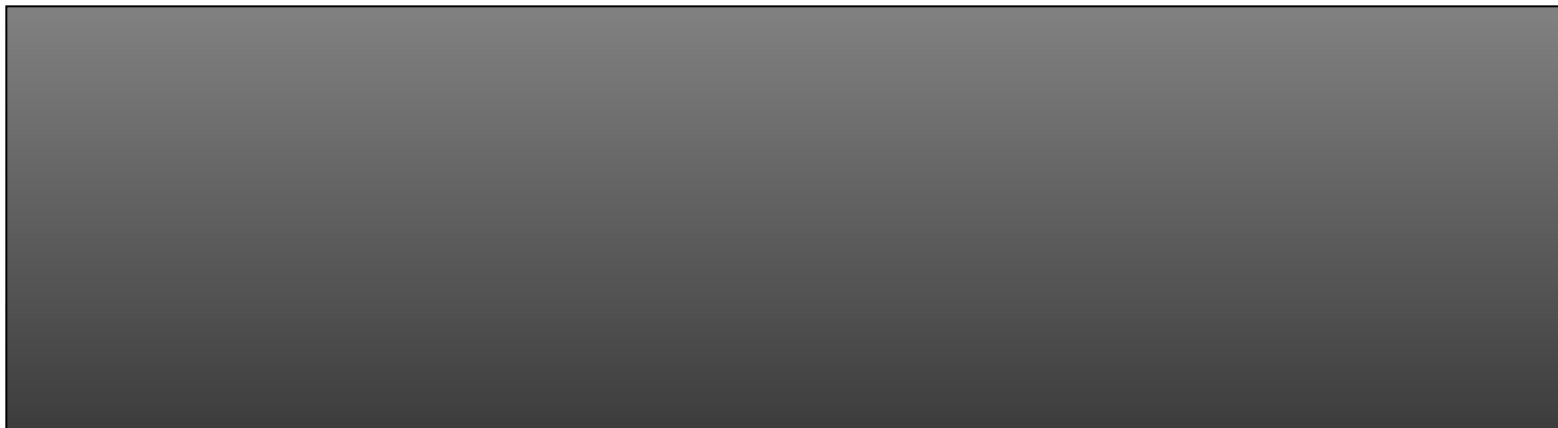
- Dependence on q^2 contained in FFs
- Even dependence on $\cos^2 \theta$ (1 γ exchange)
- No dependence on sign of FFs
- Enhancement of magnetic term

but TL form factors are complex!



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Unpolarized cross section



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Two Photon Exchange:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4q^2} \sqrt{\frac{\tau}{\tau-1}} D$$

- *Induces four new terms*
- *Odd function of θ :*
- *Does not contribute at $\theta = 90^\circ$*

$$D = (1 + \cos^2 \theta)(|G_M|^2 \text{ [redacted]}) + \frac{1}{\tau} \sin^2 \theta (|G_E|^2 \text{ [redacted]}) +$$

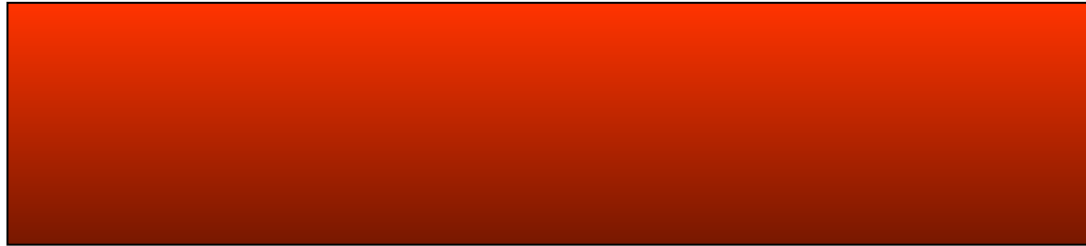
[redacted]

M.P. Rekalo and E. T.-G., EPJA 22, 331 (2004)
G.I. Gakh and E. T.-G., NPA761, 120 (2005)

Symmetry relations

- Properties of the TPE amplitudes with respect to the transformation: $\cos \theta = -\cos \theta$ i.e., $\theta \rightarrow \pi - \theta$

(equivalent to non-linearity in Rosenbluth fit)



- Based on these properties one can *remove* or *single out* TPE contribution

E. T.-G., G. Gakh, NPA (2007)

Symmetry relations (annihilation)

- *Differential cross section at complementary angles:*

The SUM cancels the 2γ contribution:

$$\frac{d\sigma_+}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(\theta) + \frac{d\sigma}{d\Omega}(\pi - \theta) = 2 \frac{d\sigma^{Born}}{d\Omega}(\theta)$$

The DIFFERENCE enhances the 2γ contribution:

$$\frac{d\sigma_-}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(\pi - \theta) = 4N \left[(1 + x^2) \text{Re}G_M \Delta G_M^* + \frac{1 - x^2}{\tau} \text{Re}G_E \Delta G_E^* + \sqrt{\tau(\tau - 1)} x (1 - x^2) \text{Re}\left(\frac{1}{\tau} G_E - G_M\right) F_3^* \right]$$

$$\tau = \frac{q^2}{4m^2}, \quad x = \cos\theta$$

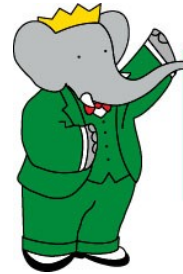
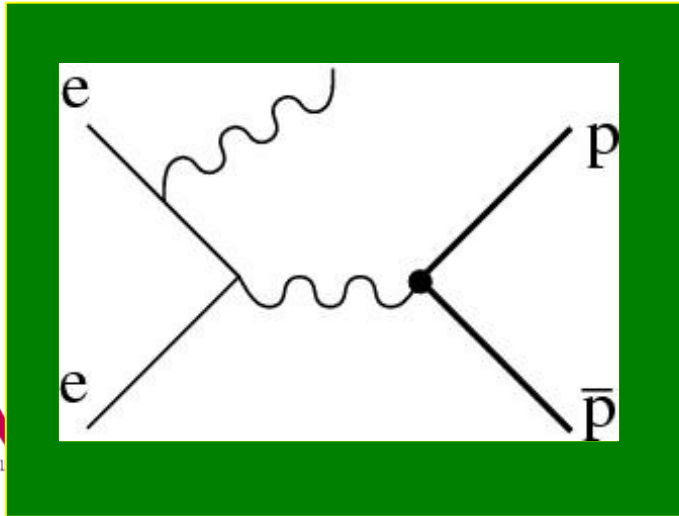
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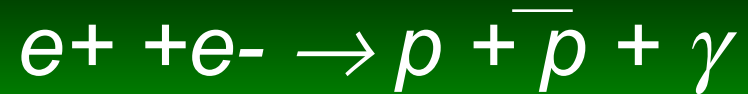
What do data say?

Radiative Return (ISR)



BABAR

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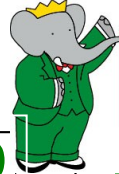


$$\frac{d\sigma(e^+ e^- \rightarrow p\bar{p}\gamma)}{dm d\cos\theta} = \frac{2m}{s} W(s, x, \theta) \sigma(e^+ e^- \rightarrow p\bar{p})(m), \quad x = \frac{2E_\gamma}{\sqrt{s}} = 1 - \frac{m^2}{s},$$

$$W(s, x, \theta) = \frac{\alpha}{\pi x} \left(\frac{2 - 2x + x^2}{\sin^2\theta} - \frac{x^2}{2} \right), \quad \theta \gg \frac{m_e}{\sqrt{s}}.$$

B. Aubert (BABAR Collaboration) Phys Rev. D73, 012005 (2006)

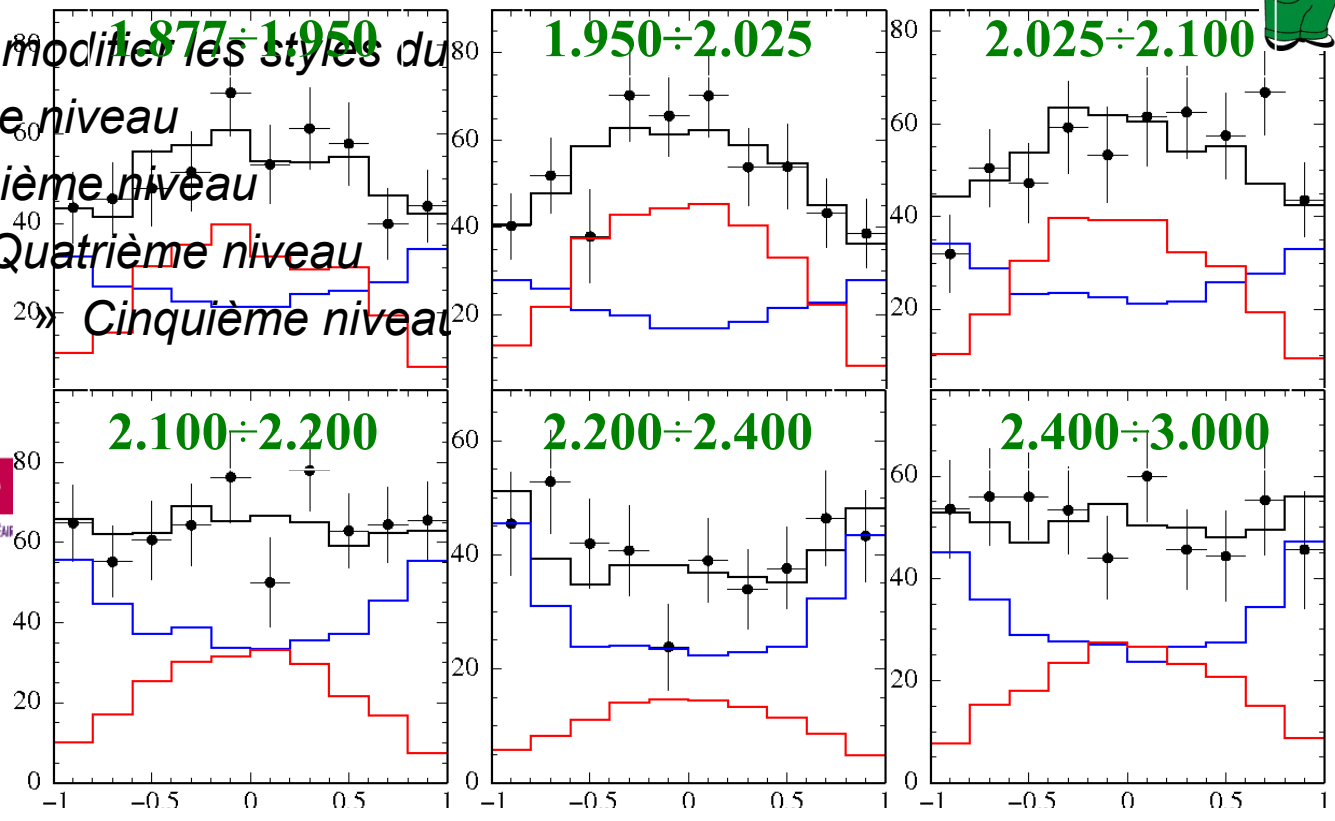
Angular distribution



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Events/0.2 vs. $\cos \theta$



$$\frac{dN}{d \cos \theta_p} = A \left[\text{[Redacted]} - \left| \frac{G_E}{G_M} \right|^2 H_E(\cos \theta, M_{pp}) \right]$$

2 γ -exchange?

pour modifier les styles du
 Deuxième niveau
 Troisième niveau
 Quatrième niveau
 Cinquième niveau



Angular Asymmetry

Cliquez pour modifier les styles du texte du masque

– Deuxième niveau

m r f u • Troisième niveau

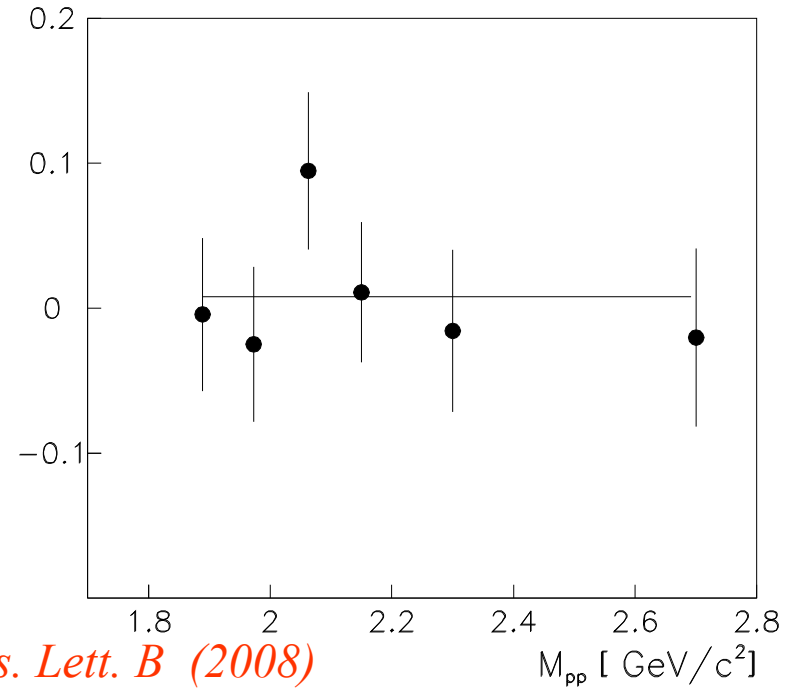
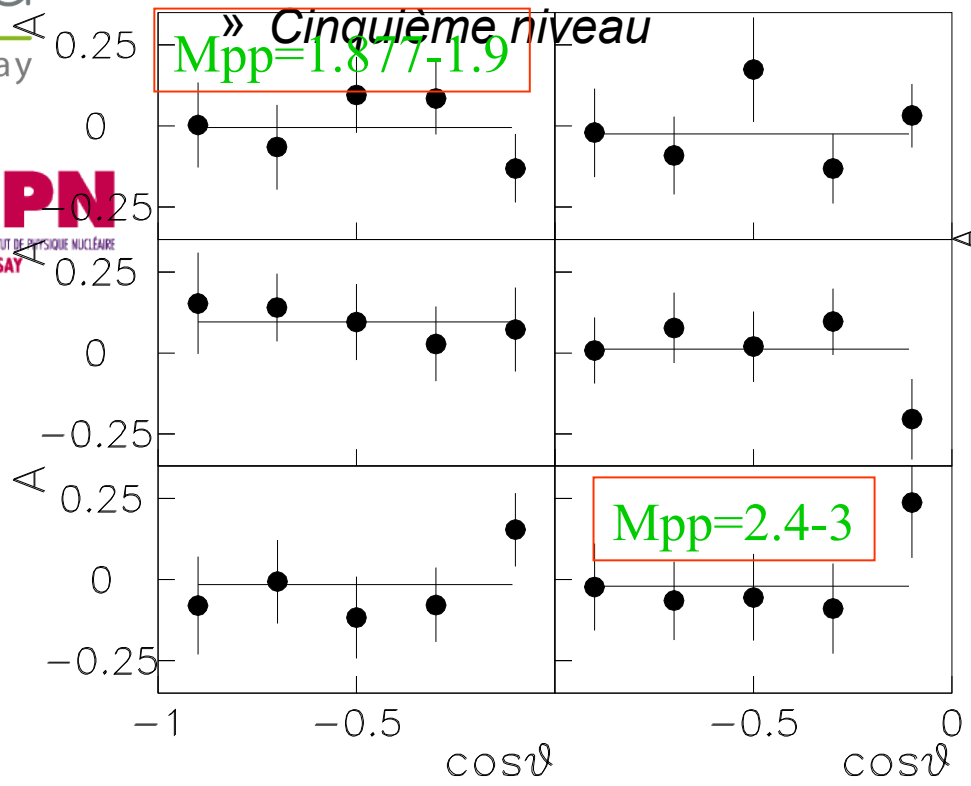
ceea Quatrième niveau

saclay » Cinquième niveau



$$A(c) = \frac{\frac{d\sigma}{d\Omega}(c) - \frac{d\sigma}{d\Omega}(-c)}{\frac{d\sigma}{d\Omega}(c) + \frac{d\sigma}{d\Omega}(-c)}$$

$$A = 0.01 \pm 0.02$$



E. T.-G., E.A. Kuraev, S. Bakmaev, S. Pacetti, Phys. Lett. B (2008)

Structure Function method

$e^+ + e^- \rightarrow p + p + \gamma$ • Cliquez pour modifier les styles du texte du n

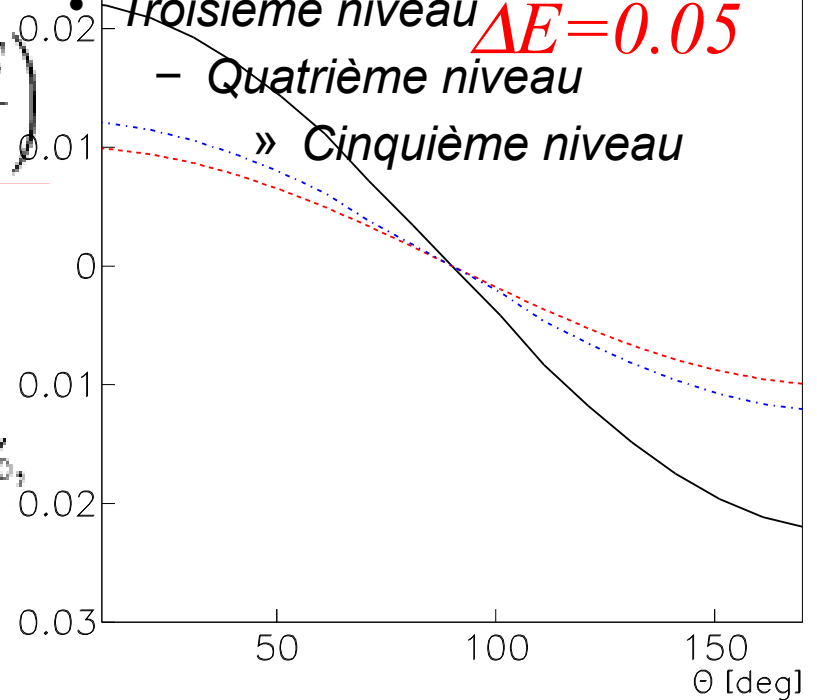
- Deuxième niveau

• Troisième niveau $\Delta E = 0.05$

- Quatrième niveau

» Cinquième niveau

$$A^{soft}(E) \simeq \frac{2\alpha}{\pi} \left(\ln \frac{1 + \beta c}{1 - \beta c} \ln \frac{\Delta E}{E} \right)$$



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$$A^{tot} = A^{soft} + A^{hard} = \frac{2\alpha}{\pi} \psi(c, \beta), \quad |A^{tot}| \leq 2\%$$

$$\frac{d\sigma}{d\Omega}(c) \pm \frac{d\sigma}{d\Omega}(-c) \sim \int dx_1 \mathcal{D}(x_1, L) \mathcal{D}(x_2, L) dx_2 \left(1 + \frac{\alpha}{\pi} K \right)$$

$$\frac{d\sigma}{d\Omega}(c) + \frac{d\sigma}{d\Omega}(-c) = 2 \frac{d\sigma_0}{d\Omega} \left[1 + \frac{\alpha}{\pi} \left(\frac{3}{2} L - 2(L-1) \ln \frac{\Delta E}{E} + \frac{\pi^2}{3} - 2 \right) \right], \quad L = \ln \frac{t}{m^2}$$

E. T.-G., E.A. Kuraev, S. Bakmaev, S. Pacetti, Phys. Lett. B (2008)

Fitting the angular distributions...

The form of the differential cross section:



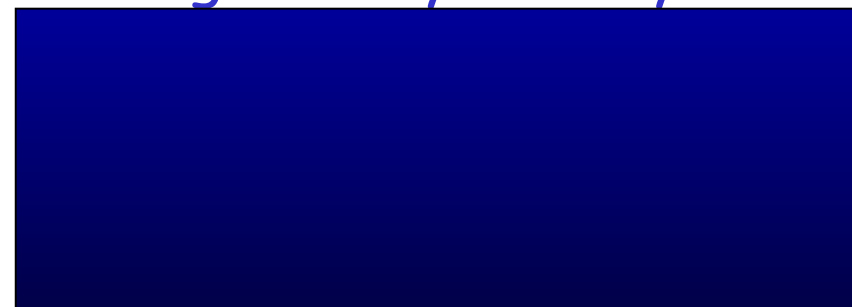
is equivalent to:



Cross section at 900

$$\sigma_0 = \frac{\alpha^2}{4q^2} \sqrt{\frac{\tau}{\tau - 1}} \left(|G_M|^2 + \frac{1}{\tau} |G_E|^2 \right)$$

Angular asymmetry



Fitting the angular distributions...

1 γ exchange:



→ Linear Fit in $\cos 2\theta$

$$y = a_0 + a_1 x \text{ with } x = \cos^2 \theta, a_0 = \sigma_0, a_1 = \sigma_0 A$$

2 γ exchange → Quadratic Fit in $x = \cos \theta$

$$y = a_0 + a_2 x + a_1 x^2$$

$$a_2 = \frac{2\sqrt{\tau(\tau-1)}(G_E - \tau G_M)F_3}{\tau|G_M|^2 + |G_E|^2}$$

E. T-G. and M. P. Rekalo, Phys. Lett. B 504, 291 (2001)

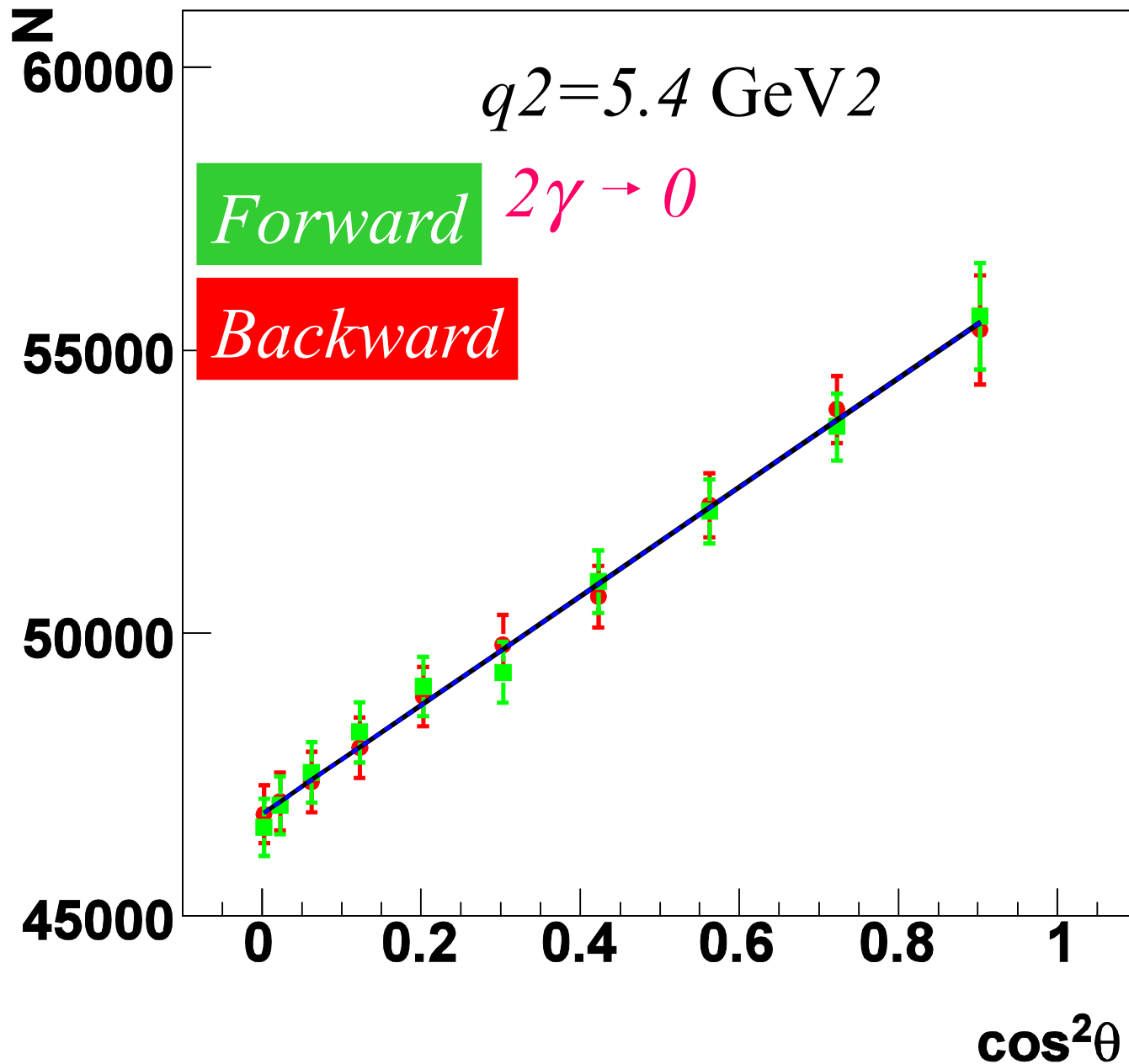
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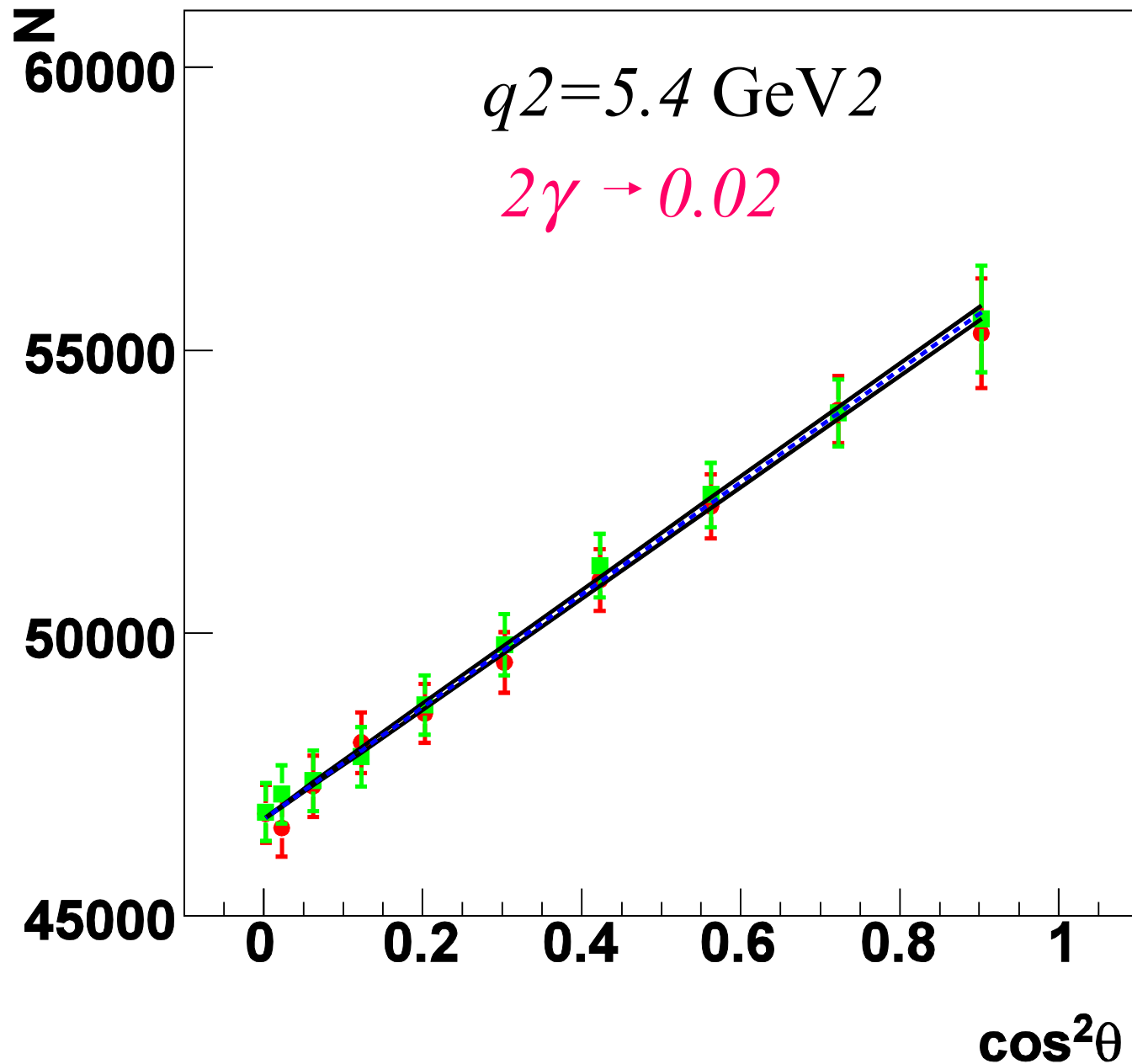
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Fitting the angular distributions...



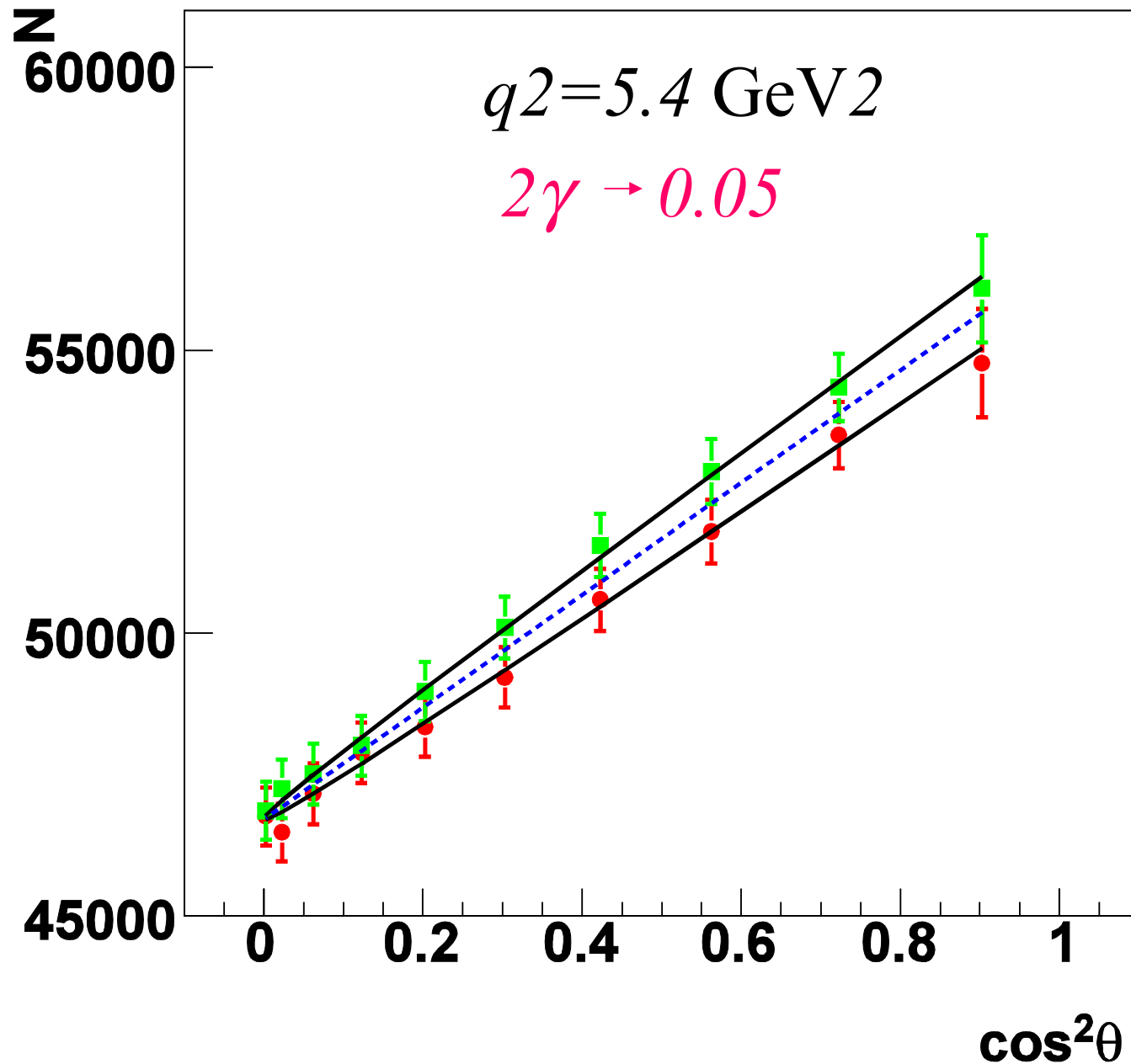
Fitting the angular distributions...



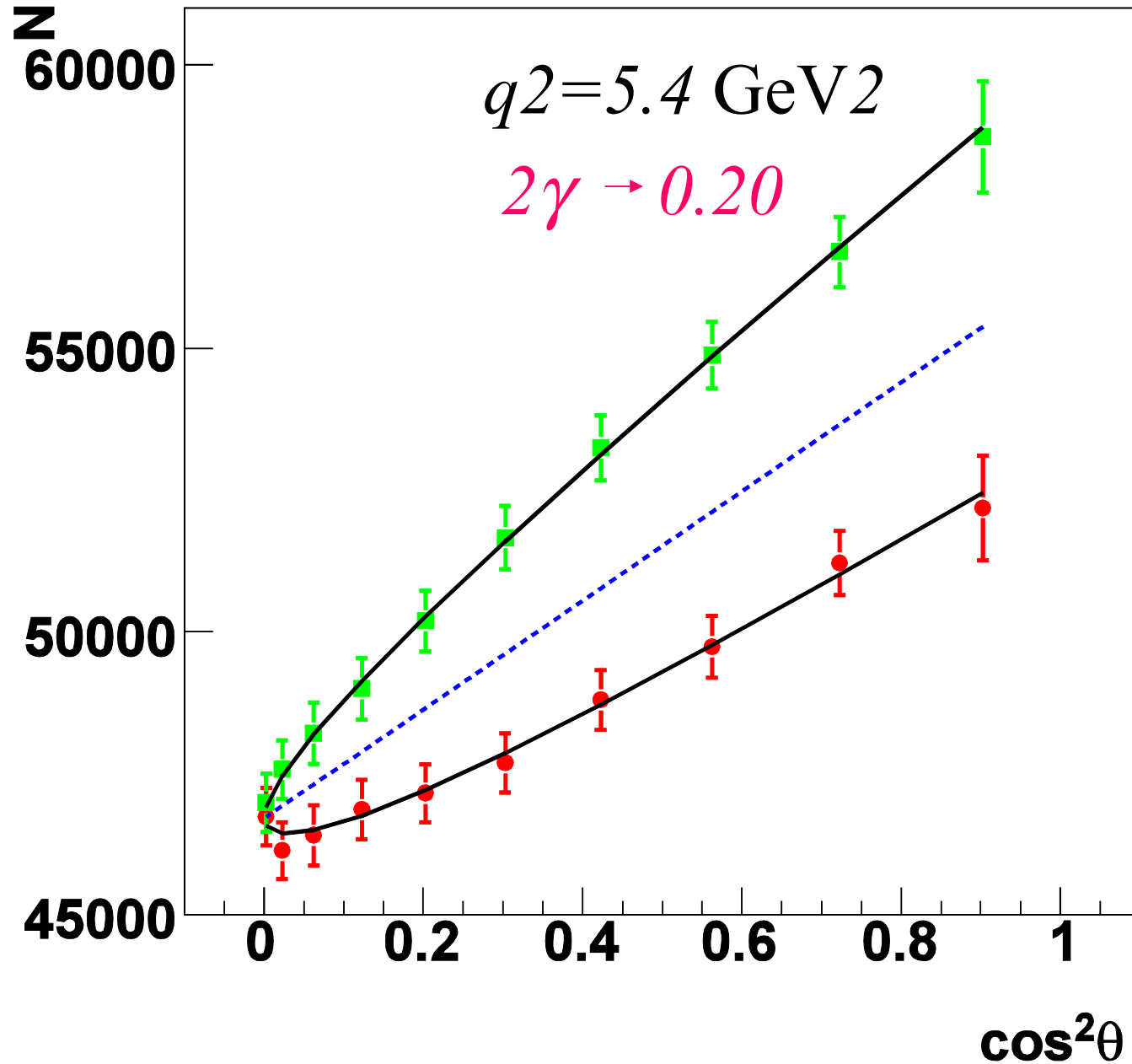
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Fitting the angular distributions...



Fitting the angular distributions ...



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ceci
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Which alternative for Gep?

Polarization experiments - Jlab

A.I. Akhiezer and M.P. Rekalo, 1967

Gep collaboration

1) "standard" dipole function for the nucleon magnetic FFs
 G_M^p and G_M^n

2) linear deviation from the dipole function for the electric proton FF
 G_E^p

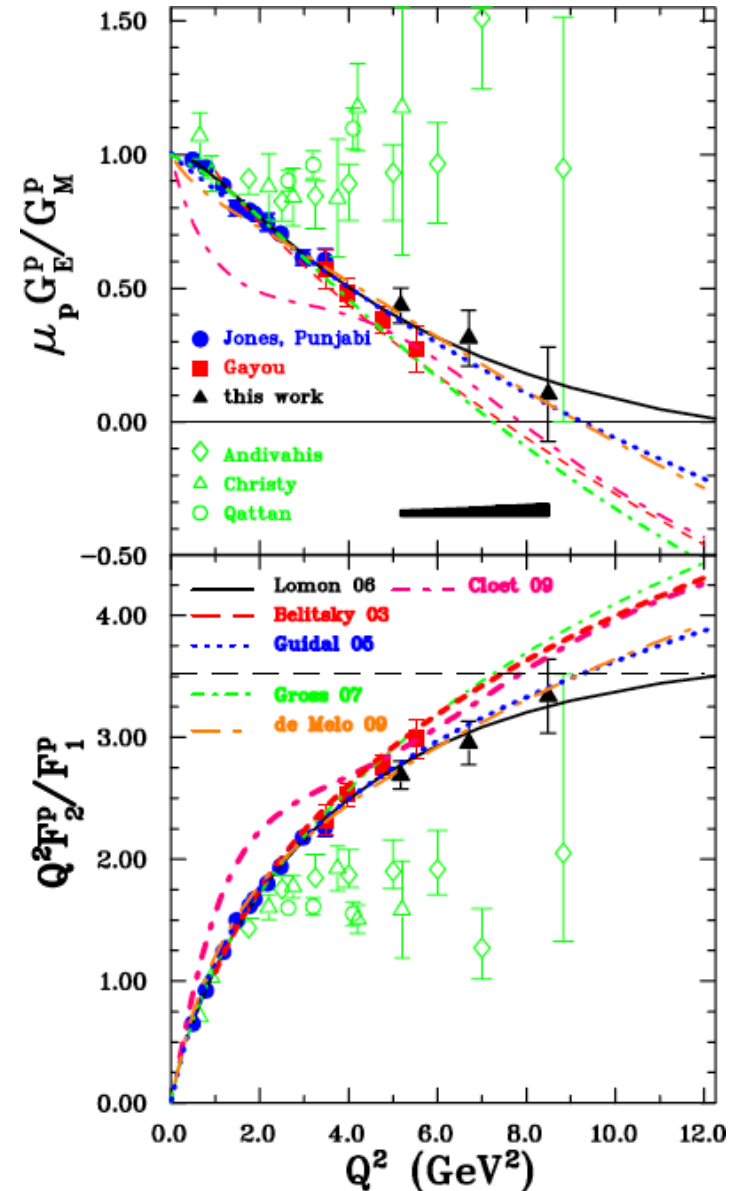
3) QCD scaling not reached

3) Zero crossing of G_E^p ?

4) contradiction between polarized and unpolarized measurements

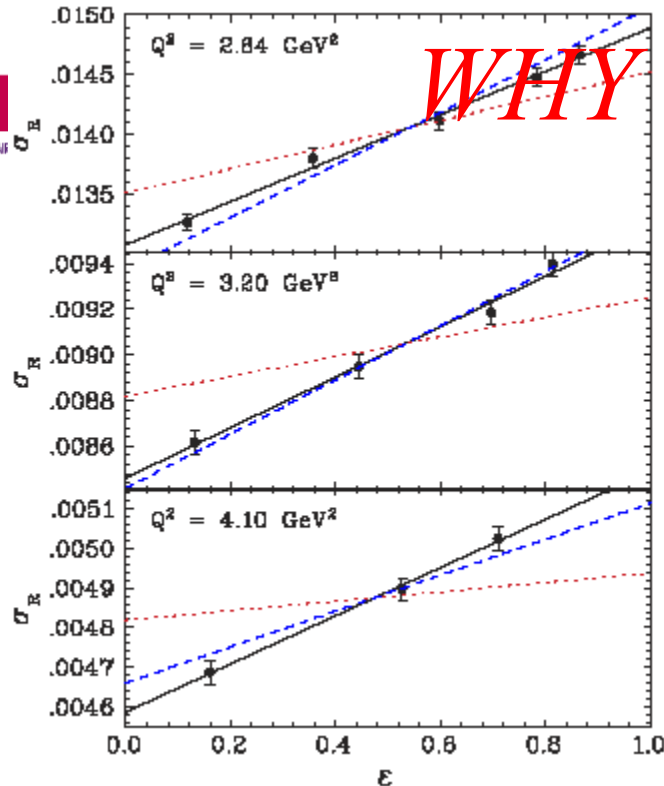
A.J.R. Puckett et al, PRL (2010)

PRC85 (2012) 045203

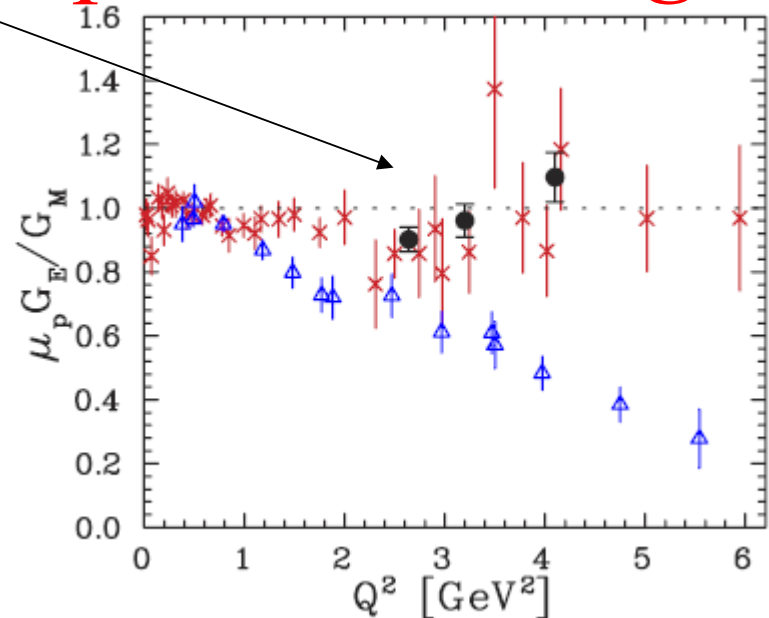


Precision Rosenbluth Measurement of the Proton Elastic Form Factors

I. A. Qattan,^{1,2} J. Arrington,² R. E. Segel,¹ X. Zheng,² K. Aniol,³ O. K. Baker,⁴ R. Beams,² E. J. Brash,⁵ J. Calarco,⁶ A. Camsonne,⁷ J.-P. Chen,⁸ M. E. Christy,⁴ D. Dutta,⁹ R. Ent,⁸ S. Frullani,¹⁰ D. Gaskell,¹¹ O. Gayou,¹² R. Gilman,^{13,8} C. Glashauser,¹³ K. Hafidi,² J.-O. Hansen,⁸ D. W. Higinbotham,⁸ W. Hinton,¹⁴ R. J. Holt,² G. M. Huber,⁵ H. Ibrahim,¹⁴ L. Jisonna,¹ M. K. Jones,⁸ C. E. Keppel,⁴ E. Kinney,¹¹ G. J. Kumbartzki,¹³ A. Lung,⁸ D. J. Margaziotis,³ K. McCormick,¹³ D. Meekins,⁸ R. Michaels,⁸ P. Monaghan,⁹ P. Moussiegt,¹⁵ L. Pentchev,¹² C. Perdrisat,¹² V. Punjabi,¹⁶ R. Ransome,¹³ J. Reinhold,¹⁷ B. Reitz,⁸ A. Saha,⁸ A. Sarty,¹⁸ E. C. Schulte,² K. Slifer,¹⁹ P. Solvignon,¹⁹ V. Sulkosky,¹² K. Wijesooriya,² and B. Zeidman²



WHY these points are aligned?



Rosenbluth separation

Contribution of the *electric term*

$$\epsilon = \frac{1}{1 + 2(1 + \tau) \tan^2 \frac{\theta_e}{2}},$$

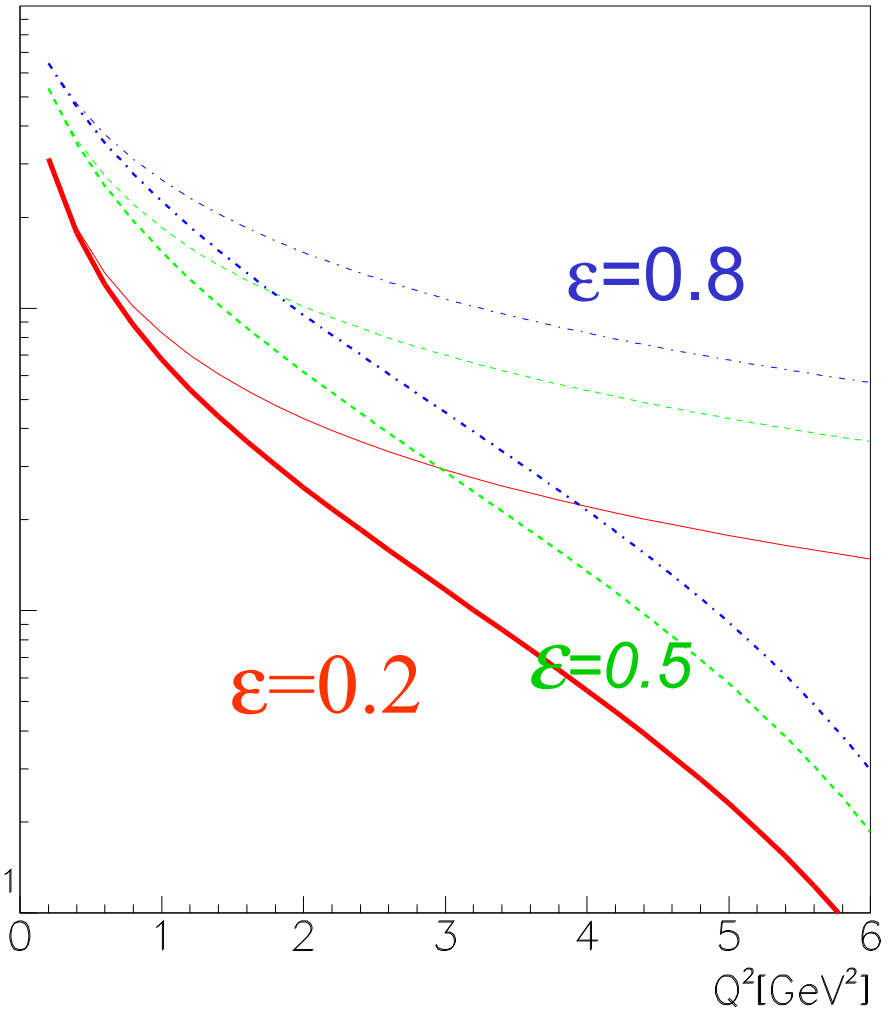
$$\tau = \frac{q^2}{4m^2}$$

$$F_E / \sigma_{red} \times 100$$

10

1

10⁻¹



$\epsilon=0.8$

$\epsilon=0.2$

$\epsilon=0.5$

$$\sigma_{red} = \tau G_{MP}^2 + \epsilon G_{EP}^2$$

...to be compared to the absolute value of the error on σ and to the size and ϵ dependence of RC

Reduced cross section and RC

• Cliquez pour modifier les styles du texte du masque

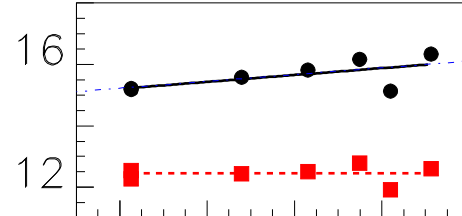
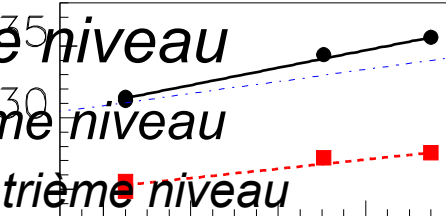
Data from J. Andivahis et al. Phys. Rev. D50, 5491 (1994)

- Deuxième niveau

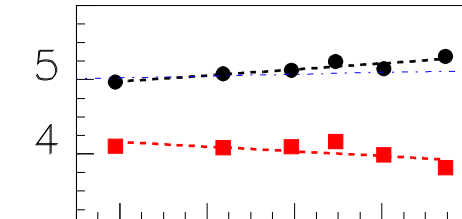
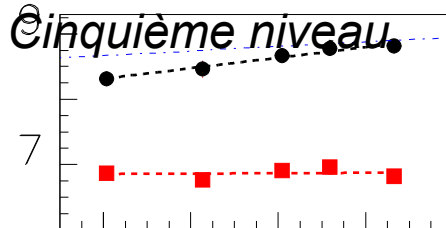
$Q^2=1.75 \text{ GeV}^2$

Troisième niveau

- Quatrième niveau



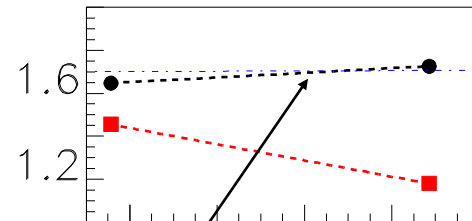
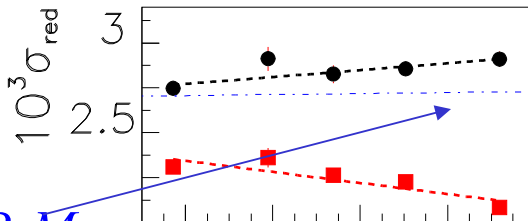
$Q^2=2.5 \text{ GeV}^2$



$Q^2=4 \text{ GeV}^2$

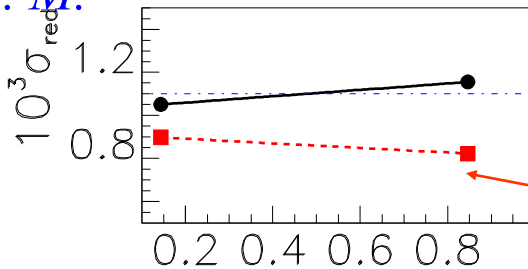
$Q^2=5 \text{ GeV}^2$

Slope from P. M.



$Q^2=6 \text{ GeV}^2$

$Q^2=7 \text{ GeV}^2$



Radiative Corrected data

Raw data without RC

E. T.-G., G. Gakh Phys. Rev. C (2005)

ϵ

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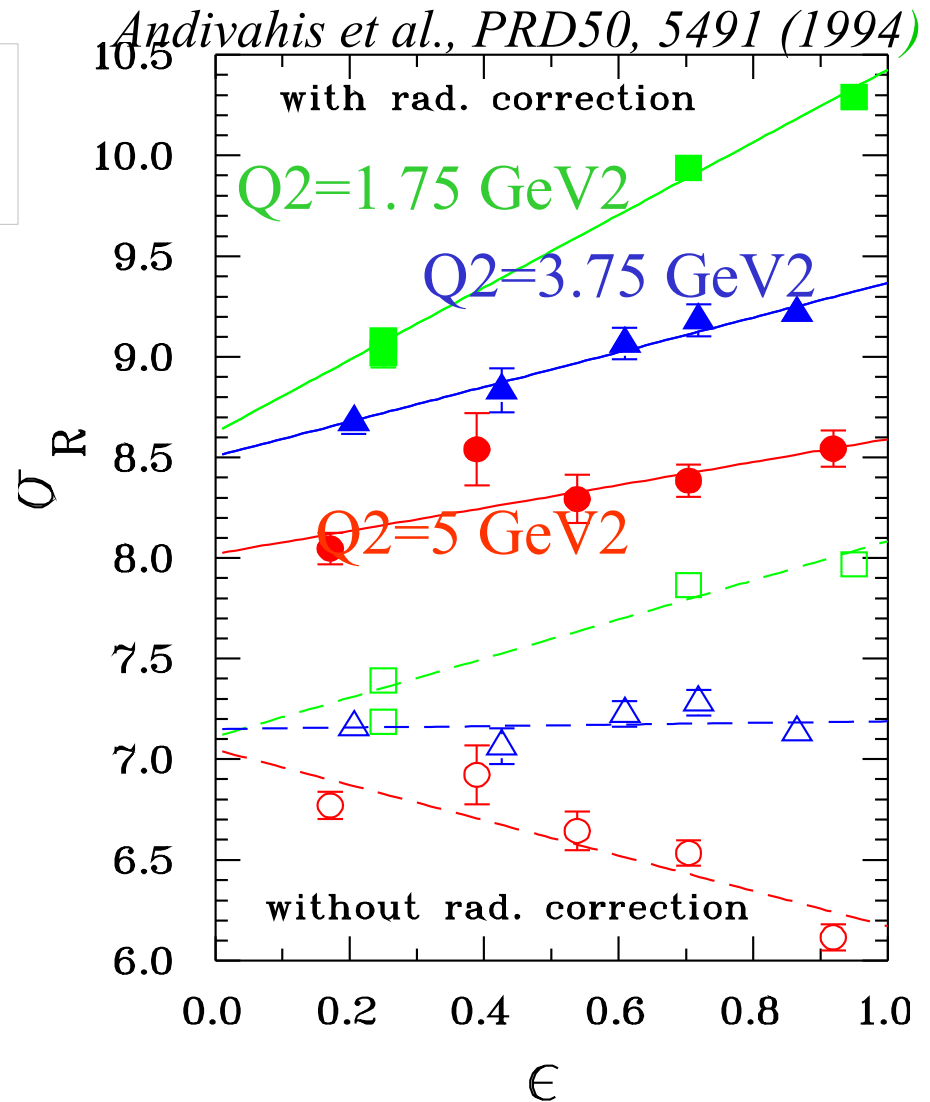


Radiative Corrections (ep)

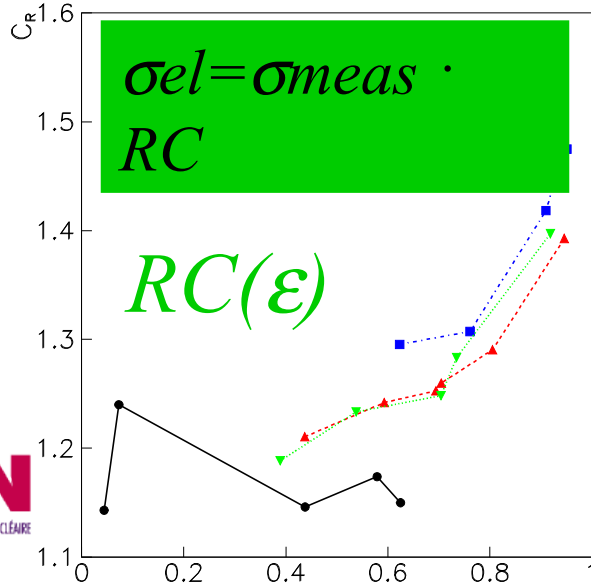
$$\sigma_R = \varepsilon G_{\frac{E}{E_0}}^2 + \tau G_M^2$$

May change
the slope of σ_R
(and even the
sign !!!)

RC to the cross section:
– large (may reach 40%)
– ε and Q^2 dependent
– calculated at first order



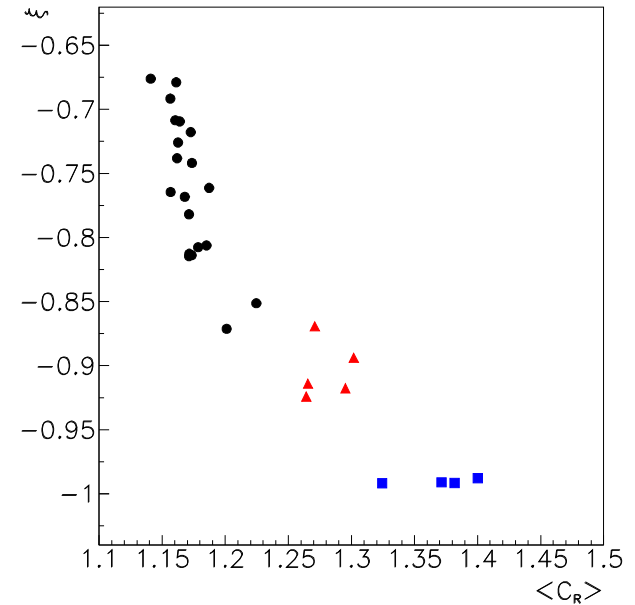
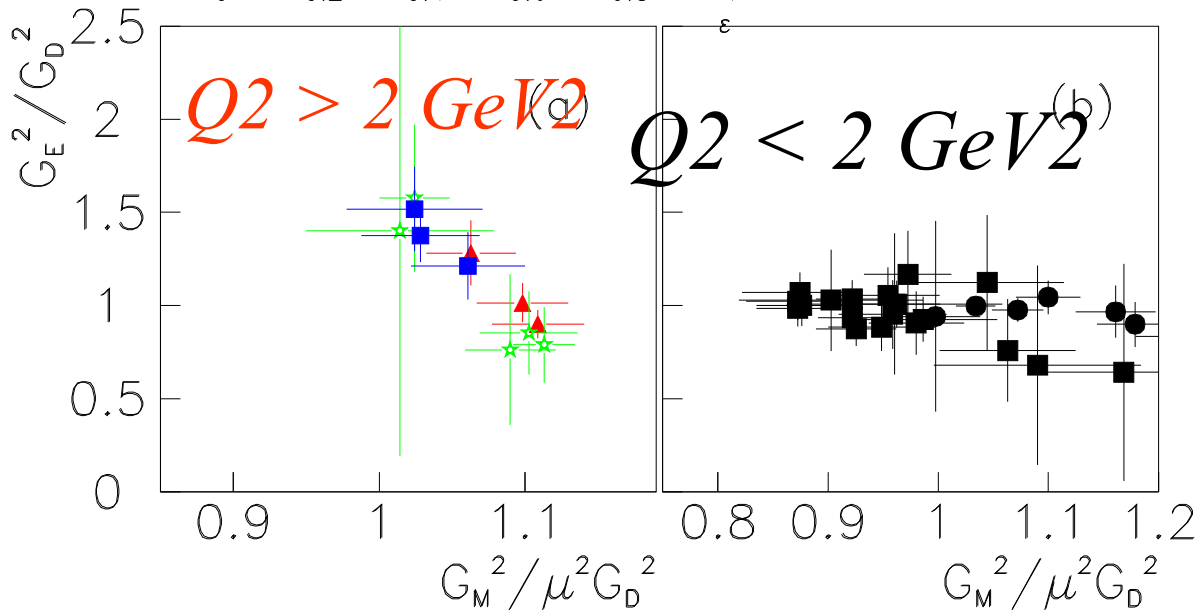
Experimental correlation



$$\sigma_{red} = \tau G_{MP}^2 + \epsilon G_{EP}^2$$

only published values!!

Correlation ($\langle RC \cdot \epsilon \rangle$)



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Scattered electron energy

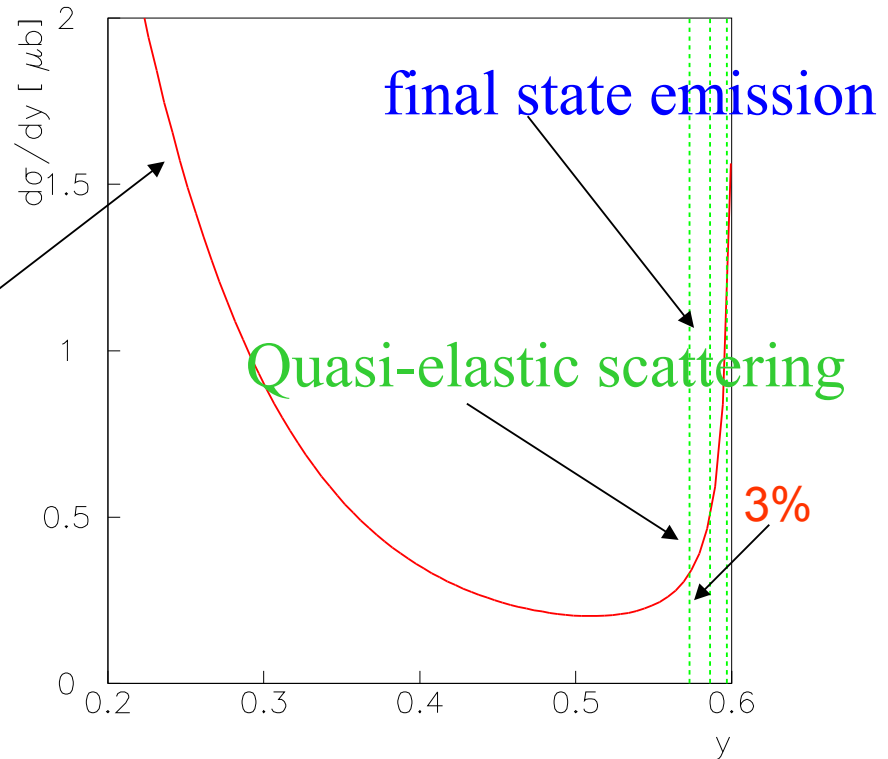
$$E'/E = \gamma ; \gamma_0 = \frac{1}{\beta}$$

$$\beta = 1 + \frac{2E}{m} \frac{\hbar^2 \theta^2}{2}$$

Initial state emission

$$\Delta \frac{d\sigma}{d\Omega} \sim \frac{d\sigma_0}{d\Omega} \cdot \frac{2}{\pi} \ln \frac{E}{\Delta E} \ln \frac{2EM}{m_e^2}$$

Not so small!



Y0

Shift to LOWER Q2

All orders of PT needed →
beyond Mo & Tsai approximation

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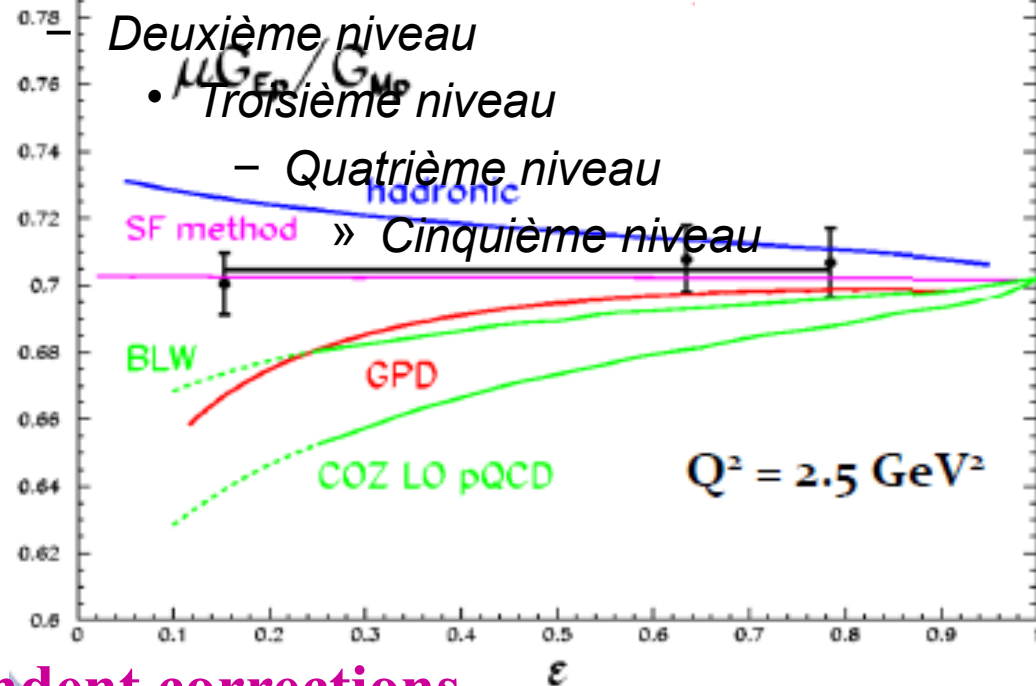
Polarization ratio (ε -dependence)

- **DATA: No evidence of ε -dependence at 1% level**

- **MODELS: large correction (opposite sign) at small ε**

- **SF method: ε -independent corrections**

- Cliquez pour modifier les styles du texte du masque



- **Theory: corrections to the Born approximation at $Q^2 = 2.5 \text{ GeV}^2$**

Y. Bystritskiy, E.A. Kuraev and E.T.-G, Phys.Rev.C75: 015207 (2007)

P. Blunden et al., Phys. Rev. C72:034612 (2005) (mainly GM)

A. Afanasev et al., Phys. Rev. D72:013008 (2005) (mainly GE)

N.Kivel and M.Vanderhaeghen, Phys. Rev. Lett.103:092004 (2009). (high Q^2)

Summary

Our Suggestion for search of 2γ effects:

- Search for *model independent statements* (M.P. Rekalo, G. Gakh..)
- Exact calculation in frame of **QED** ($p \sim \mu$)
- Prove that **QED box** is upper limit of **QCD box** diagram
- Study *analytical properties* of the Compton amplitude
- Compare *to experimental data*

Our Conclusions for elastic ep scattering

- Two photon contribution *is negligible* (real part) (E.A. Kuraev)
- Radiative corrections are huge: take into account *higher order effects* (Structure Functions method) (Yu. Bystricky)

Look for Multiple Photon Exchange in e - A scattering

- Small angle e , or p or $pbar$ - Heavy ion scattering

E.A. Kuraev, M. Shatnev, E.T-G., PRC80 (2009) 018201

2γ effects are expected to be larger in TL region
(complex nature)

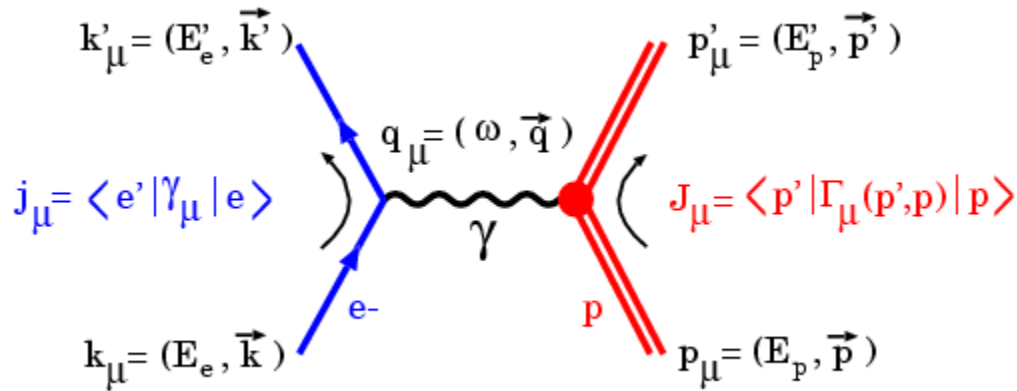
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The *Pauli* and *Dirac* Form Factors



- The electromagnetic current in terms of the **Pauli** and **Dirac** FFs:

$$\Gamma_{\mu}(p', p) = \underbrace{F_1(Q^2)}_{Dirac} \gamma_{\mu} + \frac{i\kappa_p}{2M_p} \underbrace{F_2(Q^2)}_{Pauli} \sigma_{\mu\nu} q^{\nu}$$

- Related to **the Sachs** FFs :

$$G_E(Q^2) = F_1(Q^2) - \kappa_p \frac{Q^2}{4M_p^2} F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + \kappa_p F_2(Q^2)$$

Normalization

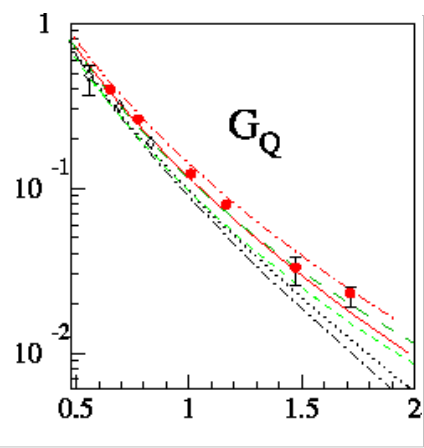
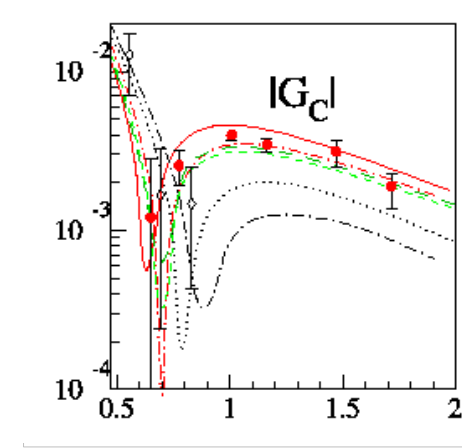
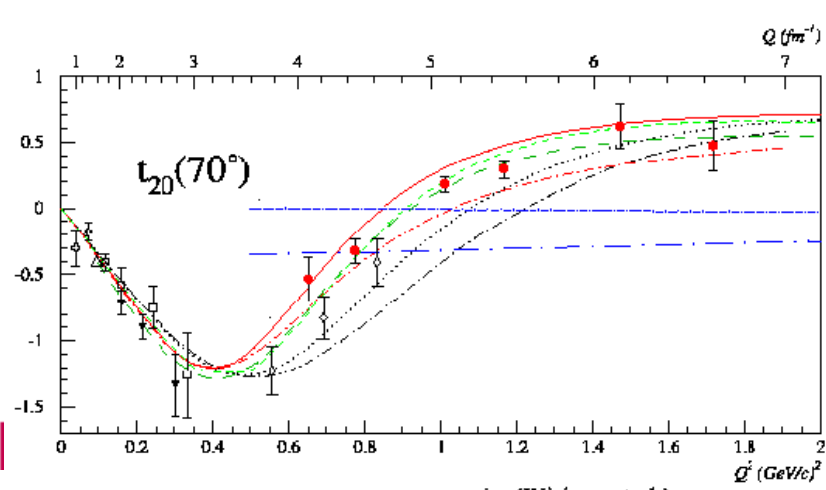
$$F_{1p}(0)=1, \quad F_{2p}(0)=\kappa_p$$

$$G_{Ep}(0)=1,$$

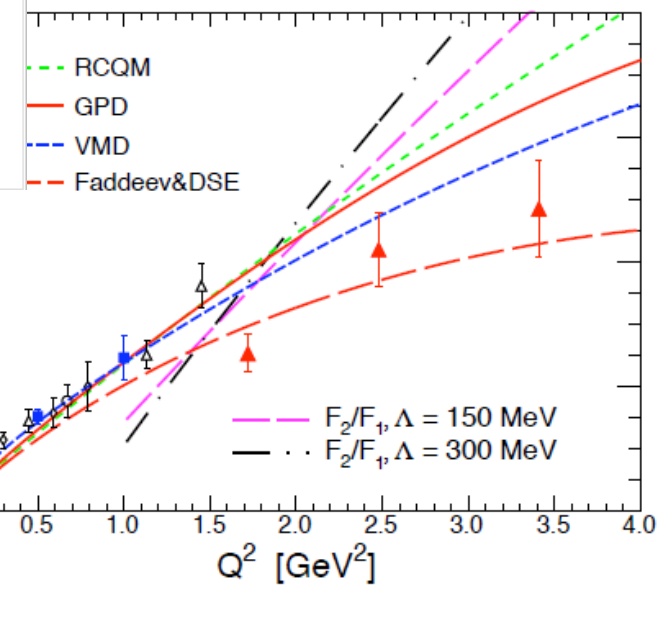
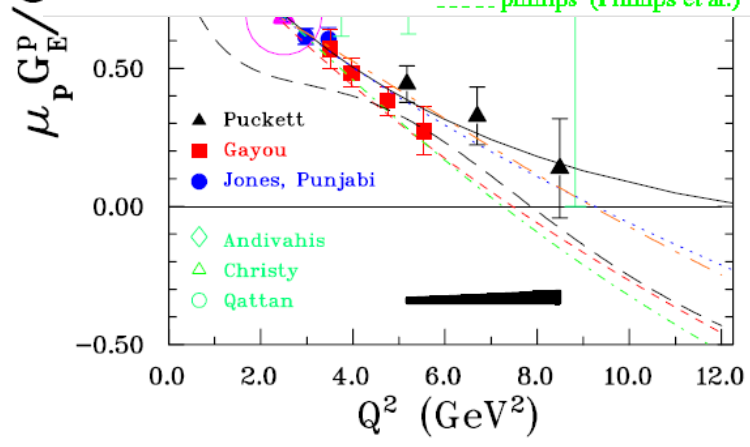
$$G_{Mp}(0)=\mu_p=2.79$$

Systematics

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- Bates (1984)
- ⊕ Novosibirsk (1985)
- Novosibirsk (1990)
- ◇ Bates (1991)
- △ Nikhef (1996)
- ▽ Nikhef (1999)
- JLab Hall C (2000)
- nria (Wiringa et al.)
- nria+mec+rc (Wiringa et al.)
- nria (Arenhovel et al.)
- - - nria+mec+rc (Arenhovel et al.)
- pqed (Brodsky et al.)
- - - pqed (Kobushkin et al.)
- lfd (Carbonell et al.)
- phillips (Phillips et al.)



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Differential cross section (SF)

$$\bar{p}(p_-) + p(p_+) \rightarrow e_+(y_+) + e_-(y_-) + (\gamma(k))$$

Energy fractions of the leptons

$$\begin{aligned} \frac{d\sigma}{dc dy_+ dy_-} &= \int dx_+ dx_- \mathcal{D}(x_+, L_s) \mathcal{D}(x_-, L_s) \\ &\times \frac{d\sigma_B(x_- p_-, x_+ p_+, z_+, z_-)}{dc} \frac{1}{|\Pi(sx_+x_-)|^2} \\ &\times \left(1 + \frac{\alpha}{\pi} K\right) \frac{1}{z_+ z_-} \mathcal{D}\left(\frac{y_+}{z_+}, L_e\right) \\ &\times \mathcal{D}\left(\frac{y_-}{z_-}, L_e\right) + \left(\frac{d\sigma}{dc}\right)^{\text{odd}}, \end{aligned}$$

K-factor

Partition function

Odd term

The structure function of the lepton

$$\mathcal{D}(x, L) = \frac{1}{2} b(1-x)^{(b/2)-1} \left(1 + \frac{3b}{8}\right)$$

$$- \frac{1}{4} b(1+x) + O(b^2),$$

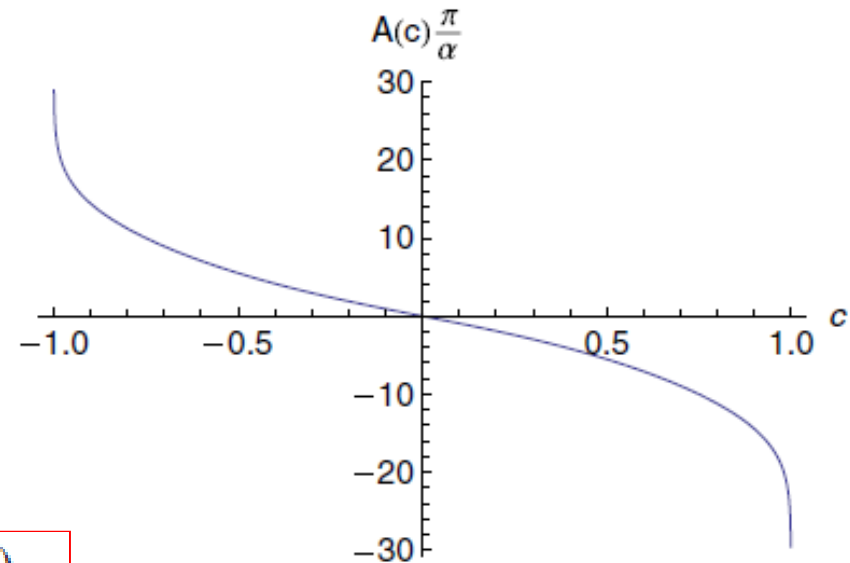
$$b = \frac{2\alpha}{\pi} (L - 1).$$

Charge Asymmetry

$$\frac{d\sigma^{\text{odd}}}{dc} = \frac{\alpha^3}{2s} F(c),$$

$$\begin{aligned} F(c) = & c \left(-6 - \frac{\pi^2}{3} + 2 \ln \frac{2}{1+c} \ln \frac{2}{1-c} + \ln \frac{4}{1-c^2} \right) \\ & + 3(1-2c^2) \ln \frac{1+c}{1-c} + \frac{6}{1-c} \\ & \times \left(-1 + \frac{2}{1-c} \ln \frac{2}{1+c} \right) - \frac{6}{1+c} \\ & \times \left(-1 + \frac{2}{1+c} \ln \frac{2}{1-c} \right) + 4(1+c^2) \\ & \times \left[\text{Li}_2 \left(\frac{1-c}{2} \right) - \text{Li}_2 \left(\frac{1+c}{2} \right) \right]. \end{aligned}$$

$$A(c) = \frac{d\sigma(c) - d\sigma(-c)}{d\sigma(c) + d\sigma(-c)} = \frac{\alpha}{\pi} \frac{F(c)}{1+c^2}$$

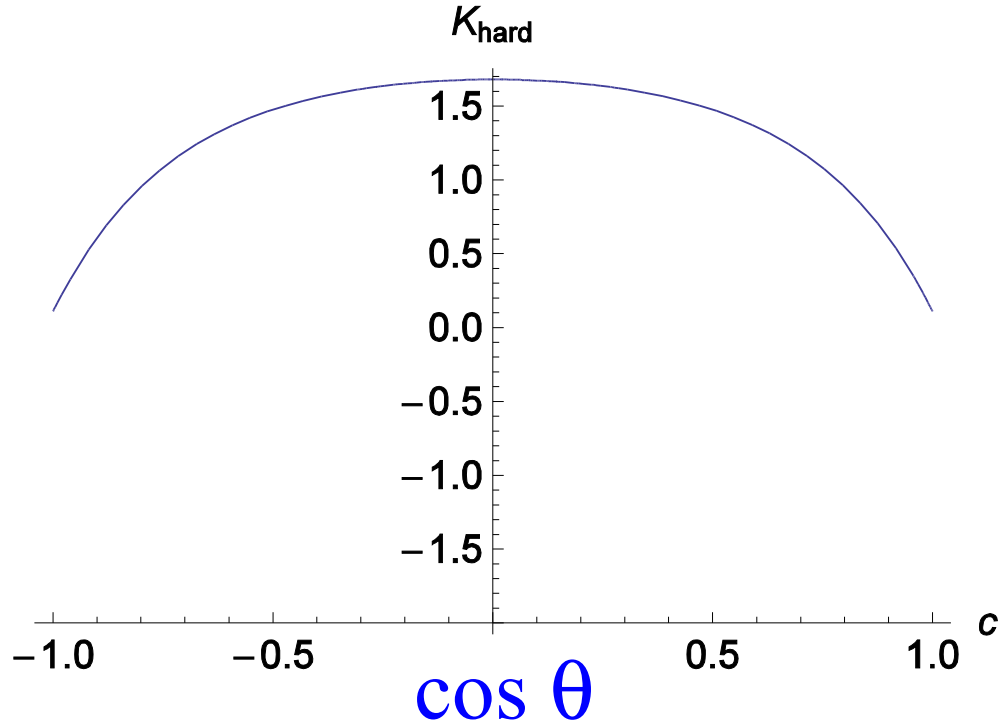


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K-factor (hard)



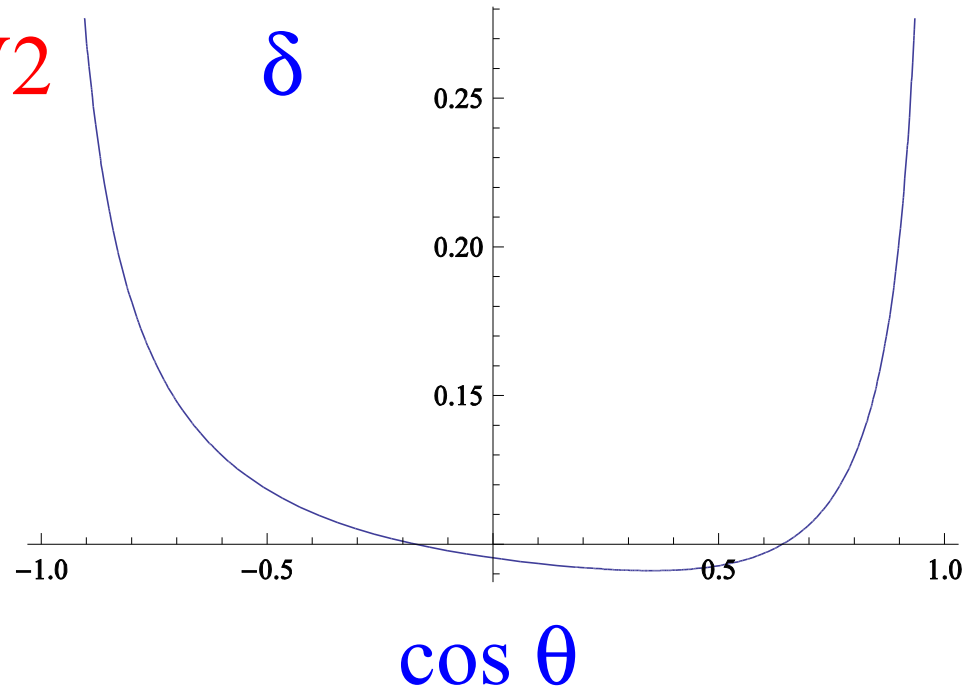
- 1. of the order of one*
- 2. from the even part of the cross section*

Radiative correction factor

$$N_{\text{corr}} = N_{\text{raw}}(1 + \delta)$$

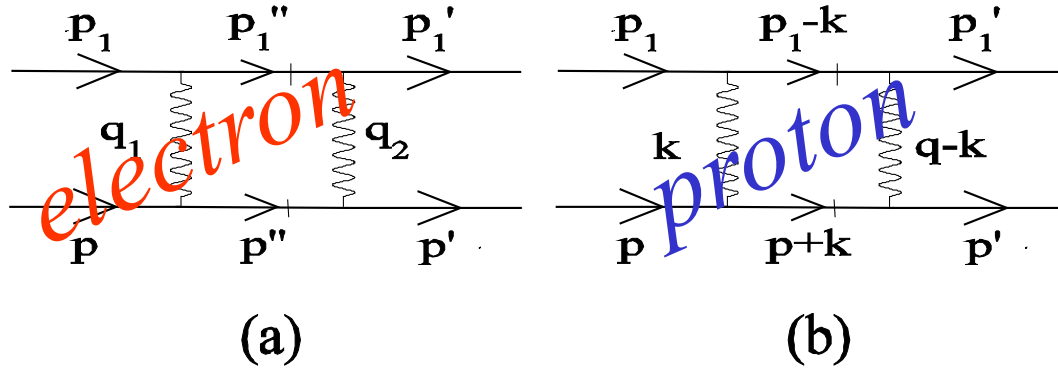
$s = 10 \text{ GeV}^2$

δ



Integrated in all phase space for γ
Proton structureless

QED versus QCD



Imaginary part of the 2γ amplitude

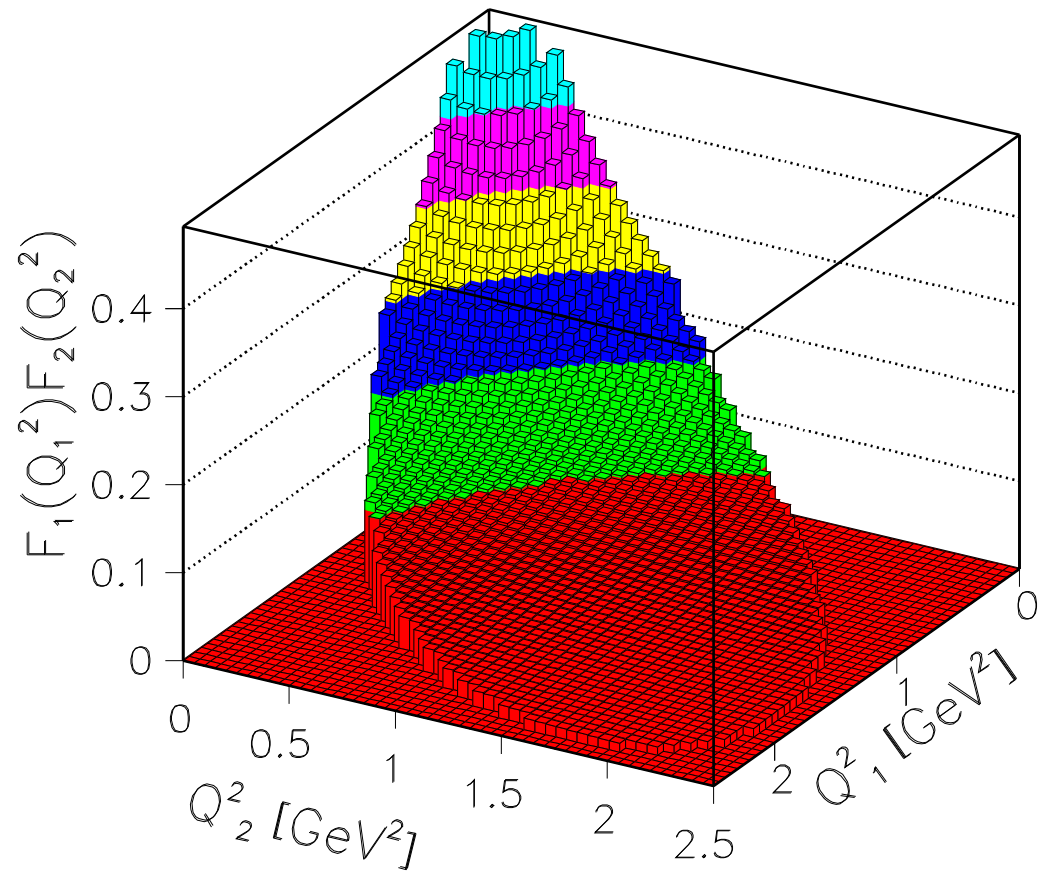
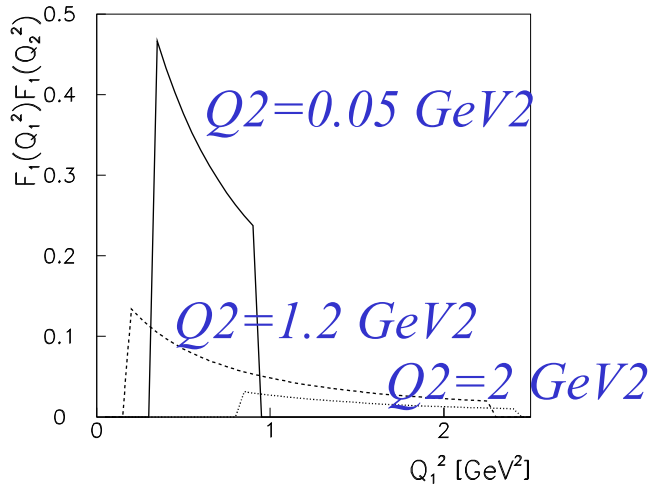
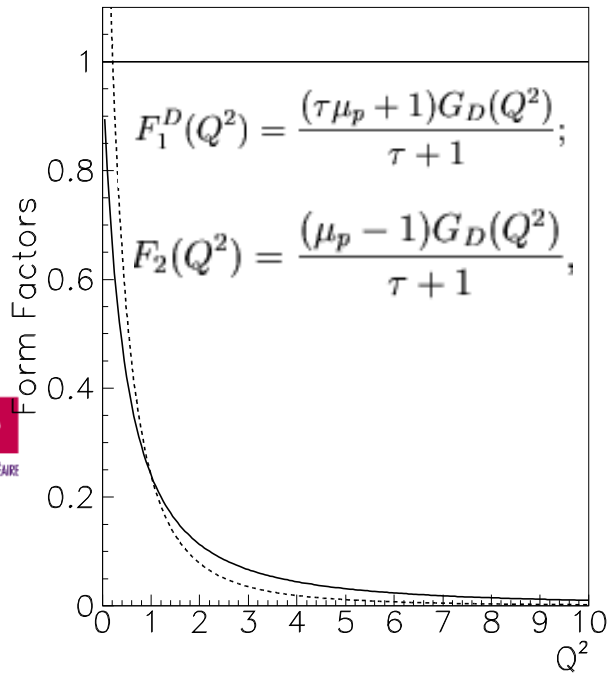
$$\mathcal{M}_{1a} = \frac{1}{\sqrt{8s}} \int \frac{dQ_1^2 dQ_2^2}{\sqrt{\mathcal{D}_1} (Q_1^2 + \lambda^2) (Q_2^2 + \lambda^2)}$$

$$\mathcal{M}_{1b} = \frac{1}{\sqrt{8s}} \int \frac{dQ_1^2 dQ_2^2 F(Q_1^2) F(Q_2^2)}{\sqrt{\mathcal{D}_1} (Q_1^2 + \lambda^2) (Q_2^2 + \lambda^2)}$$

$$dO_1'' = \frac{2dQ_1^2 dQ_2^2}{\sqrt{\mathcal{D}_1} Q_0^2}, \quad \mathcal{D}_1 = 2(Q_1^2 + Q_2^2)Q^2 Q_0^2 - 2Q^2 Q_1^2 Q_2^2 - (Q_1^2 - Q_2^2)Q_0^2 - (Q^2)^2 Q_0^2$$

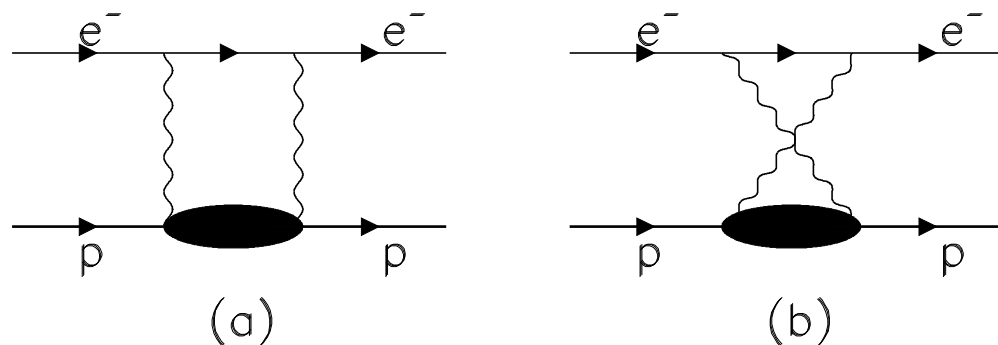
QED versus QCD

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- Cliquez pour modifier les styles du texte du ma
 - Deuxième niveau
 - Troisième niveau
 - Quatrième niveau
 - » Cinquième niveau

Interference of $1\gamma \otimes 2\gamma$ exchange



- *Explicit calculation for structureless proton*
 - *The contribution is small, for unpolarized and polarized ep scattering*
 - *Does not contain the enhancement factor L*
 - *The relevant contribution to K is ~ 1*
- *$e\mu$ (elastic) scattering is upper limit for ep*

E.A.Kuraev, V. Bytev, Yu. Bystricky, E.T-G, Phys. Rev. D74 013003 (1076)

Simulations

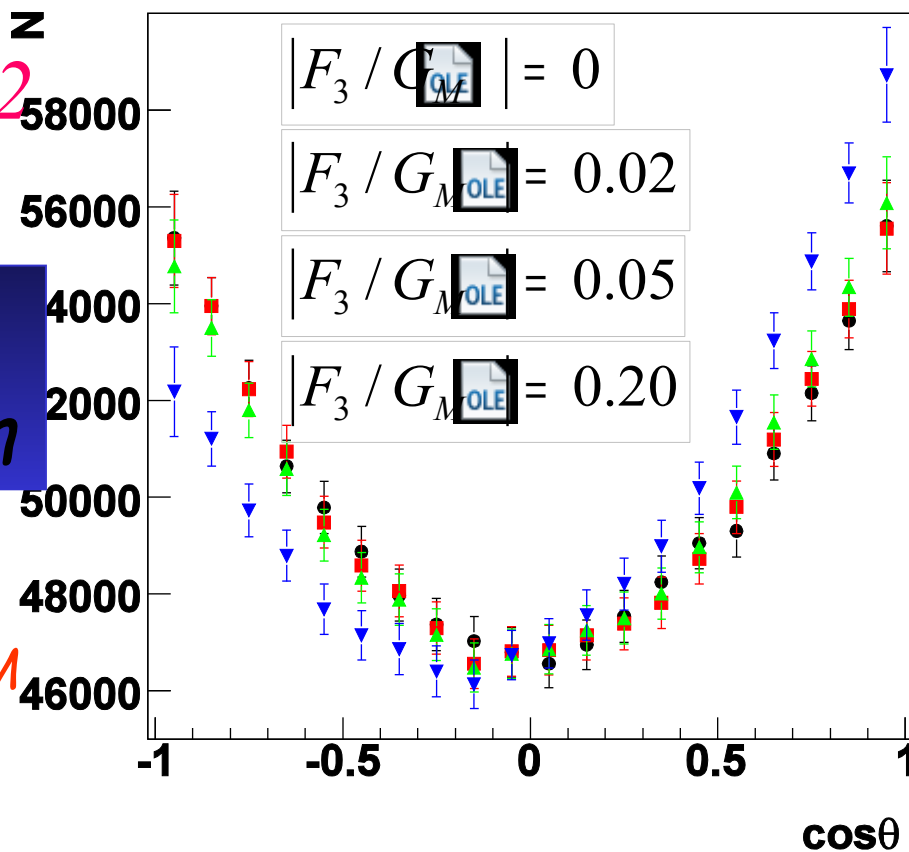
$$D = (1 + \cos^2 \theta)(|G_M|^2 + 2\text{Re}G_M \Delta G_M^*) + \frac{1}{\tau} \sin^2 \theta(|G_E|^2 + 2\text{Re}G_E \Delta G_E^*) + 2\sqrt{\tau(\tau - 1)} \cos \theta \text{Re}\left(\frac{1}{\tau}G_E - G_M\right)F_3^*$$

$q^2 = 5.4, 8.2, 13.8 \text{ GeV}^2$

Main effect:
odd $\cos\theta$ -distribution

Approximations:

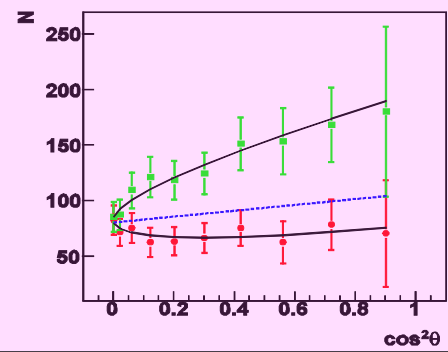
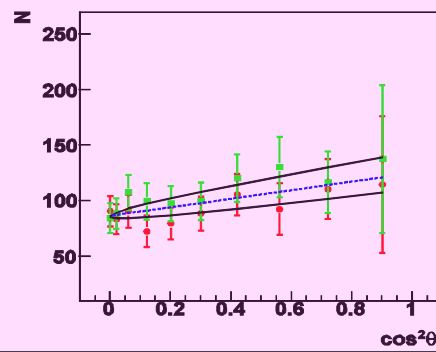
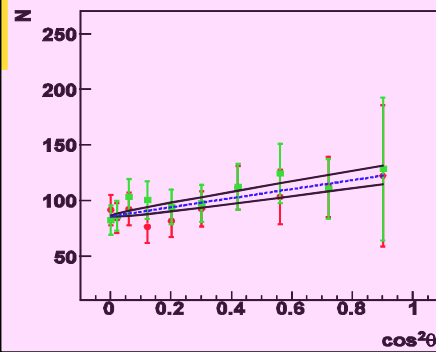
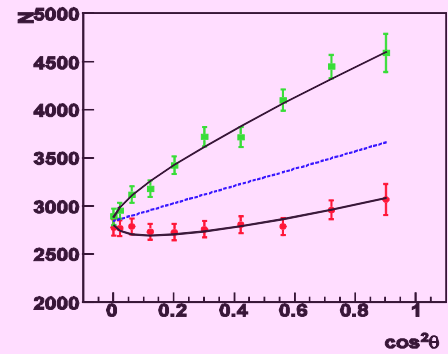
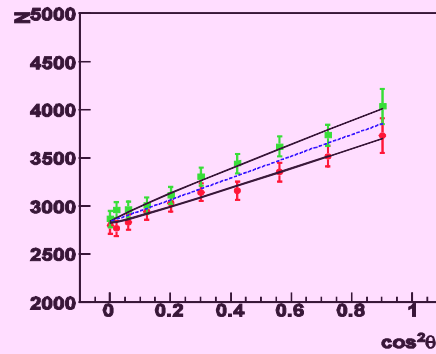
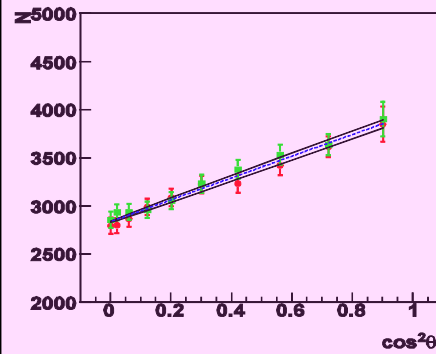
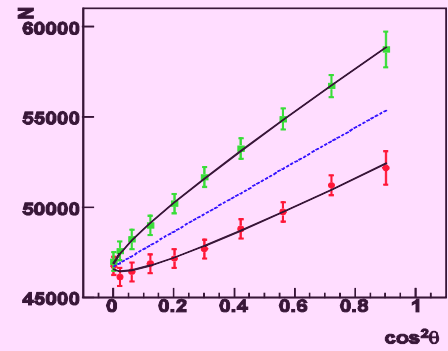
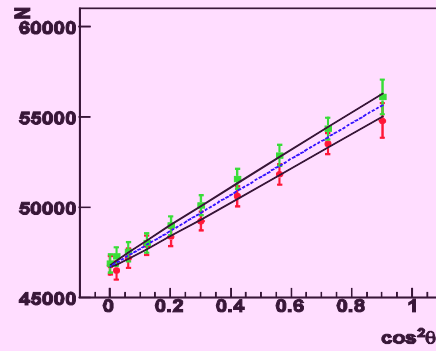
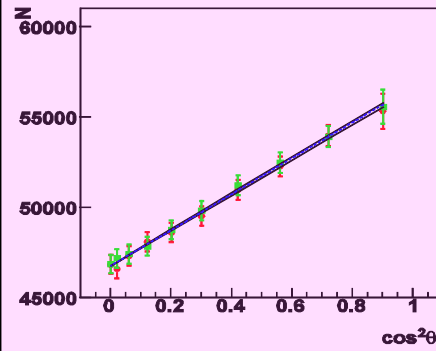
- Neglect contributions to G_E, G_M
- Consider only real part



$2\gamma 0.02$

0.05

0.20



hi
n
a

$$N = a_0 + a_2 \cos \theta \sin \theta + a_1 \cos 2\theta, \quad a_2 \sim 2\gamma$$

q^2 (GeV ²)	case	a_0		a_2	χ^2	χ^2/N_f	\mathcal{R}	\mathcal{A}
5.4		46798 ± 182		9927 ± 485	1.94	0.11	$1/00 \pm 0.017$	0.21 ± 0.01
		46713 ± 182		9926 ± 485	1.45	0.09	0.997 ± 0.017	0.21 ± 0.01
	$2\gamma \cdot 0.05$	46714 ± 182	662 ± 240	9924 ± 485	1.47	0.09	0.998 ± 0.017	0.21 ± 0.01
	$2\gamma \cdot 0.20$	46710 ± 182	3398 ± 240	9933 ± 485	1.13	0.07	0.997 ± 0.017	0.21 ± 0.01
8.2		2832 ± 30		1128 ± 85	3.66	0.22	1.001 ± 0.095	0.398 ± 0.030
		2833 ± 29		1130 ± 85	3.78	0.22	1.000 ± 0.095	0.399 ± 0.030
	$2\gamma \cdot 0.05$	2830 ± 30	163 ± 42	1136 ± 85	3.49	0.21	0.998 ± 0.096	0.401 ± 0.030
	$2\gamma \cdot 0.20$	2842 ± 30	805 ± 42	1106 ± 84	6.54	0.38	1.012 ± 0.092	0.389 ± 0.030
13.84		85 ± 5		39 ± 19	4.49	0.26	1.149 ± 1.09	0.469 ± 0.230
		86 ± 5		41 ± 19	3.36	0.19	1.133 ± 1.116	0.481 ± 0.228
	$2\gamma \cdot 0.05$	86 ± 5	16 ± 9	41 ± 19	3.67	0.22	1.137 ± 1.107	0.478 ± 0.228
	$2\gamma \cdot 0.20$	82 ± 5	59 ± 9	55 ± 18	2.12	0.12	0.848 ± 2.121	0.672 ± 0.233