

# Search for two-photon effects in charge asymmetry measurements from electron and positron scattering on nucleon and nuclei

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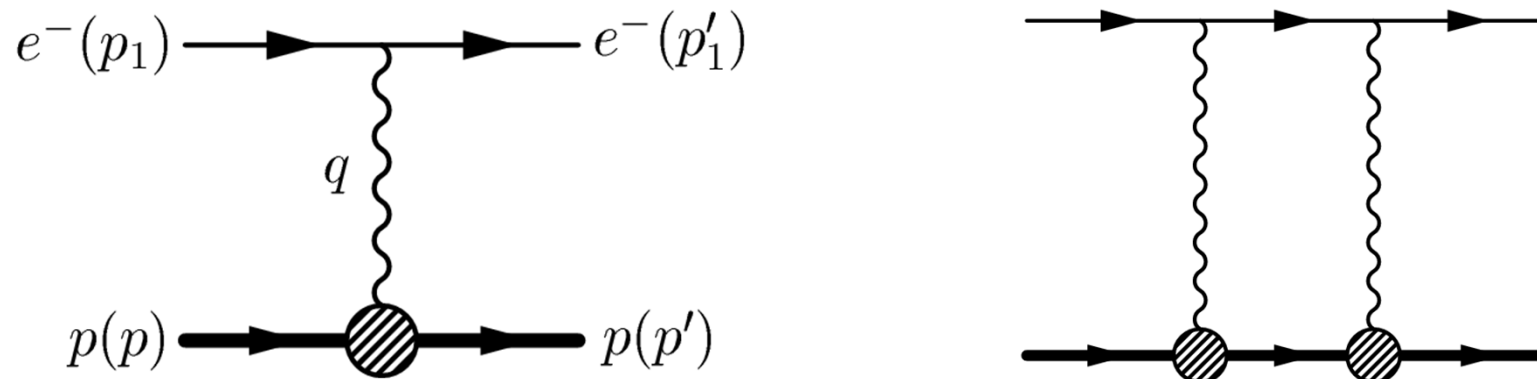
OLYMPUS Workshop

"Experimental and theoretical aspects of the proton form factors"

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# Introduction

The elastic electron-hadron scattering is one of the object of large experimental and theoretical effort since many decades. Most of the interpretation of the observables is based on the assumption that the interaction of the lepton with a hadron occurs through the exchange of one virtual photon (the so called **One-Photon-Exchange** (OPE) approximation).



Since the early sixties it was noted in the literature that the **Two (n)-Photon Exchange** (TPE) could also contribute to the observables, although the size of its amplitude would be scaled by the factor  $Z\alpha ((Z\alpha)^n)$ .

# Introduction

In order to find experimental evidence for the processes beyond OPE one has to base on various theoretical predictions.

First of all TPE is proportional to the target charge  $Z$ , hence a nuclear target is desirable.

In J. Gunion, L. Stodolsky, Phys. Rev. Lett. **30**, 345 (1973);

V. Franco, Phys. Rev. **D8**, 826 (1973);

N. Boitsov, L.A. Kondratyuk, and V. B. Kopeliovich, Sov. J. Nucl. Phys. **16**, 287 (1973);

F. M. Lev, Sov. J. Nucl. Phys. **21**, 45 (1973) it was suggested that a **possible large effect from the TPE could arise at large  $Q^2$** . In this kinematics the factor  $\alpha$  may be compensated by the steep increase of proton form factors (FFs), if the transferred momentum is equally shared between the two photons.

# Introduction

Suggested processes for the search of TPE contribution are the elastic lepton scattering off the proton or nuclei and the crossed channels (annihilation processes  $e^+ + e^- \rightarrow p + \bar{p}$  and  $p + \bar{p} \rightarrow e^+ + e^-$ ).

The unpolarized elastic scattering provides signatures which give model independent information on the presence of TPE:

1. Non-linearities in the Rosenbluth plot, i.e., in the reduced cross section versus  $\epsilon$  at fixed  $Q^2$  where  $\epsilon^{-1} = 1 + 2(1 + \tau) \tan^2(\theta/2)$  is the linear polarization of the virtual photon,  $\tau = Q^2/(4M^2)$ ,  $M$  is the proton mass, and  $\theta$  is the angle of the scattered electron in the laboratory (lab) reference frame.

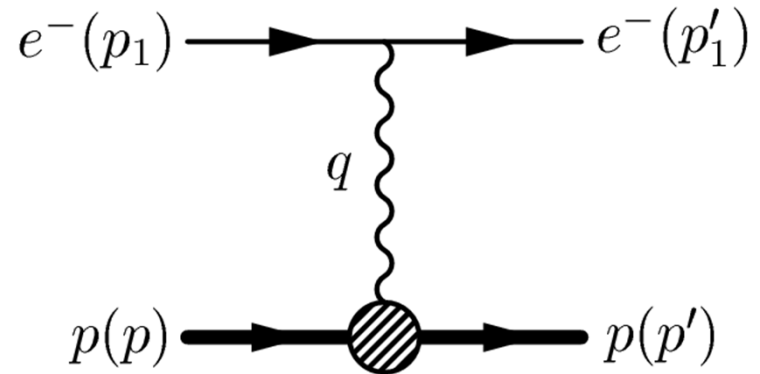
2. Non-vanishing lepton-charge asymmetry which is defined as:

$$A^{odd} = \frac{\sigma(e^- p \rightarrow e^- p) - \sigma(e^+ p \rightarrow e^+ p)}{\sigma(e^- p \rightarrow e^- p) + \sigma(e^+ p \rightarrow e^+ p)}$$

# Introduction: Notations

## 1. Rosenbluth method:

$$\sigma_M = \frac{\alpha^2 \cos^2 \frac{\theta'_1}{2}}{4E_1^2 \sin^4 \frac{\theta'_1}{2}}, \quad \tau = \frac{-q^2}{4M^2}$$

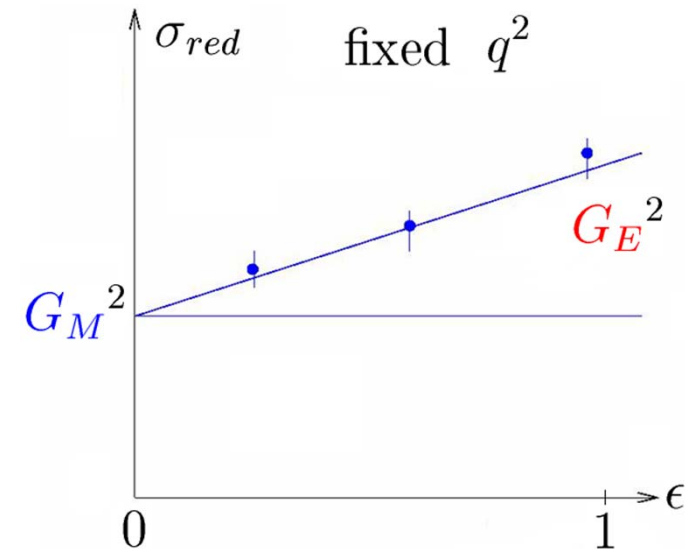


$$\frac{d\sigma}{d\Omega'_1} = \sigma_M \frac{E'_1}{E_1} \left[ \frac{G_E^2 + \tau G_M^2}{1 + \tau} + \tau G_M^2 \tan^2 \frac{\theta'_1}{2} \right],$$

$$\sigma_{red} = \frac{d\sigma}{d\Omega'_1} \left( \sigma_M \frac{E'_1}{E_1} \right)^{-1}$$

$$\sigma_{red} = \epsilon G_E^2(q^2) + \tau G_M^2(q^2),$$

$$\epsilon^{-1} = 1 + 2(1 + \tau) \tan^2 (\theta'_1/2)$$



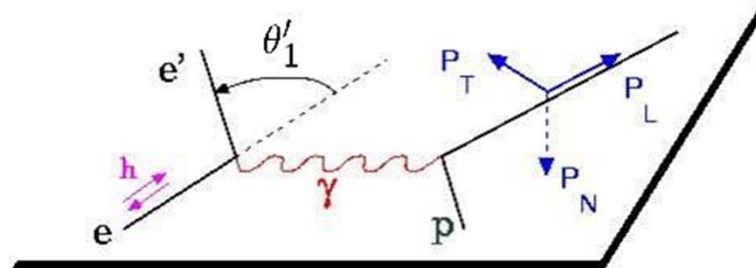
## Introduction: Notations

### 2. Polarized cross section:

$$\left(\frac{d\sigma}{d\Omega'_1}\right)_T = -h \frac{\alpha^2 E_1^2}{-q^2 E_1'^2} \sqrt{\frac{\tau}{\tan^2(\theta'_1/2)(1+\tau)}} G_E G_M$$

$$\left(\frac{d\sigma}{d\Omega'_1}\right)_L = h \frac{\alpha^2 E_1^2}{2M^2 E_1'^2} \sqrt{1 + \frac{1}{\tan^2(\theta'_1/2)(1+\tau)}} G_M^2$$

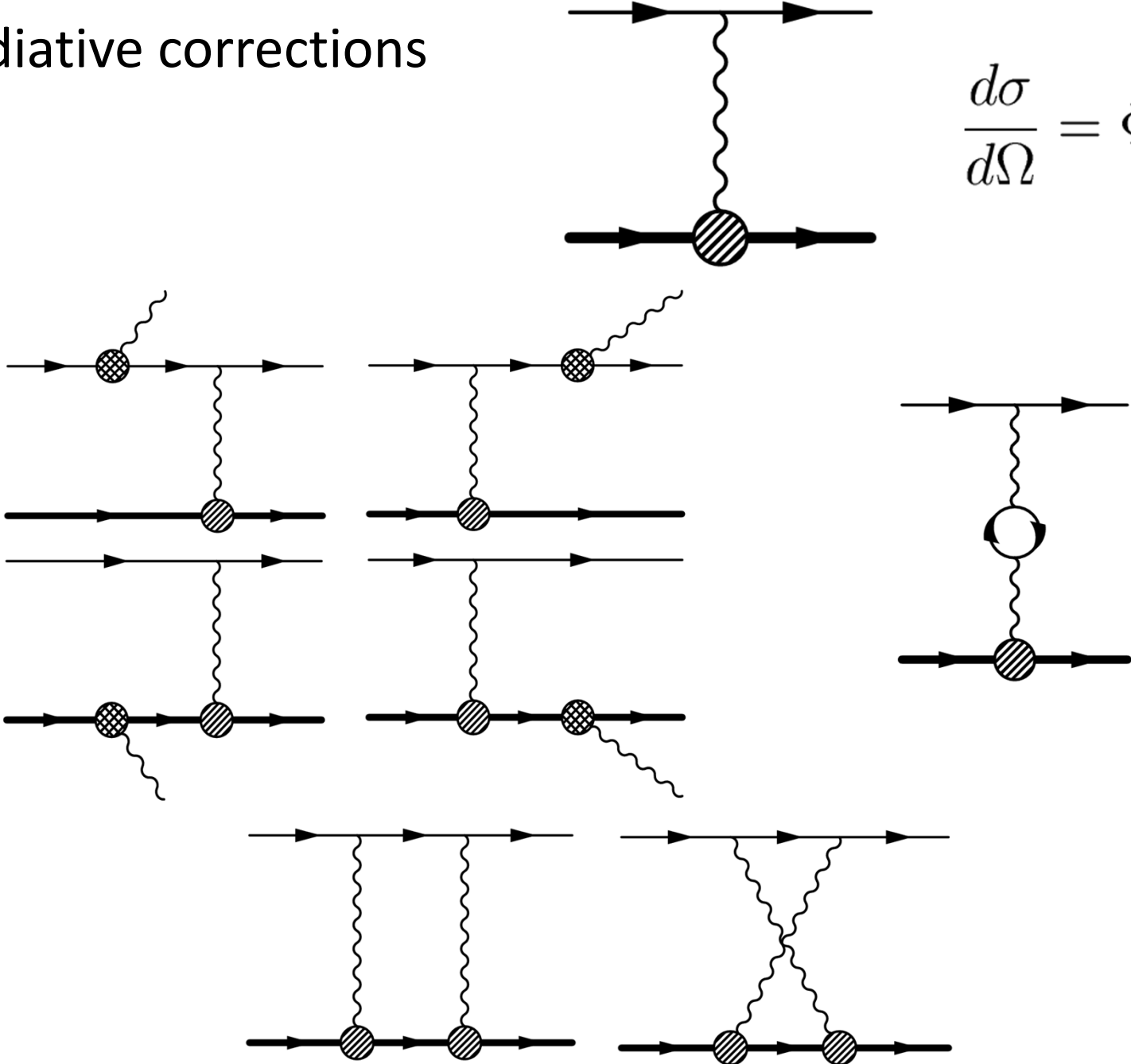
А. И. Ахиезер, М. П. Рекало,  
Доклады Академии Наук СССР,  
т. 180, № 5, стр. 1081, 1968 г.



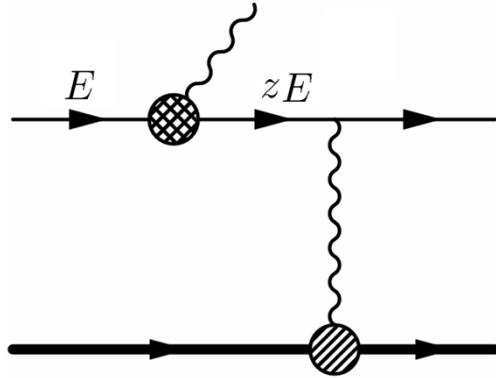
$$\frac{G_E}{G_M} = - \frac{-q^2}{2M^2} \tan \frac{\theta'_1}{2} \sqrt{1 + \frac{4M^2}{-q^2 \sin^2 \frac{\theta'_1}{2}} \frac{\left(\frac{d\sigma}{d\Omega'_1}\right)_T}{\left(\frac{d\sigma}{d\Omega'_1}\right)_L}}$$

# Radiative corrections

$$\frac{d\sigma}{d\Omega} = \Phi(E, \theta)$$



# Radiative corrections: initial electron bremsstrahlung



$$L = \ln \frac{-q^2}{m_e^2}$$

$$\left. \frac{d\sigma}{d\Omega} \right|_{Vac+Brem_1} = \int_{z_0}^1 dz \frac{\Phi(zE, \theta)}{|1 - \Pi(q_z^2)|^2} D(z, L),$$

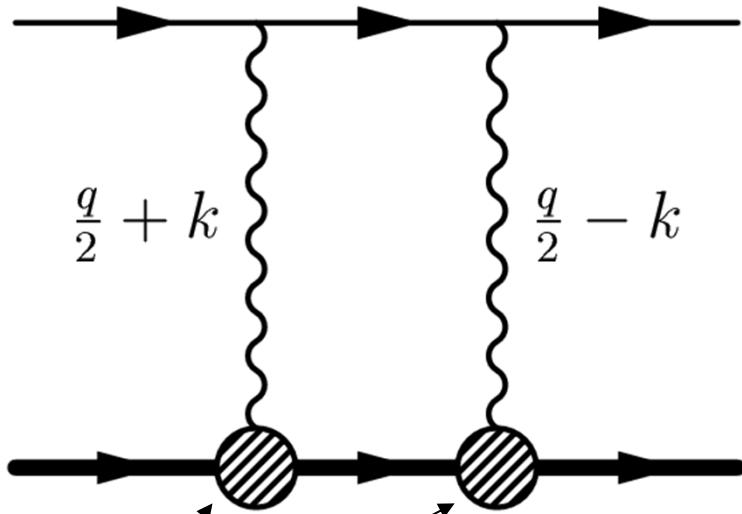
$$D(z, L) = \delta(1 - z) + \frac{\alpha}{2\pi} (L - 1) P^{(1)}(z) + \frac{1}{2!} \left( \frac{\alpha}{2\pi} \right)^2 (L - 1)^2 P^{(2)}(z) + \dots$$

$$P^{(1)}(z) = \lim_{\Delta \rightarrow 0} \left[ \frac{1 + z^2}{1 - z} \theta(1 - z - \Delta) + \delta(1 - z) \left( 2 \ln \Delta + \frac{3}{2} \right) \right]$$

$$P^{(n)}(z) = \int_z^1 \frac{dt}{t} P^{(1)}(z) P^{(n-1)}\left(\frac{z}{t}\right), \quad n = 2, 3, \dots$$



# Radiative corrections: two-photon exchange



$$P = \frac{p_1 + p_1'}{2}$$

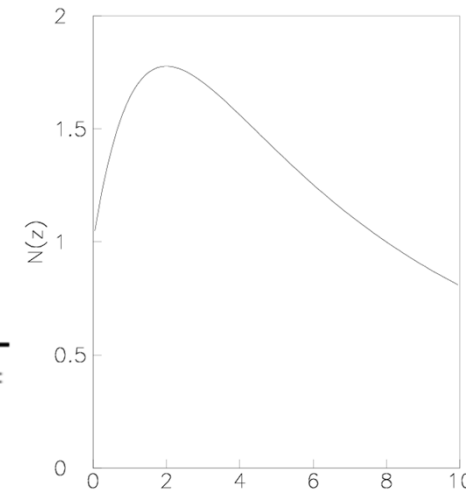
$$Q = \frac{p + p'}{2}$$

$$G_E(q) = \frac{G_M(q)}{\mu} = \frac{1}{\left(1 + \frac{-q^2}{M_0^2}\right)^2}$$

$$M_0^2 = 0.71 \text{ GeV}^2$$

$$\mu = 2.79$$

$$N(z) = \frac{(z + 1)^2}{\left(\left(\frac{z}{4}\right) + 1\right)^4}$$

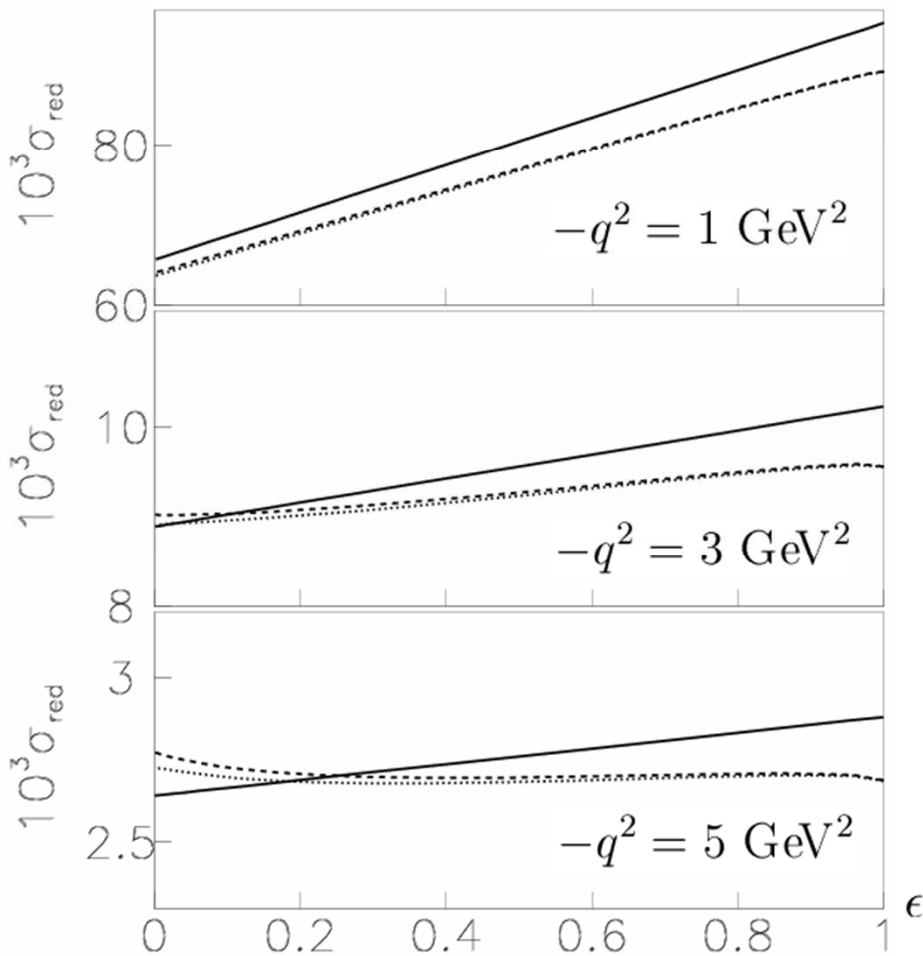


$$K_{2\gamma} = N\left(\frac{-q^2}{M_0^2}\right) \frac{1}{\Phi} \int \frac{d^4k}{i\pi^2} \frac{U(P, Q)}{\left((k + P)^2 - m_e^2\right) \left((k + Q)^2 - M^2\right)} \theta\left(M^2\tau - |k^2|\right)$$

## Contribution to unpolarized cross section

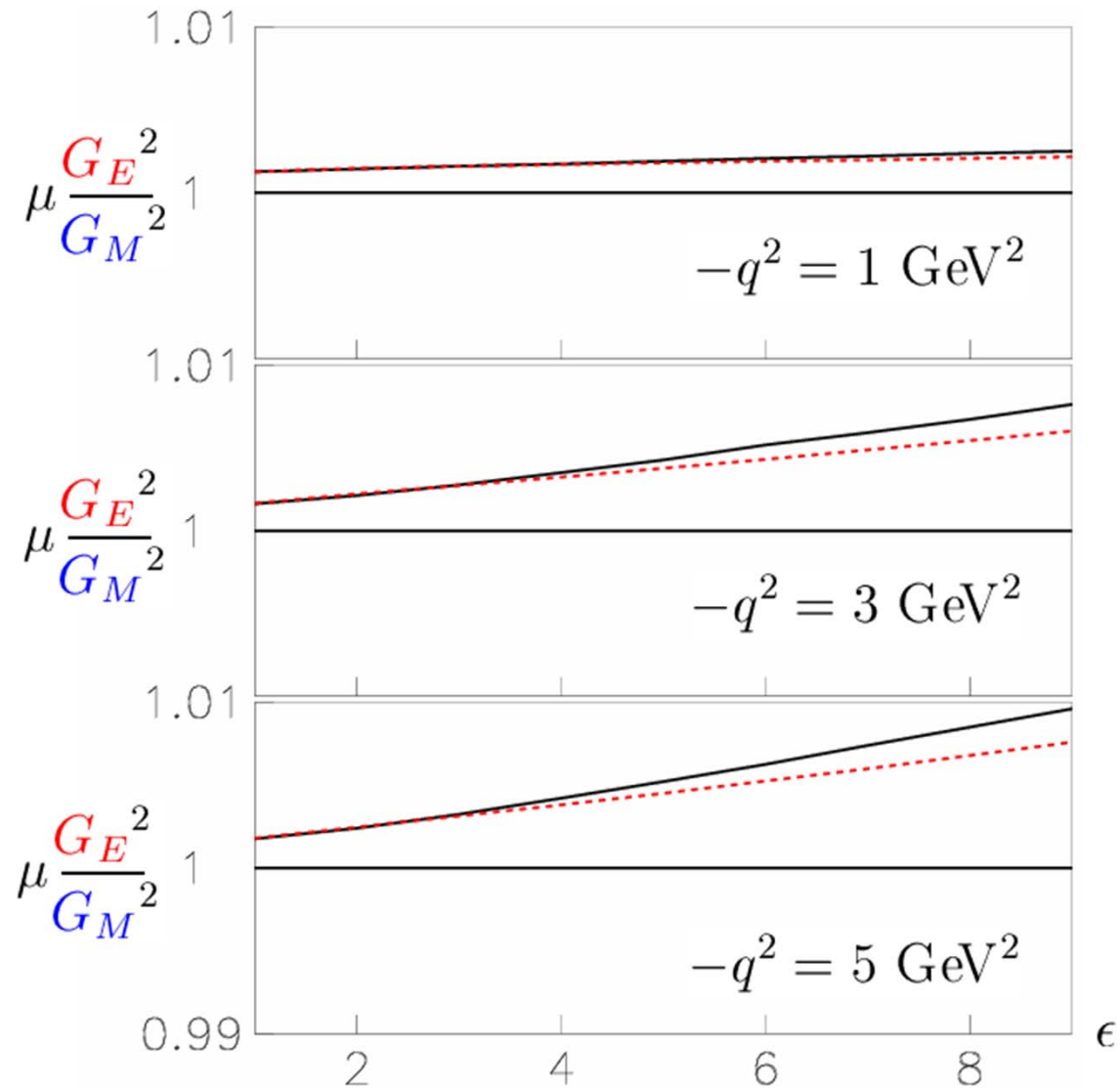
$$\left. \frac{d\sigma}{d\Omega} \right|_{RC} = \int_{z_0}^1 dz \frac{\Phi(zE, \theta)}{|1 - \Pi(q_z^2)|^2} D(z, L) \left( 1 + \frac{\alpha}{\pi} K \right)$$

$$K = K_{Brems} + K_{2\gamma}$$



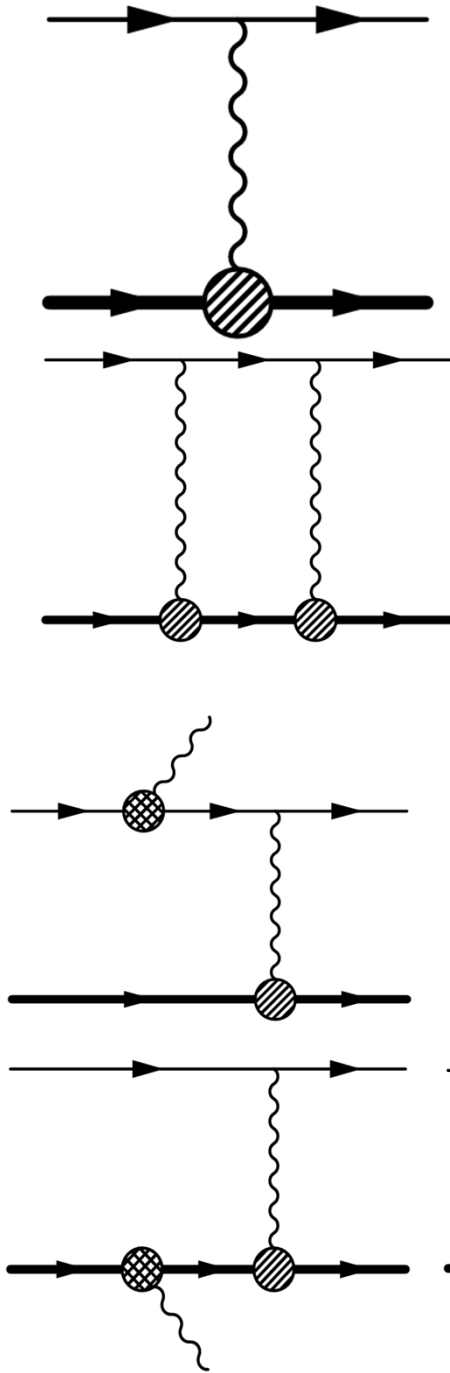
$$\sigma_{red} = \epsilon G_E^2(q^2) + \tau G_M^2(q^2),$$

# Contribution to ratio of polarized cross sections



# Charge asymmetry

The calculation of C-odd contributions into lepton-proton scattering was done in [E.A. Kuraev, V.V. Bytev, S. Bakmaev E. Tomasi-Gustafsson, Phys. Rev. C 78, 015205 \(2008\)](#). For details of calculation listen the Talk of Prof. Kuraev



$$A^{odd} = \frac{d\sigma^{e^-p} - d\sigma^{e^+p}}{d\sigma^{e^-p} + d\sigma^{e^+p}} = \frac{d\sigma^{e^-p} - d\sigma^{e^+p}}{2d\sigma^B(1 + \delta^{even})} =$$

$$= \frac{2\alpha}{\pi(1 + \delta^{even})} \times$$

$$\times \left[ \ln \rho \ln \frac{(2\Delta E')^2}{ME} + \frac{5}{2} \ln^2 \rho - \ln x \ln \rho - \right.$$

$$\left. -\text{Li}_2 \left( 1 - \frac{1}{\rho x} \right) + \text{Li}_2 \left( 1 - \frac{\rho}{x} \right) \right],$$

$$E' > E_3 - \Delta E'$$

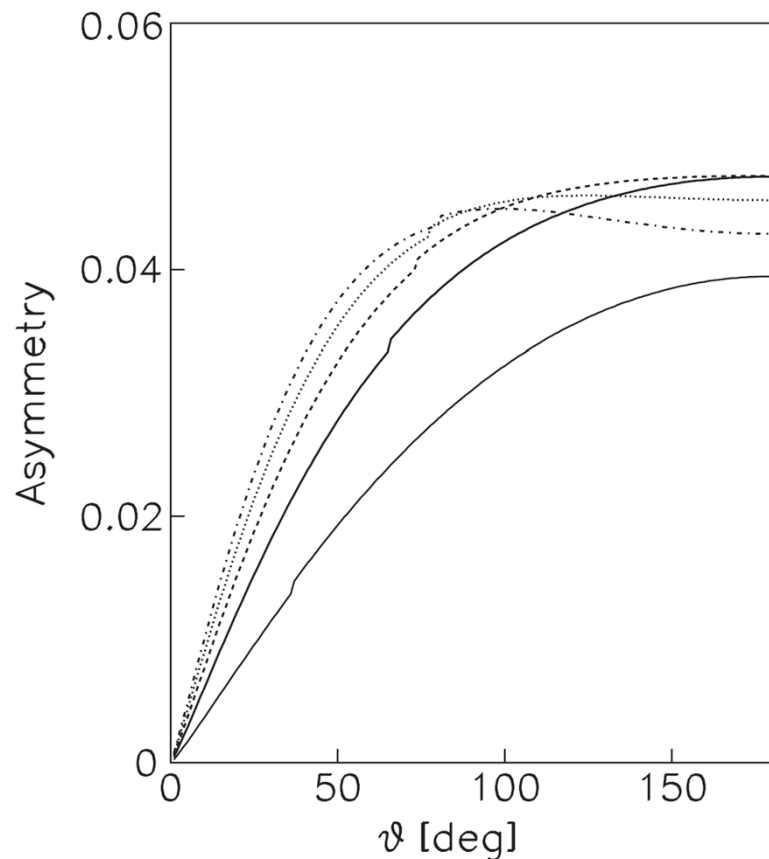
$$\rho = \left( 1 - \frac{Q^2}{s} \right)^{-1}$$

$$x = \frac{\sqrt{1 + \tau} + \sqrt{\tau}}{\sqrt{1 + \tau} - \sqrt{\tau}}$$

# Charge asymmetry

$$\begin{aligned}
 Q^2 &= 9 \text{ GeV}^2 \\
 Q^2 &= 7 \text{ GeV}^2 \\
 Q^2 &= 5 \text{ GeV}^2 \\
 Q^2 &= 3 \text{ GeV}^2 \\
 Q^2 &= 1 \text{ GeV}^2
 \end{aligned}$$

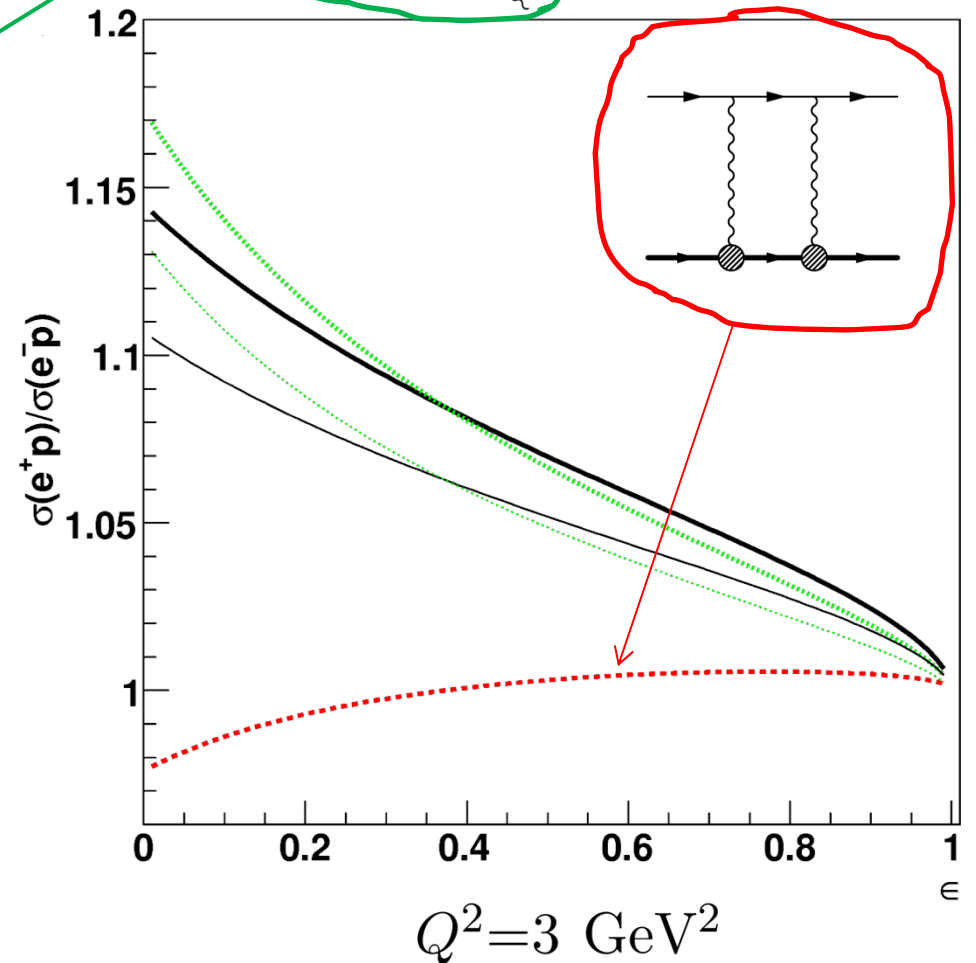
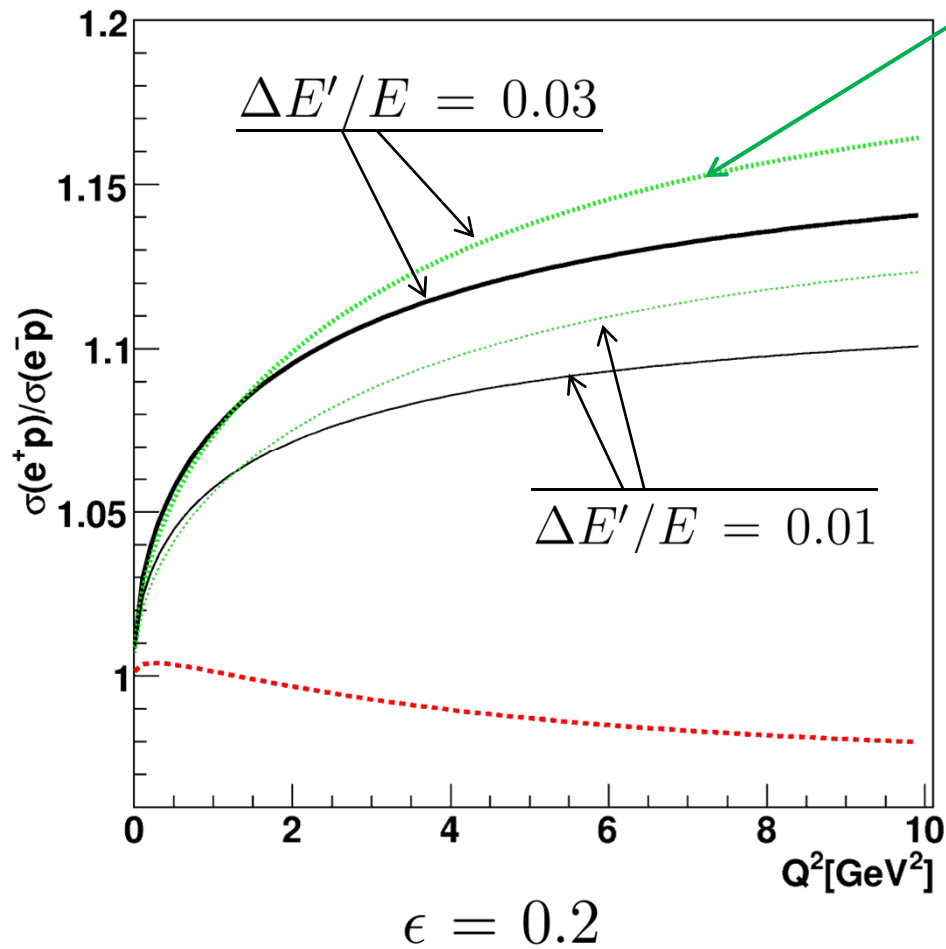
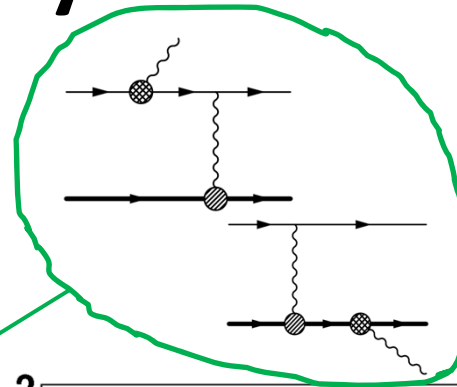
The calculation of C-odd contributions into lepton-proton scattering was done in [E.A. Kuraev, V.V. Bytev, S. Bakmaev, E. Tomasi-Gustafsson, Phys. Rev. C 78, 015205 \(2008\)](#). For details of calculation listen the Talk of Prof. Kuraev



$$\begin{aligned}
 A^{odd} &= \frac{d\sigma^{e^-p} - d\sigma^{e^+p}}{d\sigma^{e^-p} + d\sigma^{e^+p}} = \frac{d\sigma^{e^-p} - d\sigma^{e^+p}}{2d\sigma^B(1 + \delta^{even})} = \\
 &= \frac{2\alpha}{\pi(1 + \delta^{even})} \times \\
 &\times \left[ \ln \rho \ln \frac{(2\Delta E')^2}{ME} + \frac{5}{2} \ln^2 \rho - \ln x \ln \rho - \right. \\
 &\quad \left. - \text{Li}_2 \left( 1 - \frac{1}{\rho x} \right) + \text{Li}_2 \left( 1 - \frac{\rho}{x} \right) \right], \\
 &E' > E_3 - \Delta E' \\
 &\rho = \left( 1 - \frac{Q^2}{s} \right)^{-1} \quad x = \frac{\sqrt{1 + \tau} + \sqrt{\tau}}{\sqrt{1 + \tau} - \sqrt{\tau}}
 \end{aligned}$$

# Charge asymmetry

$$R = \frac{\sigma(e^+h \rightarrow e^+h)}{\sigma(e^-h \rightarrow e^-h)} = \frac{1 - A^{odd}}{1 + A^{odd}}$$



# Charge asymmetry data: elastic scattering

Yount'1962

Browman'1965

Anderson'1966

Cassiday'1967

Bartel'1967

Mar'1968

Anderson'1968

Bouquet'1968

Camilleri'1969

Jostlein'1974

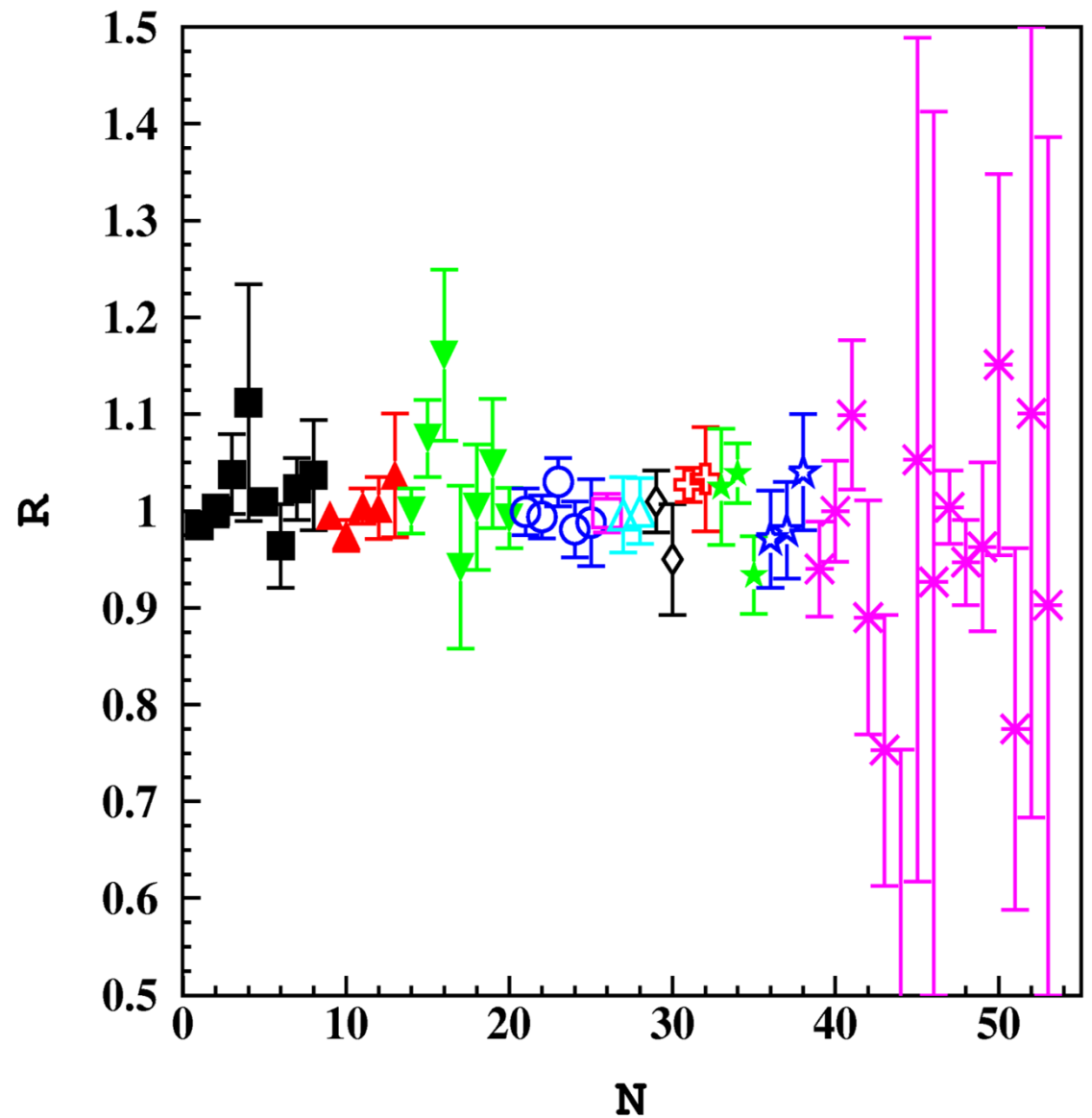
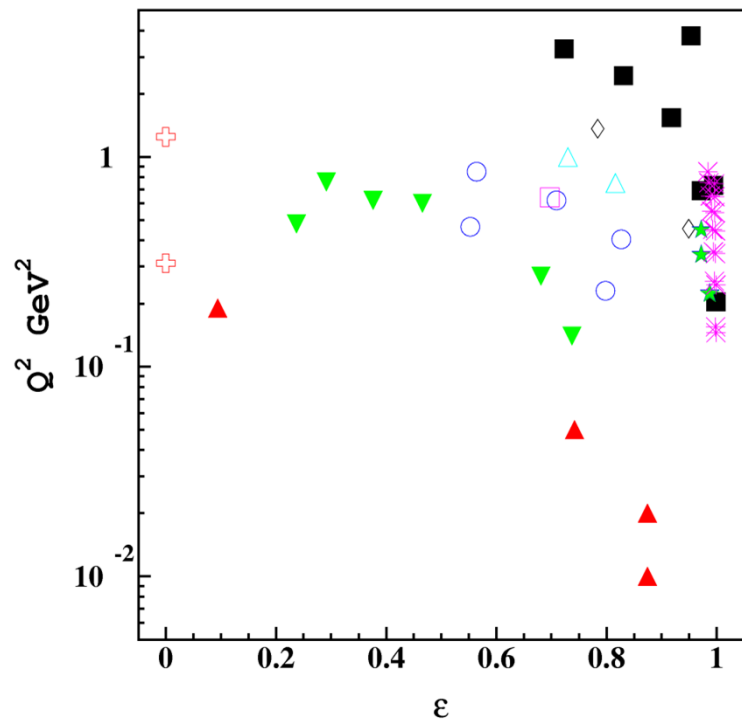
Hartwig'1975

Hartwig'1976

Hartwig'1979

Rochester'1976

Fancher'1976



# Charge asymmetry data: inelastic scattering

Mar'1968

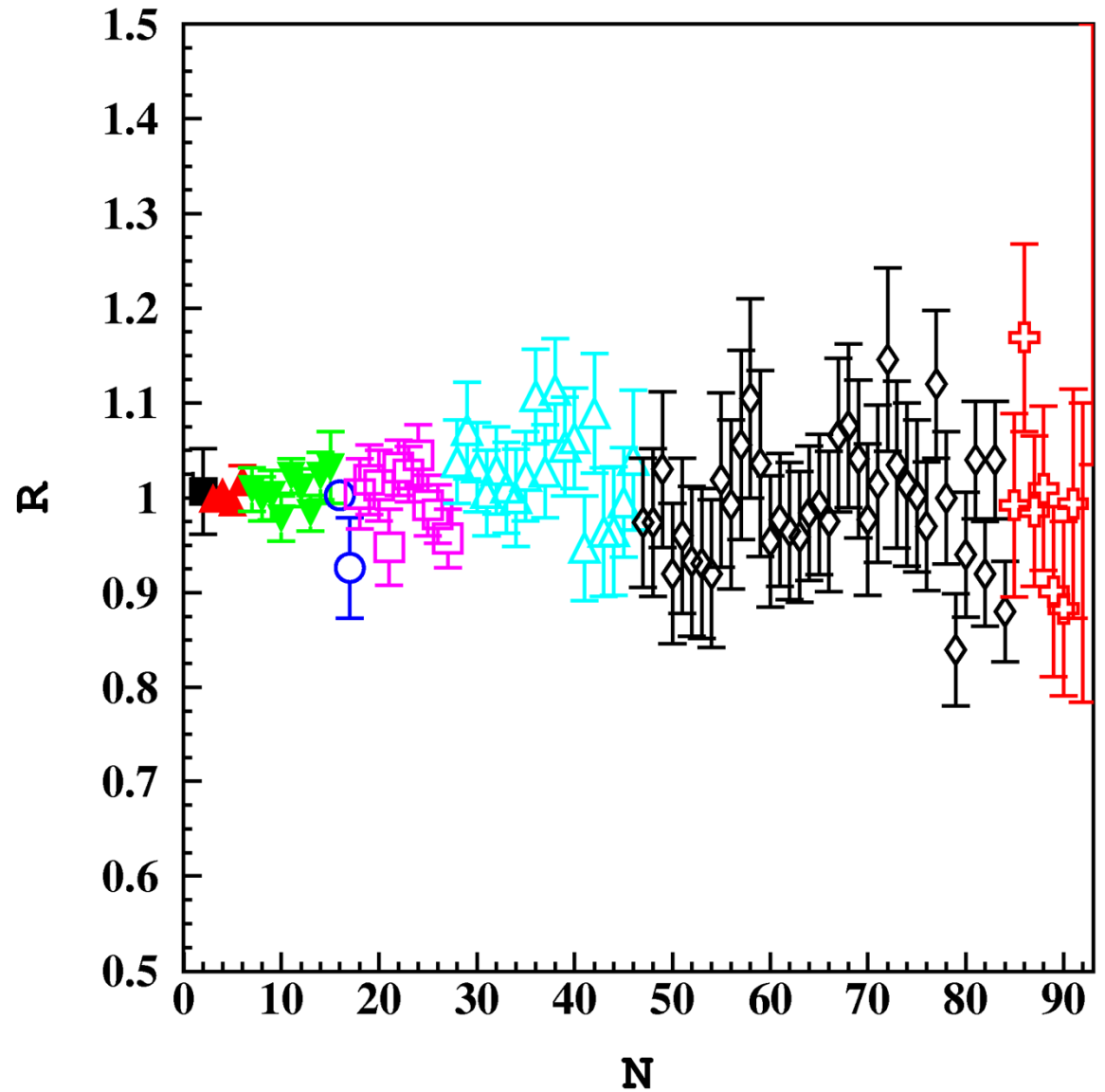
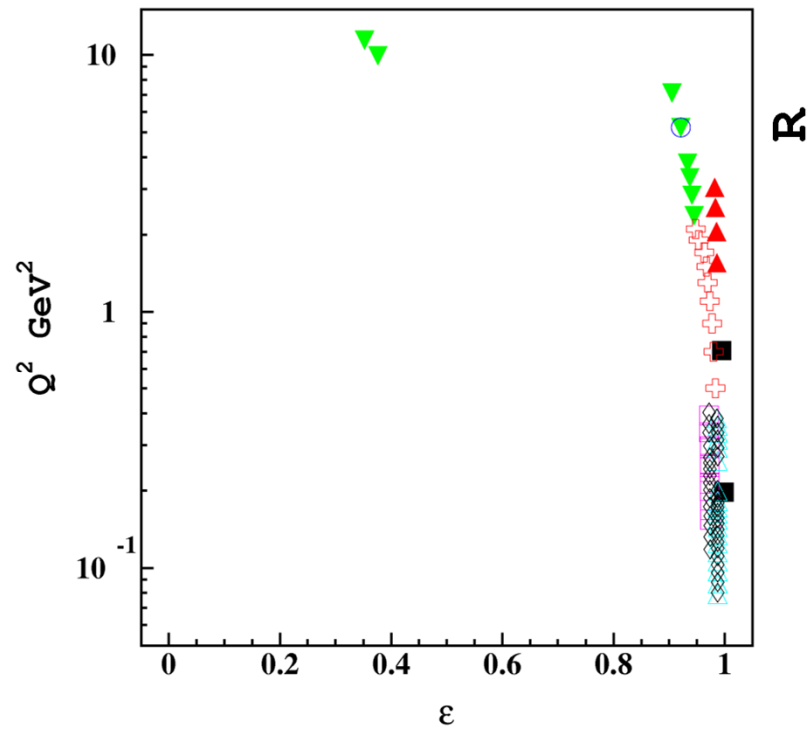
Fancher'1976

Rochester'1976

Hartwig'1979

Hartwig'1976

Jostlein'1974





# $G_E/G_M$ Puzzle

Formfactors obtained by polarization transfer and Rosenbluth techniques are inconsistent.

It has been argued that two-photon exchange may reconcile these measurements

(P.A.M. Guichon and M. Vanderhaeghen, PRL **91**, 142303 (2003);

P.G. Blunden, W. Melnitchouk and J.A. Tjon, PRC **72**, 034612 (2005);

A.V. Afanasev, S.J. Brodsky, C.E. Carlson, Y.C. Chen,

M. Vanderhaeghen, PRD **72**, 013008 (2005)).

$$\begin{aligned}\sigma^{red} &= \epsilon \left(G_E^{PT}\right)^2 + \tau \left(G_M^{PT}\right)^2 + C_{2\gamma} = \\ &= \epsilon \left(G_E^R\right)^2 + \tau \left(G_M^R\right)^2\end{aligned}$$

## $G_E/G_M$ Puzzle

$$(G_M^{PT})^2 \left[ \tau + \frac{\epsilon}{\mu^2} \left( \frac{\mu G_E^{PT}}{G_M^{PT}} \right)^2 \right] + C_{2\gamma} = (G_M^R)^2 \left[ \tau + \frac{\epsilon}{\mu^2} \left( \frac{\mu G_E^R}{G_M^R} \right)^2 \right]$$

We assume that  $\mu G_E^R/G_M^R = 1$

$$\mu G_E^{PT}/G_M^{PT} = 1 - S \frac{Q^2}{M^2}$$

And thus can evaluate the two-photon effects as

$$C_{2\gamma} = (\epsilon - 1) \tau (G_M^R)^2 \frac{1 - (\mu G_E^{PT}/G_M^{PT})^2}{\mu^2 \tau + (\mu G_E^{PT}/G_M^{PT})^2}$$

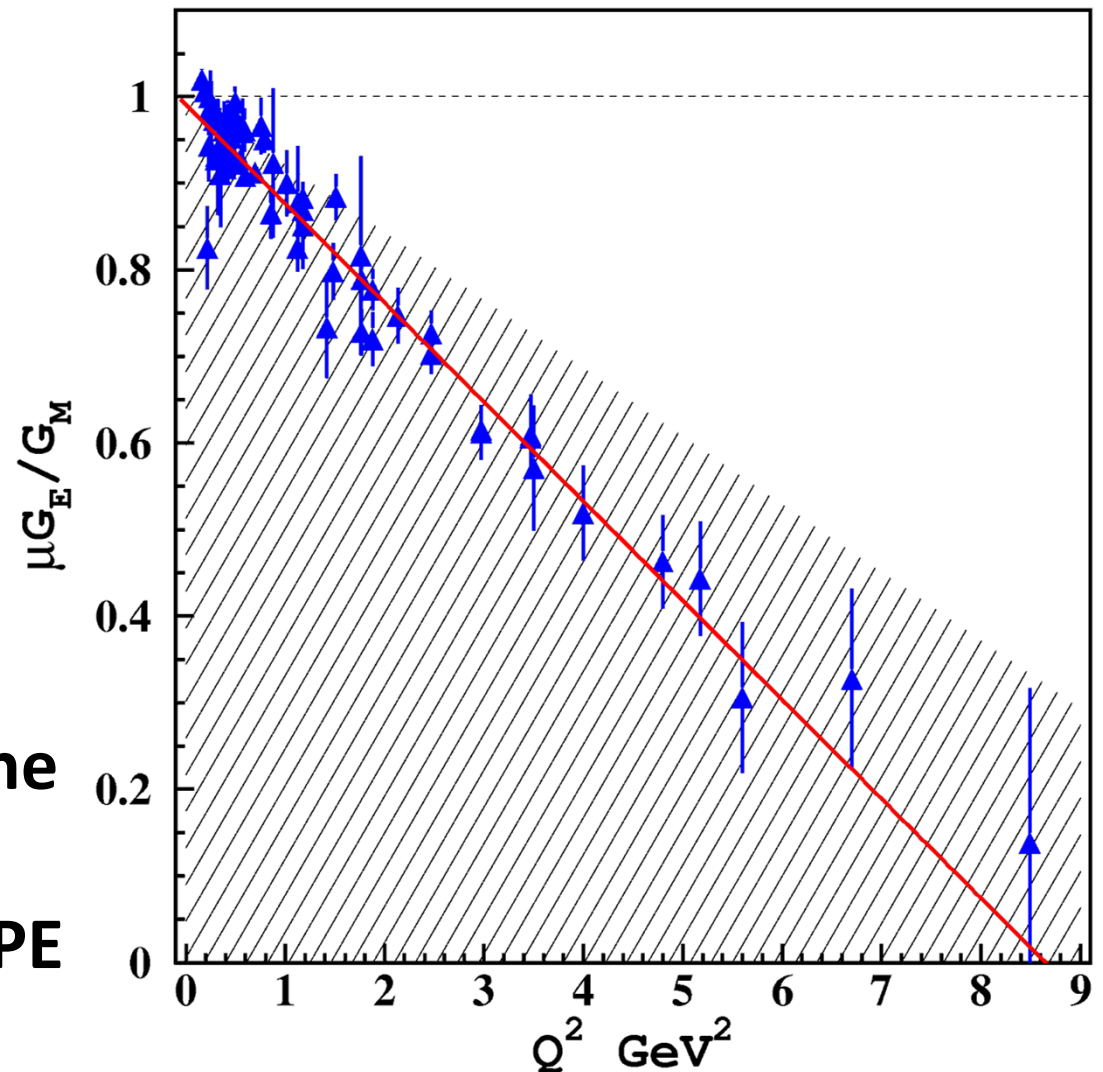
# $G_E/G_M$ Puzzle

$$A_{2\gamma}^{odd} = \frac{C_{2\gamma}}{\sigma^- - C_{2\gamma}} = (\epsilon - 1) \left[ 1 - \epsilon + \left( 1 + \frac{\epsilon}{\mu^2\tau} \right) \frac{\mu^2\tau + (\mu G_E^{PT}/G_M^{PT})^2}{1 - (\mu G_E^{PT}/G_M^{PT})^2} \right]^{-1}$$

Using this evaluation of two-photon effects (TPE) we can test the hypothesis that the difference between polarization transfer and Rosenbluth extraction is due to TPE contribution.

Assuming **significance level of 0.05** we found the region excluded by this procedure.

**Within our assumptions the data at  $Q^2 > 2 \text{ GeV}^2$  cannot be reconciled by TPE contribution.**



## Conclusions

1. Two-photon exchange is small ( $< 1.5\%$ ).
2. **Experimental data** on the C-odd asymmetry in lepton-proton scattering **demonstrates small contributions from two-photon effects.**
3. These possible **two-photon effects cannot explain the difference of proton formfactors** extracted in Rosenbluth and polarized setups.