Physics at a future e+e- Z-boson factory

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Outline

1. Motivation
2. e^+e^- colliders
3. QED
4. Higher order logs
5. SANC Project
6. Outlook

General motivation

- The Standard Model is the most successful physical model ever
- But there are still many open questions to it
- We believe that it is only an effective theory, but its applicability domain might be limited just by the Planck mass scale
- The primary goal of HEP is to study the physics of our actual microworld
- Discovering physics beyond SM is our hope
- In any case, the research in HEP will not stop by the end of LHC
- Logically, the next step should be a e^+e^- collider

Future e^+e^- collider projects

Linear Colliders

- ILC, CLIC

E_{tot}

- ILC: 91; 250 GeV — 1 TeV
- CLIC: 500 GeV — 3 TeV

$\mathcal{L} \approx 2 \cdot 10^{34}$ cm^{-2}s^{-1}

Stat. uncertainty $\sim 10^{-4}$

Beam polarization:
e^-beam: $P = 80 - 90\%$
e^+beam: $P = 30 - 60\%$

Circular Colliders

- FCC-ee, CEPC
- Z-factory
- Super Charm-Tau Factory
- $\mu^+\mu^-$ collider (μTRISTAN)

E_{tot}

- 91; 160; 240; 350 GeV

$\mathcal{L} \approx 2 \cdot 10^{36}$ cm^{-2}s^{-1} (4 exp.)

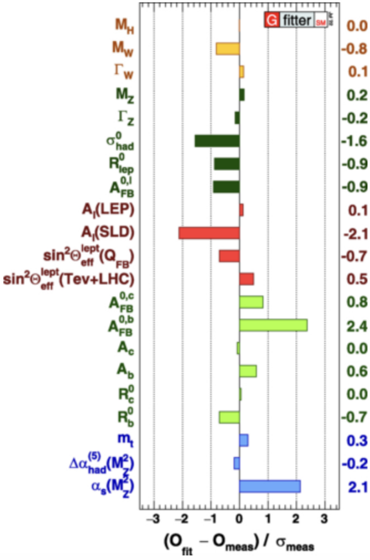
Stat. uncertainty $\sim 10^{-6}$

Tera-Z mode! Beam polarization: desirable

Physics possibilities at the Z peak

- Indeep verification of the EW sector of SM
- Unique possibilities for QCD at the EW scale
- Photon-photon physics
- Properties of tau lepton
- Physics of (exotic) mesons
- Searches for new physics of SMEFT and other types

Where are we now



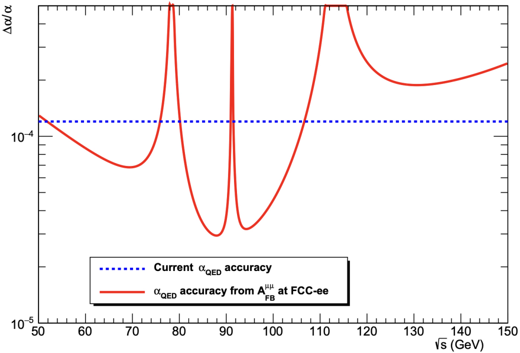
Weak mixing angle

An experimental precision better than 5×10^{-6} is therefore a robust target for the measurement of $\sin^2\theta_W^{\text{eff}}$ at FCC-ee, corresponding to more than a thirty-fold improvement with respect to the current precision of 1.6×10^{-4}.

Individual measurements of leptonic and heavy quark couplings are achievable, with a factor of several hundred improvement on statistical errors and, with the help of detectors providing better particle identification and vertexing, by up to two orders of magnitude on systematic uncertainties.

[FCC Coll. EPJC'2019]

$\alpha_{QED}(m_Z^2)$



An experimental relative accuracy of 3×10^{-5} on $\alpha_{QED}(m_Z^2)$ can be achieved at FCC-ee, from the measurement of the muon forward-backward asymmetry at energies ~ 3 GeV below and ~ 3 GeV above the Z pole. The corresponding parametric uncertainties on other SM parameters and observables will be reduced. [FCC Coll. EPJC'2019]

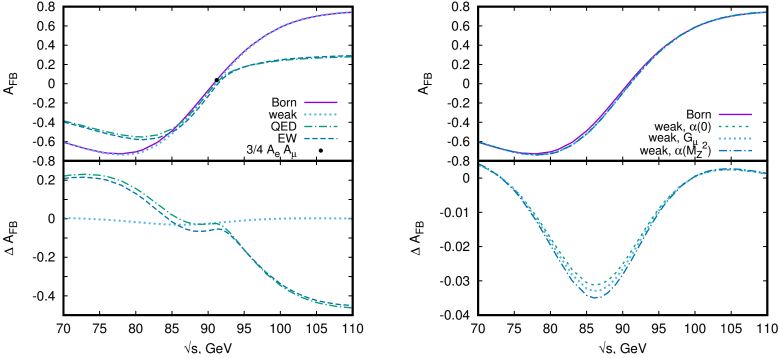
Z boson mass and width; R_l

Overall experimental uncertainties of 0.1 MeV or better are achievable for the Z mass and width measurements at FCC-ee. The corresponding parametric uncertainties on $\sin^2\theta_W^{\mathrm{eff}}$ and m_W SM predictions are accordingly reduced to 6×10^{-7} and 0.12 MeV, respectively.

An absolute (relative) uncertainty of 0.001 (5×10^{-5}) on the ratio of the Z hadronic-to-leptonic partial widths (R_l) can be reached. The same relative uncertainty is expected for the ratios of the Z leptonic widths, which allows a stringent test of lepton universality.

[FCC Coll. EPJC'2019]

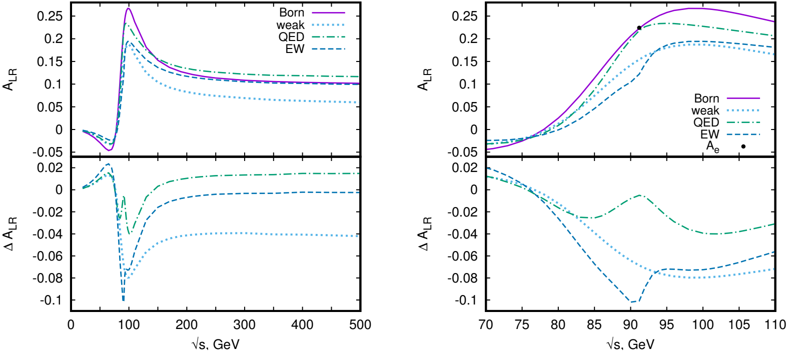
Forward-Backward Asymmetry



$$A_f = \frac{2g_V g_A}{g_V^2 + g_A^2} \qquad \text{for the given fermion } f$$

[A.A., S.Bondarenko, L.Kalinovskaya, Symmetry'2020]

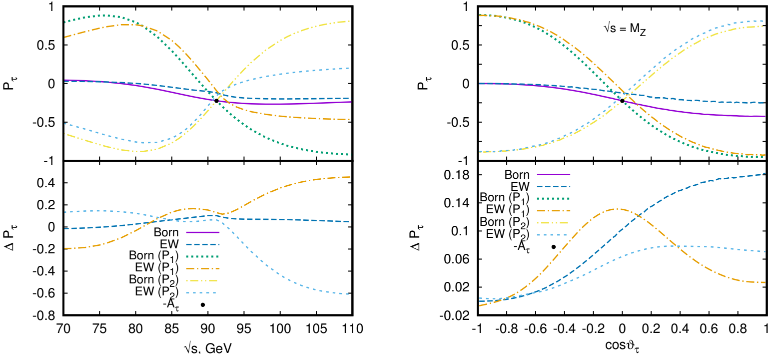
Left-Right Asymmetry



$$A_{LR} = \frac{1}{P_{\text{eff}}} \frac{\sigma(-P_{\text{eff}}) - \sigma(P_{\text{eff}})}{\sigma(-P_{\text{eff}}) + \sigma(P_{\text{eff}})}, \qquad P_{\text{eff}} \equiv \frac{P_{e^-} - P_{e^+}}{1 - P_{e^-}P_{e^+}}$$

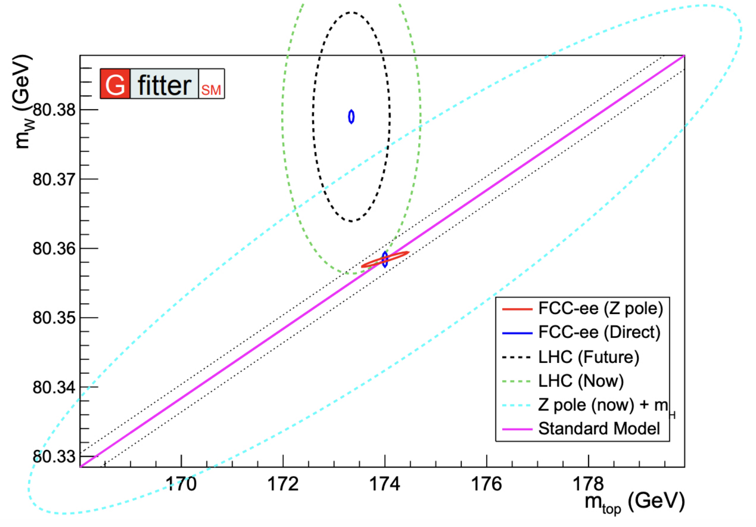
[A.A., S.Bondarenko, L.Kalinovskaya, Symmetry'2020]

Tau lepton polarization



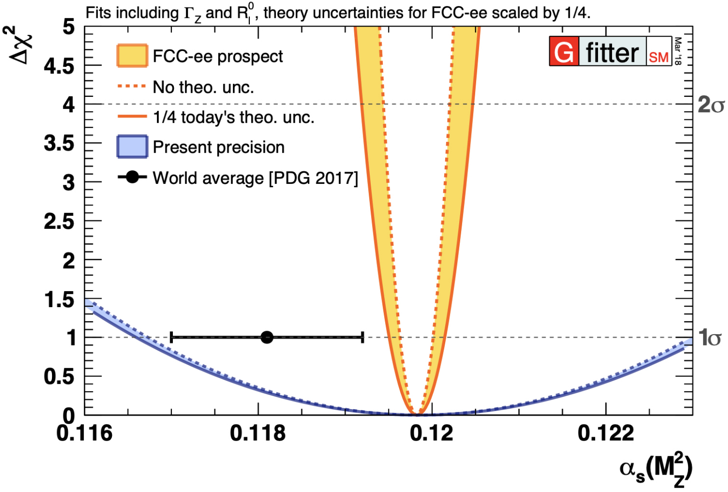
[A.A., S.Bondarenko, L.Kalinovskaya, Symmetry'2020]

Indirect measurements



[FCC Coll. EPJC'2019]

Alpha QCD



[FCC Coll. EPJC'2019]

EW quasi observables (I)

Observable	Present value	±	error	FCC-ee (statistical)	FCC-ee (systematic)	Source and dominant experimental error
m_Z (keV/c^2)	91 186 700	±	2200	5	100	Z line shape scan Beam energy calibration
Γ_Z (keV)	2 495 200	±	2300	8	100	Z line shape scan Beam energy calibration
R_ℓ^Z ($\times 10^3$)	20 767	±	25	0.06	1	Ratio of hadrons to leptons Acceptance for leptons
$\alpha_s(m_Z)$ ($\times 10^4$)	1196	±	30	0.1	1.6	R_ℓ^Z above
R_b ($\times 10^6$)	216 290	±	660	0.3	<60	Ratio of $b\bar{b}$ to hadrons Stat. extrapol. from SLD [7]
σ^0_{had} ($\times 10^3$) (nb)	41 541	±	37	0.1	4	Peak hadronic cross-section Luminosity measurement
N_ν($\times 10^3$)	2991	±	7	0.005	1	Z peak cross-sections Luminosity measurement
$\sin^2\theta_W^{eff}$($\times 10^6$)	231 480	±	160	3	2–5	$A_{FB}^{\mu\mu}$ at Z peak Beam energy calibration
$1/\alpha_{QED}(m_Z)$($\times 10^3$)	128 952	±	14	4	Small	$A_{FB}^{\mu\mu}$ off peak
$A_{FB}^{b,0}$ ($\times 10^4$)	992	±	16	0.02	<1	b quark asymmetry at Z pole Jet charge

[A.Blondel et al., CERN YR 2019]

EW quasi observables (II)

Observable	Present value	±	error	FCC-ee (statistical)	FCC-ee (systematic)	Source and dominant experimental error
$A_{FB}^{pol,\tau}$ ($\times 10^4$)	1498	±	49	0.15	<2	τ polar. and charge asymm. τ decay physics
m_W (keV/c^2)	803 500	±	15 000	600	300	WW threshold scan Beam energy calibration
Γ_W (keV)	208 500	±	42 000	1500	300	WW threshold scan Beam energy calibration
$\alpha_s(m_W)(\times 10^4)$	1170	±	420	3	Small	R_ℓ^W
$N_\nu(\times 10^3)$	2920	±	50	0.8	Small	Ratio of invis. to leptonic in radiative Z returns
m_{top} (MeV/c^2)	172 740	±	500	20	Small	$t\bar{t}$ threshold scan QCD errors dominate
Γ_{top} (MeV/c^2)	1410	±	190	40	Small	$t\bar{t}$ threshold scan QCD errors dominate
$\lambda_{top}/\lambda_{top}^{SM}$	m = 1.2	±	0.3	0.08	Small	$t\bar{t}$ threshold scan QCD errors dominate
$t\bar{t}Z$ couplings		±	30%	<2%	Small	E_{CM} = 365 GeV run

[A.Blondel et al., CERN YR 2019]

SMEFT

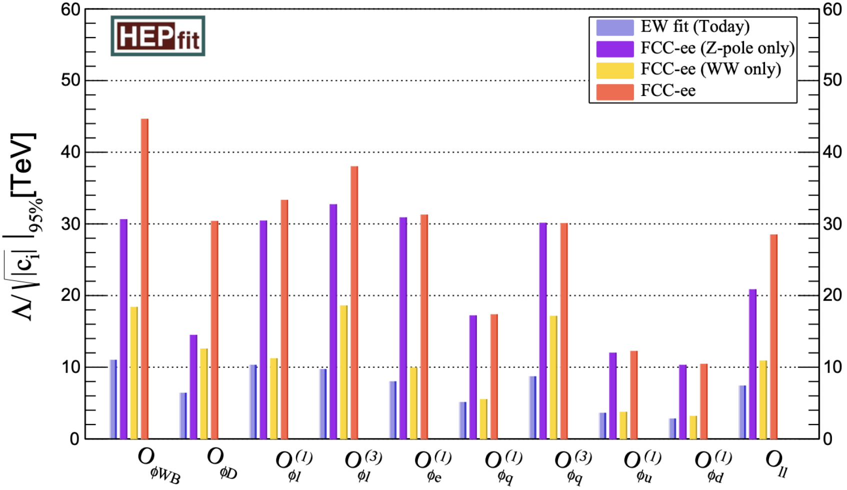
Possible deviations from SM predictions in differential and inclusive observables to be fit within SMEFT extension of the SM by 6 dim. operators

Remind three oblique Peskin–Takeuchi parameters used at LEP.
At a Z-factory one can (should) do a much more detailed study

Scenarios of specific new physics models can be also verified

N.B. Having polarized beams would help a lot

Sensitivity to new physics scale



[FCC Coll. EPJC'2019]

To-do list for QED

- Compute 2-loop QED radiative corrections to differential distributions of key processes: Bhabha scattering, muon decay, $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to \pi^+\pi^-$, $e^+e^- \to ZH$ etc.
- Estimate higher-order contributions within some approximations
- Account for interplay with QCD and electroweak effects
- Construct reliable Monte Carlo codes

Perturbative QED (I)

Fortunately, in our case the general perturbation theory can be applied:

$$\frac{\alpha}{2\pi} \approx 1.2 \cdot 10^{-3}, \quad \left(\frac{\alpha}{2\pi}\right)^2 \approx 1.4 \cdot 10^{-6}$$

Moreover, other effects: hadronic vacuum polarization, (electro)weak contributions, hadronic pair emission, etc. are small in, e.g., Bhabha scattering and can be treated one-by-one separately

Nevertheless, there are some enhancement factors:

1) First of all, the large logarithm $L \equiv \ln \frac{\Lambda^2}{m_e^2}$ where $\Lambda^2 \sim Q^2$ is the momentum transferred squared, e.g., $L(\Lambda = 1\,\text{GeV}) \approx 16$ and $L(\Lambda = M_Z) \approx 24$.

2) The energy region at the Z boson peak ($s \sim M_Z^2$) requires a special treatment since factor M_Z/Γ_Z appears in the annihilation channel

Perturbative QED (II)

Methods of resummation of higher-order QED corrections

- Resummation of vacuum polarization corrections (geometric series)
- Yennie–Frautschi–Suura (YFS) soft photon exponentiation and its extensions, see, e.g., PHOTOS
- Resummation of leading logarithms via QED structure functions or QED PDFs (E.Kuraev and V.Fadin 1985;
A. De Rujula, R. Petronzio, A. Savoy-Navarro 1979)

N.B. Resummation of real photon radiation is good for sufficiently inclusive observables…

Leading and next-to-leading logs in QED

The QED leading (LO) logarithmic corrections

$$\sim \left(\frac{\alpha}{2\pi}\right)^n \ln^n \frac{s}{m_e^2}$$

were relevant for LEP measurements of Bhabha, $e^+e^- \to \mu^+\mu^-$ etc. for $n \leq 3$ since $\ln(M_Z^2/m_e^2) \approx 24$

NLO contributions

$$\sim \left(\frac{\alpha}{2\pi}\right)^n \ln^{n-1} \frac{s}{m_e^2}$$

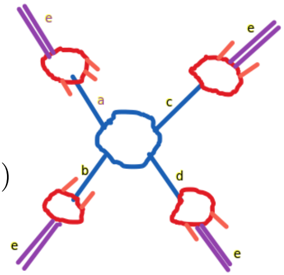
with at least $n = 3, 4$ are required for future e^+e^- colliders

In the collinear approximation we can get them within the NLO QED structure function formalism

- F.A.Berends, W.L. van Neerven, G.J.Burgers, NPB'1988
- A.A., K.Melnikov, PRD'2002; A.A. JHEP'2003

QED NLO master formula

The NLO Bhabha cross section reads



$$d\sigma = \sum_{a,b,c,d=e,\bar{e},\gamma} \int_{\bar{z}_1}^{1} dz_1 \int_{\bar{z}_2}^{1} dz_2 \mathcal{D}^{\rm str}_{ae}(z_1)\mathcal{D}^{\rm str}_{b\bar{e}}(z_2)$$
$$\times \left[d\sigma^{(0)}_{ab\to cd}(z_1,z_2) + d\bar{\sigma}^{(1)}_{ab\to cd}(z_1,z_2) \right]$$
$$\times \int_{\bar{y}_1}^{1} \frac{dy_1}{Y_1} \int_{\bar{y}_2}^{1} \frac{dy_2}{Y_2} \mathcal{D}^{\rm frg}_{ec}\left(\frac{y_1}{Y_1}\right) \mathcal{D}^{\rm frg}_{\bar{e}d}\left(\frac{y_2}{Y_2}\right)$$
$$+\mathcal{O}\left(\alpha^n L^{n-2}, \frac{m_e^2}{s}\right)$$

$\alpha^2 L^2$ and $\alpha^2 L^1$ terms are completely reproduced [A.A., E.Scherbakova, JETP Lett. 2006; PLB 2008] || $\bar{e} \equiv e^+$

High-order ISR in e^+e^- annihilation

$$\frac{d\sigma_{e^+e^-\to\gamma^*}}{ds'} = \frac{1}{s}\sigma^{(0)}(s') \sum_{a,b=e^-,\gamma,e^+} D_{ae^-} \otimes \tilde{\sigma}_{ab\to\gamma^*} \otimes D_{be^+}$$

$a\backslash b$	e^+	γ	e^-
e^-	$D_{e^-e^-}D_{e^+e^+}\sigma_{e^-e^+}$ LO (1)	$D_{\gamma e^-}D_{e^-e^-}\sigma_{e^-\gamma}$ NLO ($\alpha^2 L$)	$D_{e^-e^-}D_{e^-e^+}\sigma_{e^-e^-}$ NNLO ($\alpha^4 L^2$)
γ	$D_{\gamma e^-}D_{e^+e^+}\sigma_{e^+\gamma}$ NLO ($\alpha^2 L$)	$D_{\gamma e^-}D_{\gamma e^+}\sigma_{\gamma\gamma}$ NNLO ($\alpha^4 L^2$)	$D_{\gamma e^-}D_{e^-e^+}\sigma_{e^-\gamma}$ NLO ($\alpha^4 L^3$)
e^+	$D_{e^+e^-}D_{e^+e^+}\sigma_{e^+e^+}$ NNLO ($\alpha^4 L^2$)	$D_{e^+e^-}D_{\gamma e^+}\sigma_{e^+\gamma}$ NLO ($\alpha^4 L^3$)	$D_{e^+e^-}D_{e^-e^+}\sigma_{e^+e^-}$ LO ($\alpha^4 L^4$)

Contributions from $D_{e^-e^+}$ and $D_{e^+e^-}$ are missed in [J. Ablinger, J. Blümlein, A. De Freitas and K. Schönwald, “Subleading Logarithmic QED Initial State Corrections to $e^+e^-\to\gamma^*/Z^{0^*}$ to $O(\alpha^6 L^5)$,” NPB 955 (2020) 115045]

QED NLO DGLAP evolution equations

$$\mathcal{D}_{ba}\left(x, \frac{\mu_R}{\mu_F}\right) = \delta_{ab}\delta(1-x) + \sum_{c=e,\gamma,\bar{e}} \int\limits_{\mu_R^2}^{\mu_F^2} \frac{dt}{t} \int\limits_{x}^{1} \frac{dy}{y} P_{bc}(y,t) \mathcal{D}_{ca}\left(\frac{x}{y}, \frac{\mu_R^2}{t}\right)$$

μ_F is a factorization (energy) scale

μ_R is a renormalization (energy) scale

D_{ba} is a parton density function (PDF)

P_{bc} is a splitting function or kernel of the DGLAP equation

N.B. In QED $\mu_R = m_e \approx 0$ is the natural choice

Running coupling constant

Compare QED-like

$$\bar{\alpha}(t) = \alpha\left\{1 + \frac{\alpha}{2\pi}\left(-\frac{10}{9} + \frac{2}{3}L\right) + \left(\frac{\alpha}{2\pi}\right)^2\left(-\frac{13}{27}L + \frac{4}{9}L^2 + \ldots\right) + \ldots\right.$$

and QCD-like

$$\bar{\alpha}(t) = \frac{4\pi}{\beta_0 \ln(t/\Lambda^2)}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\ln[\ln(t/\Lambda^2)]}{\ln(t/\Lambda^2)} + \ldots\right]$$

Note that "$-10/9$" could have been hidden into Λ

In QED $\beta_0 = -4/3$ and $\beta_1 = -4$

$\mathcal{O}(\alpha)$ matching

The expansion of the master formula for ISR gives

$$d\sigma^{(1)}_{e\bar{e}\to\gamma^*} = \frac{\alpha}{2\pi}\left\{2LP^{(0)} \otimes d\sigma^{(0)}_{e\bar{e}\to\gamma^*} + 2d^{(1)}_{ee} \otimes d\sigma^{(0)}_{e\bar{e}\to\gamma^*}\right\} + d\,\bar{\sigma}^{(1)}_{e\bar{e}\to\gamma^*} + \mathcal{O}\left(\alpha^2\right)$$

We know the massive $d\sigma^{(1)}$ and massless $d\,\bar{\sigma}^{(1)}$ ($m_e \to 0$ with $\overline{\text{MS}}$ subtraction) results in $\mathcal{O}(\alpha)$. E.g.

$$\frac{d\sigma^{(1)}_{e\bar{e}\to\gamma^*}}{d\sigma^{(0)}_{e\bar{e}\to\gamma^*}} = \frac{\alpha}{\pi}\left[\frac{1+z^2}{1-z}\right]_+ \left(\ln\frac{s}{m_e^2} - 1\right) + \delta(1-z)(\ldots), \quad z \equiv \frac{s'}{s}$$

Scheme dependence comes from here

Factorization scale dependence is also from here

N.B. "Massification procedure”

Factorization scale choice

We apply the BLM-like prescription, i.e., hide the bulk of one-loop correction into the scale

For e^+e^- annihilation

$$\frac{d\sigma^{(1)}_{e\bar{e}\to\gamma^*}}{d\sigma^{(0)}_{e\bar{e}\to\gamma^*}} = \frac{\alpha}{\pi}\left[\frac{1+z^2}{1-z}\right]_+ \left(\ln\frac{s}{m_e^2} - 1\right) + \delta(1-z)(...) \Rightarrow \mu_F^2 = s \quad \text{or } \mu_F^2 = \frac{s}{e}$$

Remind Drell-Yan where we usually take $\mu_F^2 = s' \equiv zs$, i.e., the enegry scale of the hard subprocess (?!)

For muon decay $\mu_F = m_\mu$ is good, but $\mu_F = m_\mu z(1-z)$ is better. It was cross-checked with the help of (partially) known two-loop results [K.Melnikov et al. JHEP'2007]

Iterative solution

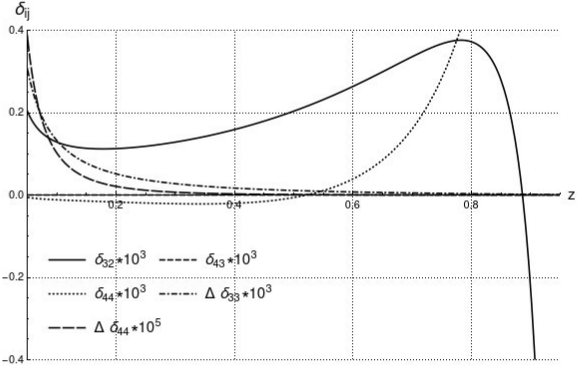
The NLO "electron in electron" PDF reads [A.A., U.Voznaya, JPG 2023]

$$
\begin{aligned}
\mathcal{D}_{ee}(x,\mu_F,m_e) &= \delta(1-x) + \frac{\alpha}{2\pi} L P^{(0)}_{ee}(x) + \frac{\alpha}{2\pi} d^{(1)}_{ee}(x,m_e,m_e) \\
&+ \left(\frac{\alpha}{2\pi}\right)^2 L^2 \left(\frac{1}{2} P^{(0)}_{ee} \otimes P^{(0)}_{ee}(x) + \frac{1}{2} P^{(0)}_{ee}(x) + \frac{1}{2} P^{(0)}_{e\gamma} \otimes P^{(0)}_{\gamma e}(x) \right) \\
&+ \left(\frac{\alpha}{2\pi}\right)^2 L \left(P^{(0)}_{e\gamma} \otimes d^{(1)}_{\gamma e}(x,m_e,m_e) + P^{(0)}_{ee} \otimes d^{(1)}_{ee}(x,m_e,m_e) - \frac{10}{9} P^{(0)}_{ee}(x) + P^{(1)}_{ee}(x) \right) \\
&+ \left(\frac{\alpha}{2\pi}\right)^3 L^3 \left(\frac{1}{6} P^{(0)}_{ee} \otimes P^{(0)}_{ee} \otimes P^{(0)}_{ee}(x) + \frac{1}{6} P^{(0)}_{e\gamma} \otimes P^{(0)}_{\gamma\gamma} \otimes P^{(0)}_{\gamma e}(x) + \dots \right) \\
&+ \left(\frac{\alpha}{2\pi}\right)^3 L^2 \left(P^{(0)}_{ee} \otimes P^{(1)}_{ee}(x) + P^{(0)}_{ee} \otimes P^{(0)}_{ee} \otimes d^{(1)}_{ee}(x,m_e,m_e) + \frac{1}{3} P^{(1)}_{ee}(x) - \frac{10}{9} P^{(0)}_{ee} \otimes P^{(0)}_{ee}(x) + \dots \right) \\
&+ \mathcal{O}(\alpha^2 L^0, \alpha^3 L^1)
\end{aligned}
$$

The large logarithm $L \equiv \ln \frac{\mu_F^2}{\mu_R^2}$ with factorization scale $\mu_F^2 \sim s$ or $\sim -t$; and renormalization scale $\mu_R = m_e$.

Higher-order effects in e^+e^- annihilation

$$d\sigma^{\text{NLO}}_{e\bar{e}\to\gamma^*} = d\sigma^{(0)}_{e\bar{e}\to\gamma^*}\left\{1+\sum_{k=1}^{\infty}\left(\frac{\alpha}{2\pi}\right)^k\sum_{l=k-1}^{k}\delta_{kl}L^l+\mathcal{O}(\alpha^k L^{k-2})\right\}$$



[A.A., U.Voznaya, PRD'2024]

ISR corrections to $e^+e^- \to Z(\gamma^*)$ ($\sqrt{s} = M_Z$)

LO $\mathcal{O}(\alpha^n L^n)$ and NLO $\mathcal{O}(\alpha^n L^{n-1})$ ISR corrections in % at the Z-peak for $z_{\min} = 0.1$

Type / n	1	2	3	4	5
LO γ	−32.7365	4.8843	−0.3776	0.0034	0.0032
NLO γ	2.0017	−0.5952	0.0710	−0.0019	
LO pair	—	−0.3057	0.0875	0.0016	−0.0001
NLO pair	—	0.1585	−0.0460	0.0038	
Σ	−30.7348	4.1419	−0.2651	0.0069	0.0031

N.B. $\mathcal{O}(\alpha^2 L^0)$ ISR corrections are known [Berends; Blümlein]

Impact of new corrections on LEP results?!

PRELIMINARY NUMBERS

QED PDFs vs. QCD ones

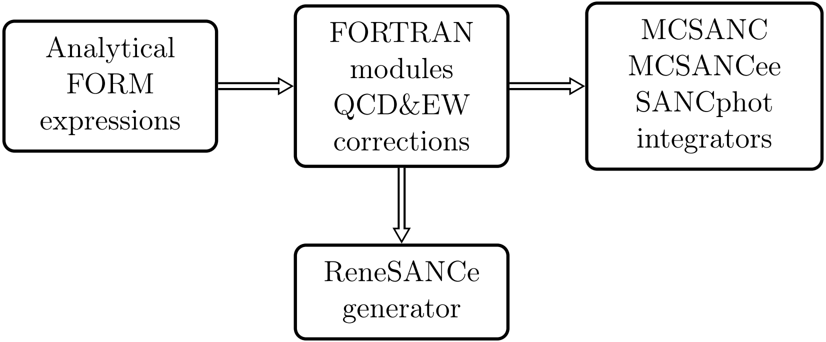
Common properties:

- QED splitting functions = abelian part of QCD ones
- The same structure of DGLAP evolution equations
- The same Drell-Yan-like master formula with factorization
- Factorization scale and scheme dependence

Peculiar properties:

- QED PDFs are calculable
- QED PDFs are less inclusive
- QED renormalization scale $\mu_R = m_e$ is preferable
- QED PDFs can (do) lead to huge corrections
- Massification procedure

The SANC framework and products family



Publications:

SANC – CPC 174 481-517
MCSANC – CPC 184 2343-2350; JETP Letters 103, 131-136
SANCphot – CPC 294 108929
ReneSANCe – CPC 256 107445; CPC 285 108646

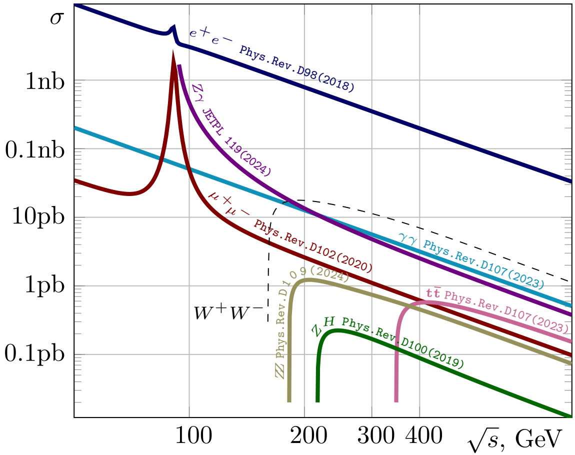
SANC products are available at http://sanc.jinr.ru/download.php

ReneSANCe is also available at http://renesance.hepforge.org

SANC advantages:

- full one-loop electroweak corrections
- leading higher order corrections
- massive case
- accounting for polarization effects
- full phase space operation
- results of ReneSANCe event generator and SANC integrators are thoroughly cross checked

Basic processes of SM for e^+e^- annihilation



The cross sections are given for polar angles between $10^\circ < \theta < 170^\circ$ in the final state.

ReneSANCe Monte Carlo event generator

- Based on the SANC modules
- Complete one-loop and some higher-order electroweak radiative corrections
- Unweighted events in ROOT and LHE format
- Thoroughly cross checked against MCSANC integrator

Outlook

- A new high-energy e^+e^- collider is well motivated by the necessity to study SM in more detail
- Complementarity to hadron-hadron machines is essential
- A Z-factory provides unique possibilities for progress in HEP
- New theoretical calculations of higher-order corrections are required
- Chains of interfaced Monte Carlo codes to be developed
- The work is started, but there are still many tasks

Electron is as inexhaustible as atom (1909)

Thank you for attention!