Multiplicity distributions in the eikonal and the U-matrix unitarization schemes

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We compare the 'eikonal' and the U-matrix' unitarization schemes for the Pomeron with $\alpha_P(0) > 1$ and using the AGK cutting rules caculate the multiplicity distributions expected inn both approaches.

The data prefers the eikonal

Plan

1. Eikonal and U-matr. unitarizations

2. AGK cutting rules

3. Multiplicity distribution

4. Disadvantages of U-matr. at high energy

1. Unitarization

$$2\mathrm{Im}\mathcal{A}(s,b) = |\mathcal{A}(s,b)|^2 + G_{inel}(s,b)$$

Two solutions for $\mathrm{Im}\mathcal{A}$:

$$\mathrm{Im}\mathcal{A}(s,b) = \frac{1 \pm \sqrt{1-(1+\rho^2)G_{inel}(s,b)}}{1+\rho^2};$$

$\rho(s,b) = \mathrm{Re}/\mathrm{Im}$

$$\mathcal{A}(s,t) = s\int bdbJ_0(bq)\mathcal{A}(s,b)$$

Eikonal

$$\mathcal{A}(s,b) \;=\; i[1-e^{i\chi(s,b)}] = -i\sum_{n=1}^{\infty}\frac{[i\chi(s,b)]^n}{n!}$$

U-matr.

$$\mathcal{A}(s,b) \;=\; \frac{\hat{\chi}(s,b)}{1-i\hat{\chi}(s,b)/2} = -2i\sum_{n=1}^{\infty}\frac{[i\hat{\chi}(s,b)]^n}{2^n}$$

Note - the first two terms are the SAME

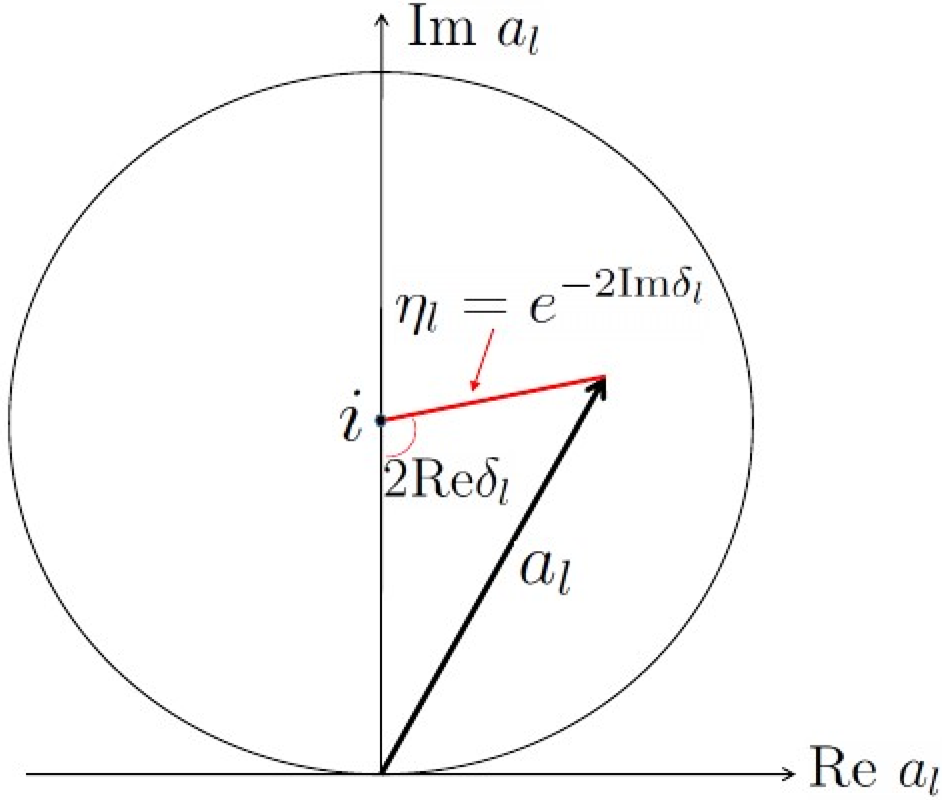
Inelastic contribution:

$$G_{inel}^{eik}(s,b) = 1 - e^{-2\mathrm{Im}\chi(s,b)} \to 1$$

$$G_{inel}^{U}(s,b) = \; 2\mathrm{Im}\mathcal{A}(s,b) - |\mathcal{A}(s,b)|^2$$

$$= \; \frac{2\mathrm{Im}\hat{\chi}(s,b)}{(1 - i\hat{\chi}(s,b)/2)(1 + i\hat{\chi}^*(s,b)/2)} \to 0$$

The hole in center at $|\hat{\chi}| \to \infty$



The partial wave amplitude, $a_l \ = \ i(1 - e^{2i\delta_l})$,

2. AGK rules

$$\text{unitarity} - \quad 2\text{Im}\mathcal{A}_{ij} = disc\mathcal{A}_{ij} = \sum_m \mathcal{A}^*_{im}\mathcal{A}_{mj}$$

Im$\mathcal{A}$ is given by the cut of diagr. over state m

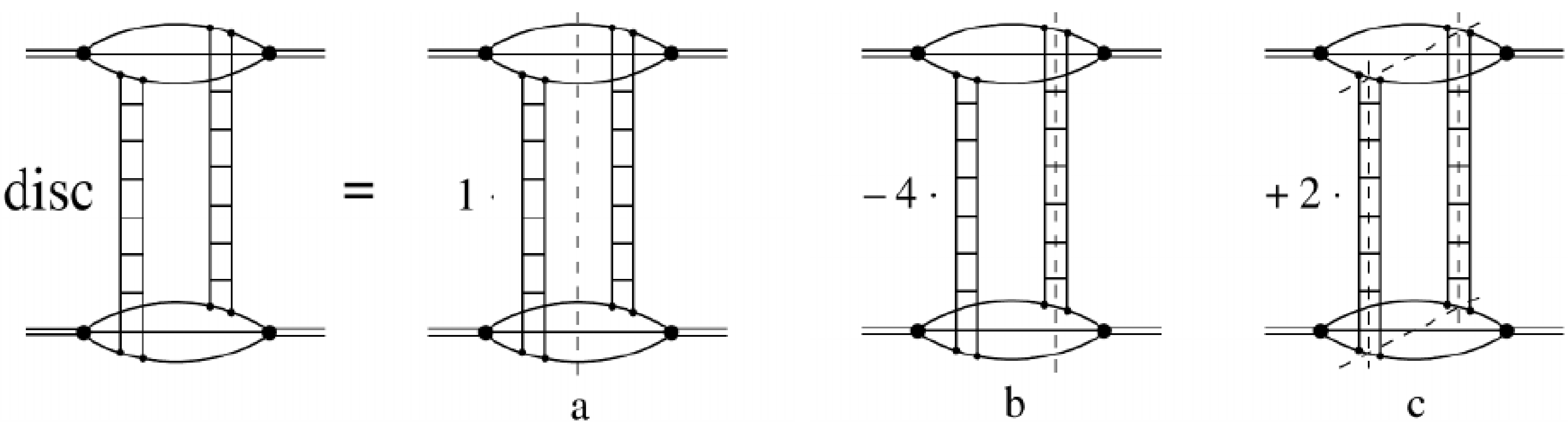


FIG. 1. Two-Pomeron exchange in the t channel expressed as a sum over all intermediate states in the s-channel.

Pomeron exchange gives $\mathrm{Im}\chi = \mathbb{P}/2$

($\mathbb{P}$ = cut Pomeron)

In the cut Pomeron we deal with discontinuity ($disc = 2\mathrm{Im}A$) while the uncut Pomeron can be to the left or the right of the cut (this is the origin of factor 2 in (9)) and this way the real part canceled.

$$\mathrm{Im}\mathcal{A}_{(\mathrm{cut}\,\mathbb{P})}(s, t = 0) = s \sum C_n (-1)^{n-1} \mathbb{P}^n$$

cutting k Pomerons from the term $C_n(-1)^{n-1}\mathbb{P}^n$

$$c_n^{k \neq 0} = (-1)^{n-k} 2^{n-1} \frac{n!}{k!(n-k)!} \, , \qquad (9)$$

$$c_n^{k=0} = (-1)^n (2^{n-1} - 1)$$

$$\sigma^k(s, b) = 2 \sum_n C_n \cdot (-1)^{n-k} 2^{n-1} \frac{n! [\mathbb{P}(s, b)]^n}{k!(n-k)!}$$

$$\frac{n!\mathbb{P}^n}{(n-k)!} = \mathbb{P}^k \left(\frac{d}{d\mathbb{P}}\right)^k \mathbb{P}^n$$

$$\sigma^k_{eik}(s) = \int d^2b\, \frac{[\mathrm{Re}\Omega(s,b)]^k}{k!} \exp(-\mathrm{Re}\Omega(s,b))$$

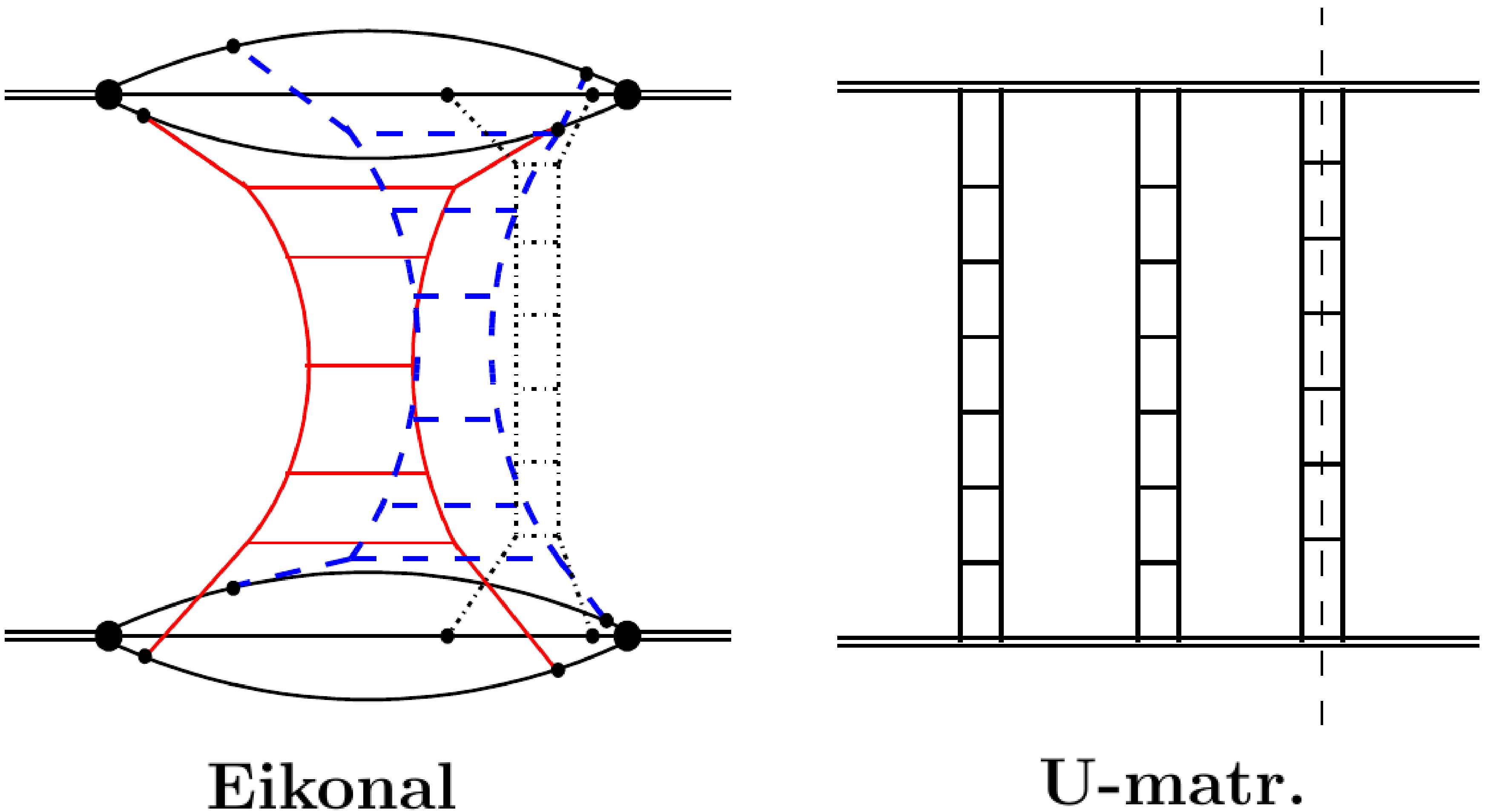
$$\Omega(s,b) \equiv -2i\chi(s,b) \qquad\qquad \mathrm{Re}\Omega = \mathbb{P}$$

$$\sigma^k_U(s) = 2\int d^2b \left[\frac{\mathrm{Im}\hat{\chi}(s,b)}{1+\mathrm{Im}\hat{\chi}(s,b)}\right]^k \frac{1}{1+\mathrm{Im}\hat{\chi}(s,b)}$$

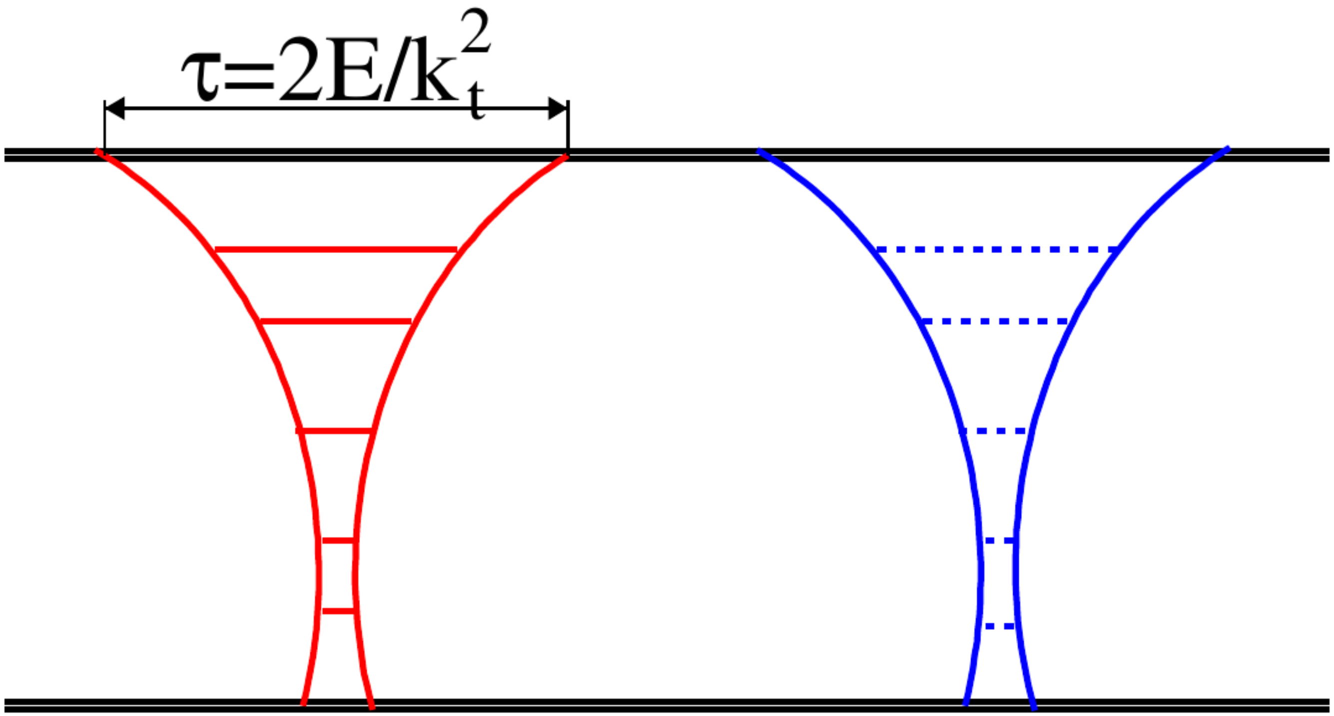
$$\text{Problem} - \sum_k \sigma^k_U(s,b) = 2\frac{\mathrm{Im}\hat{\chi}(s,b)}{1+\mathrm{Im}\hat{\chi}(s,b))} \to 2 \neq 0$$

$$2\mathrm{Im}\mathcal{A}(s,b) = |\mathcal{A}(s,b)|^2 + G_{inel}(s,b)$$

U-matr. is inconsistent with AGK -???



In U-matr. case one can cut only one object; it is impossible to cut a few "quasi potentials" (Pomerons) simultaneously.



$\tau=2E/k_t^2$

3. Multiplicity distribution

For numerical estimates, we take the parameters of the Pomeron trajectory and the Pomeron-proton coupling from 2402.11385.

We assume that one Pomeron produces the Poisson distribution in $N = N^{\mathbb{P}}_{ch}/C$.

[C accounts for "short range correlations" and denotes the mean charged multiplicity of a cluster (resonance or minijet). Due to electric charge conservation, we expect $C > 2$.]

value of $N^{\mathbb{P}}_{ch}$ is chosen to reproduce the particle density $dN_{ch}/d\eta$

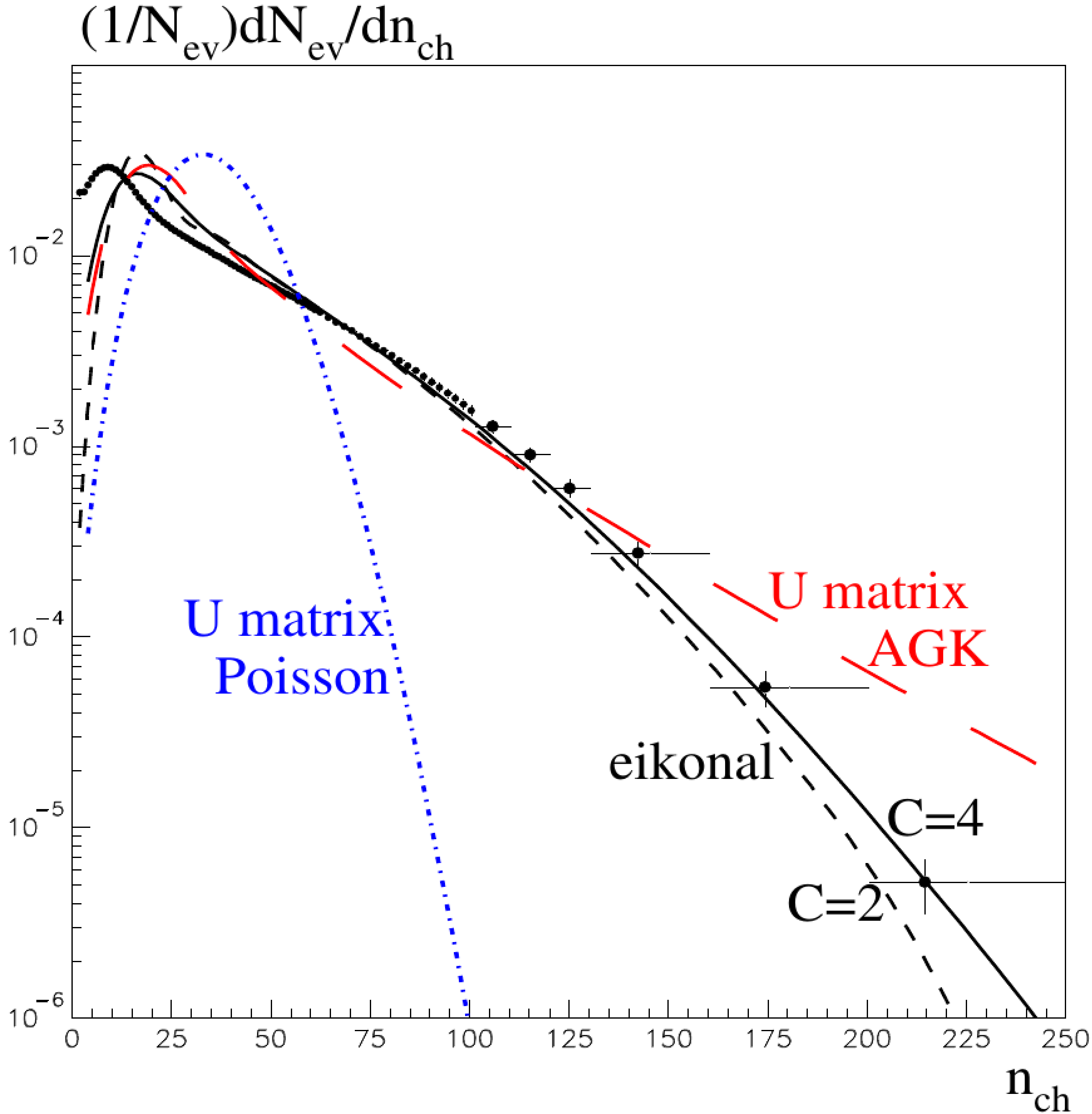
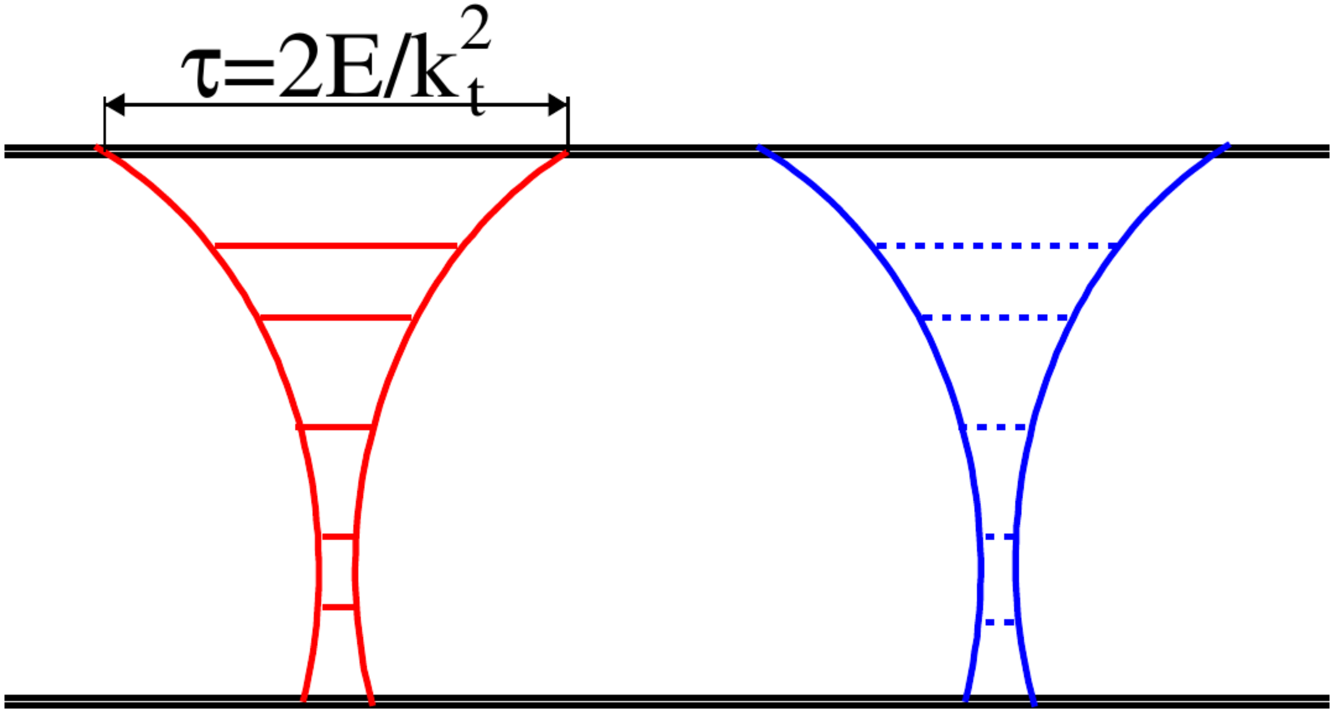


FIG. 3. Charged particle central region ($-2.5 < \eta < 2.5$) multiplicity distribution in the eikonal (black continuous and short dashed curves) and the U-matrix (red long dashed and blue dot-dashed curves; $C = 4$) unitarization schemes. The data are from [13].

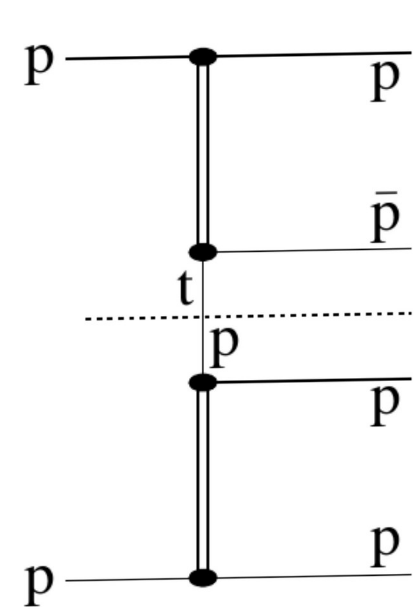
4. Disadvantages of U-matr. at high energy

a) S. Mandelstam, Nuovo Cim. 30, 1148 (1963)
and space-time picture



b) $\sigma(pp \to p + (\bar{p}p) + p) \propto \ln s$

on contrary to $G^{U}_{inel}(s,b) \to 0$



c) $dN/d\eta \propto s^{0.23}$

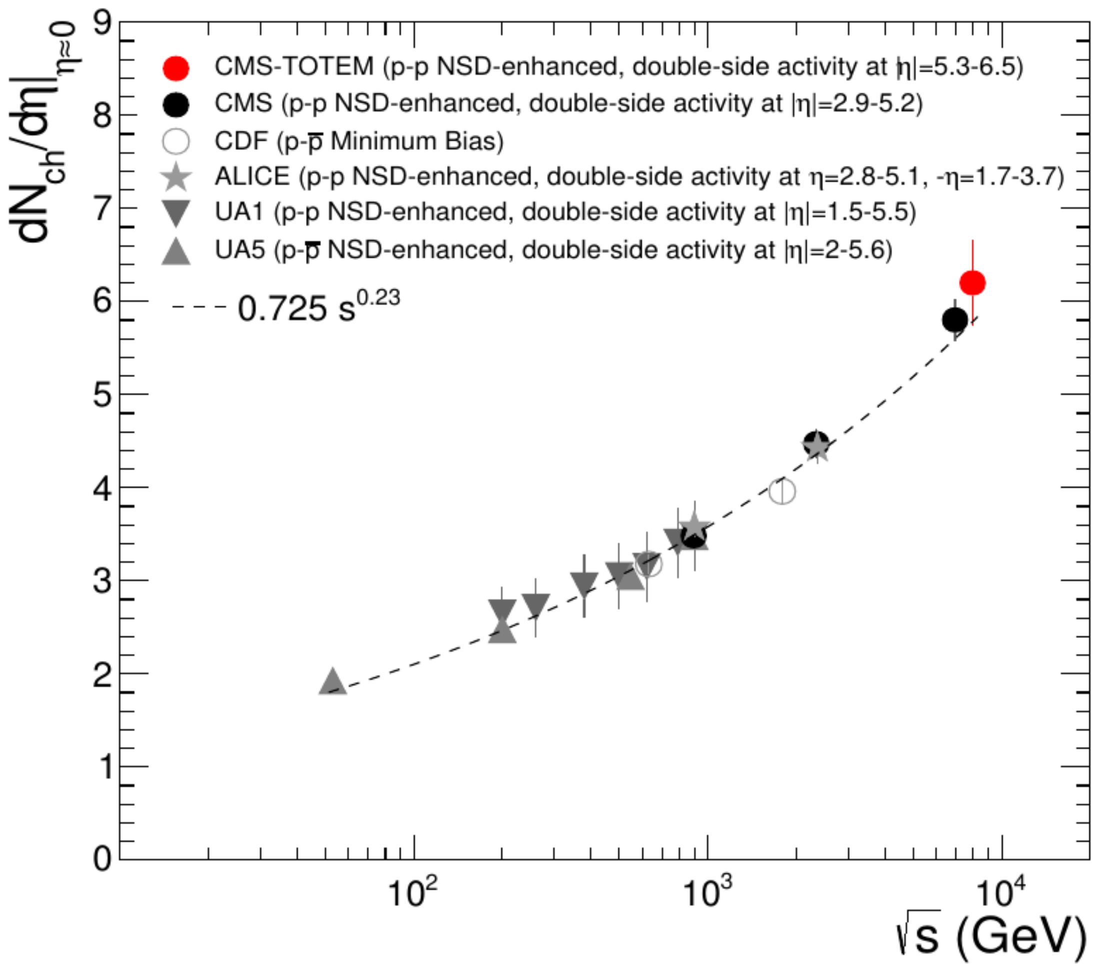


Figure 7: Value of $dN_{ch}/d\eta$ at $\eta \approx 0$ as a function of the centre-of-mass energy in pp and $p\overline{p}$ collisions. Shown are measurements performed with different NSD event selections from UA1 [12], UA5 [14], CDF [10, 11], ALICE [6], and CMS [4]. The dashed line is a power-law fit to the data.

THANK YOU