4. Alpha Transition Probabilities

Preston[1] has derived the solution for $t^{\alpha}_{1/2}$, in a one-body model, for the transfer of l-units of angular momentum to the alpha particle from a parent of atomic number Z, as

$$t^{\alpha}_{1/2} = \ln 2 \frac{r_0}{2v} \frac{\mu^2(H_l^2 + K_l^2) + \tan^2\alpha_0(C_l^2 + S_l^2) + 2\mu\tan\alpha_0(C_l K_l - S_l H_l)}{\mu^2\tan\alpha_0(H_l C_l + K_l S_l)Q_l} e^{+2\omega_0} \quad (1)$$

where

$$\mu = -\tan\alpha_0 \frac{S_l \tan(\mu k r_0) + C_l}{K_l - H_l \tan(\mu k r_0)} \quad (2)$$

and

$$\alpha_0 = \cos^{-1}(mv^2 r_0/4e^2 Z)^{1/2} \quad (3)$$

$$k = mv/\hbar$$

$$\omega_0 = \frac{4e^2 Z}{\hbar v}(\alpha_0 - \sin\alpha_0\cos\alpha_0).$$

Q_l is a rational function of $\tan\alpha_0$ and the daughter nuclear radius r_0, and C_l, S_l, H_l, and K_l are polynomials in $M = \frac{1}{\mu k r_0}$, defined in Table 6.

Table 6.

l-transfer	Parameters
l=0	$Q_0=1$, $S_0=1$, $C_0=0$, $H_0=0$, $K_0=1$
l=1	$Q_1=\frac{(\kappa-2\tan\alpha_0)}{(\kappa+2\tan\alpha_0)}$, $S_1=M$, $C_1=-1$, $H_1=M^2-1$, $K_1=M$
l=2	$Q_2=\frac{(\kappa+10\mu M-6\tan\alpha_0)}{(\kappa+10\mu M+6\tan\alpha_0)}$, $S_2=3M^2-1$, $C_2=-3M$, $H_2=3M(2M^2-1)$, $K_2=6M^2-1$
l=3	$Q_3=\frac{\kappa+28\mu M-12\tan\alpha_0+(44/\kappa)\tan^2\alpha_0}{\kappa+28\mu M+12\tan\alpha_0+(44/\kappa)\tan^2\alpha_0}$, $S_3=-6M+15M^3$, $C_3=1-15M^2$, $H_3=1-21M^2+45M^4$, $K_3=-6M+45M^3$
l=4	$Q_4=\frac{\kappa(\kappa+60\mu M)-20\tan\alpha_0(40\mu M+\kappa)+140}{\kappa(\kappa+60\mu M)+20\tan\alpha_0(40\mu M+\kappa)+140}$, $S_4=1-45M^2+105M^4$, $C_4=10M-105M^3$, $H_4=10M-195M^3+420M^5$, $K_4=1-55M^2+420M^4$

In the equations above, m is the alpha-particle mass, $v=\sqrt{2E_\alpha/m}$ is the alpha-particle velocity where the α particle energy (in laboratory coordinates) is given by

$$E_\alpha = Q'_\alpha \frac{M_{\text{recoil}}}{M_{\text{recoil}} + m}, \quad (4)$$

$\kappa=4e^2Z/hv$, and Q'_α is the alpha decay energy, in MeV, corrected for screening by[2]

$$Q'_\alpha = Q_\alpha + 0.0000653 Z^{7/5} - 0.000080 Z^{2/5} \quad (5)$$

The parameters μ and r_0 are variables whose values are typically established for ground-state to ground-state transitions of even-even nuclei (l=0) by equating the right side of equation (2) to the experimental partial half-life for alpha decay, corresponding to a "hindrance factor" HF=1, and solving equations (2) and (3) simultaneously. The ground-state even-even r_0 values are used to calculate HF for transitions to excited states in the same nuclei. For odd-even and odd-odd nuclei, r_0 is interpolated from the values for neighboring even-even ground-state transitions. The l-transfer can take values from $|j_f - j_i|$ to $j_f + j_i$, where j_f, j_i are the final and initial level spins, respectively.

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[1] M.A. Preston, *Phys. Rev.* **71**, 865 (1947)

[2] J.O. Rasmussen, "Alpha Decay", in *Alpha-, Beta-, and Gamma-ray Spectroscopy*, K. Siegbahn editor, North-Holland, Amsterdam (1965).