

3. Beta Transition Probabilities

For an n^{th} -forbidden beta decay ($n=0$ for allowed), the transition probability can be written as¹

$$\frac{\ln 2}{t_{1/2}} = \frac{g^2}{2\pi^3} \int_1^{W_0} p W (W_0 - W)^2 S_n(Z, W) dW , \quad (1)$$

where g is the weak interaction coupling constant, Z is the atomic number of the daughter nucleus, W is the total energy of the β -particle in units of electron mass, $p=\sqrt{W^2 - 1}$ is the momentum of the β -particle, and W_0 is the maximum β -particle energy. For β^- decay $W_0=Q + 1$, and for β^+ decay $W_0=Q - 1$, where Q is the mass difference between initial and final states in neutral atoms. For an n^{th} -forbidden β -branch, $\Delta J=n$ (nonunique) or $\Delta J=n + 1$ (unique), and $\Delta\pi=(-1)^n$. $\Delta J=0$ is allowed for $n=1$.

$S_n(Z, W)$ is the shape factor defined as

$$S_n(Z, W) = \sum_{k_e, k_v=1}^{\infty} [\lambda_k M_L^2 + \lambda'_k m_L^2 - 2\lambda''_k M_L m_L] \quad (2)$$

where λ_k , λ'_k , and λ''_k are bilinear combinations of the radial components of the continuum electron or positron wavefunctions. M_L and m_L contain the nuclear matrix elements. L is the orbital angular momentum carried off by the electron and the neutrino. For k_e and k_v , $k=|\kappa|$, where $\kappa=I$ for $j=I-1/2$, and $\kappa=-I-1$ for $j=I+1/2$. For allowed and unique forbidden transitions, a single nuclear matrix element is dominant, while a combination of nuclear matrix elements contribute to non-unique forbidden transitions.

The partial half-life $t_{1/2}^\beta$ for an n^{th} -forbidden decay can be obtained from equation (1). A reduced half-life $f_n t$ is defined as

$$f_n t = \frac{2\pi^3 \ln 2}{g^2 \eta^2} , \quad (3)$$

where

$$f_n = \int_1^{W_0} p W (W_0 - W)^2 \frac{S_n(Z, W)}{\eta^2} dW , \quad (4)$$

and η^2 contains the nuclear matrix elements. For allowed decays,

$$\eta^2 = \frac{S_0(Z, W)}{\lambda_1(Z, W)} \quad (5)$$

and

$$f_0 = \int_1^{W_0} p W (W_0 - W)^2 \lambda_1(Z, W) dW . \quad (6)$$

More generally, for unique forbidden decays,

$$f_n = (2n + 1)! \int_1^{W_0} p W (W_0 - W)^2 \sum_{k=1}^{n+1} \frac{\lambda_k(Z, W) p^{2(k-1)} (W_0 - W)^{2(n-k+1)}}{(2k - 1)![2(n - k - 1) + 1]!} dW \quad (7)$$

For first-forbidden unique decay, equation (7) reduces to

$$f_1 = \int_1^{W_0} p W (W_0 - W)^2 [\lambda_1(Z, W) (W_0 - W)^2 + \lambda_2(Z, W) p^2] dW . \quad (8)$$

The f function can be calculated with corrections for finite nuclear size and screening by atomic electrons. Values of f_0 and f_1 for β^- decay with $Z=10-100$ are plotted in Figure 6. Electron-capture decay always competes with β^+ decay. Equation (4) applies to electron capture, except that the function f is written as a combination of the radial components of the bound-state electron wavefunctions $g_i(Z)$, $f_i(Z)$ corresponding to electron capture from i th atomic state.

¹N.B. Gove and M.J. Martin, *Nucl. Data Tables* **A10**, 205 (1971)

For allowed decay,

$$f_0^{EC} = \frac{\pi}{2} \left[q_K^2 g_K^2 B_K + q_{L1}^2 g_{L1}^2 B_{L1} + q_{L2}^2 f_{L2}^2 B_{L2} + \sum_i q_i^2 g_i^2 B_i + \sum_j q_j^2 f_j^2 B_j \right]. \quad (9)$$

B_i are the atomic exchange and overlap corrections, and $q_i = Q_{EC} - E_B$ are the neutrino energies where Q_{EC} is the mass difference between the initial and final state and E_B is the electron binding energy in the daughter nucleus. For first-forbidden unique transitions,

$$f_1^{EC} = \frac{\pi}{2} \left[q_K^4 g_K^2 B_K + q_{L1}^4 g_{L1}^2 B_{L1} + q_{L2}^4 g_{L2}^2 B_{L2} + \sum_i q_i^4 g_i^2 B_i + \sum_j q_j^4 f_j^2 B_j + \frac{9}{R^2} \left\{ q_{L3}^2 g_{L3}^2 B_{L3} + \sum_m q_m^2 g_m^2 B_m \right\} \right] \quad (10)$$

where i, j, m refer to the s, p, d electrons, respectively. The functions f_0^{EC} and f_1^{EC} are plotted for $Z=10-100$ in Figure 7.

If the available electron-capture decay energy q is greater than $2m_e c^2$ (1022 keV), both positron emission and electron-capture decay will compete. The nuclear matrix elements are identical for both modes and cancel in the ratio $\frac{EC}{\beta^+} = \frac{f_n^{EC}}{f_n^{\beta^+}}$ for unique transitions. The electron-capture to positron decay ratios for allowed and first forbidden transitions are plotted in Figure 8.

Because ft values vary over several orders of magnitude, it is customary to consider the quantity $\log ft$ rather than ft . The $\log ft$ values for non-unique forbidden transitions cannot be calculated directly, so it is conventional to use the allowed $\log ft$ value for systematic comparisons. Table 5 outlines the systematics of beta transitions^{2,3} as a function of $\log ft$.

Table 5. Systematics of Logft Values

Logft	Transitions
<3.5	$0^+ \rightarrow 0^+$ Superallowed ($\Delta T=0$) and $n, {}^3H, {}^6He, {}^{18}Ne$ decays
3.6-5.9 [†]	Allowed ($\Delta J=0,1$; no parity change)
≤ 6.4	Not $0^+ \rightarrow 0^+$ ($\Delta T>0$)
≤ 8.5 ^{1u}	$\Delta J=0,1$
<11.0	$\Delta J=0,1; \Delta J=2$, parity change
<12.8	$\Delta J=0,1,2$

[†]For heavier nuclei, higher-order nuclear matrix elements, especially those proportional to αZ^2 , become important. This leads to anomalously low $\log ft$ values for forbidden transitions. For the $Z=82$ mass region, the upper limit for allowed transitions should be lowered to ~5.1.

² S. Raman and N.B. Gove, *Phys. Rev. C7*, 1995 (1973).

³ M.J. Martin, *Nucl. Data Sheets* **74**, ix (1995).

Figure 6. Allowed and first-forbidden unique f^{β^-} function for Z=10-100

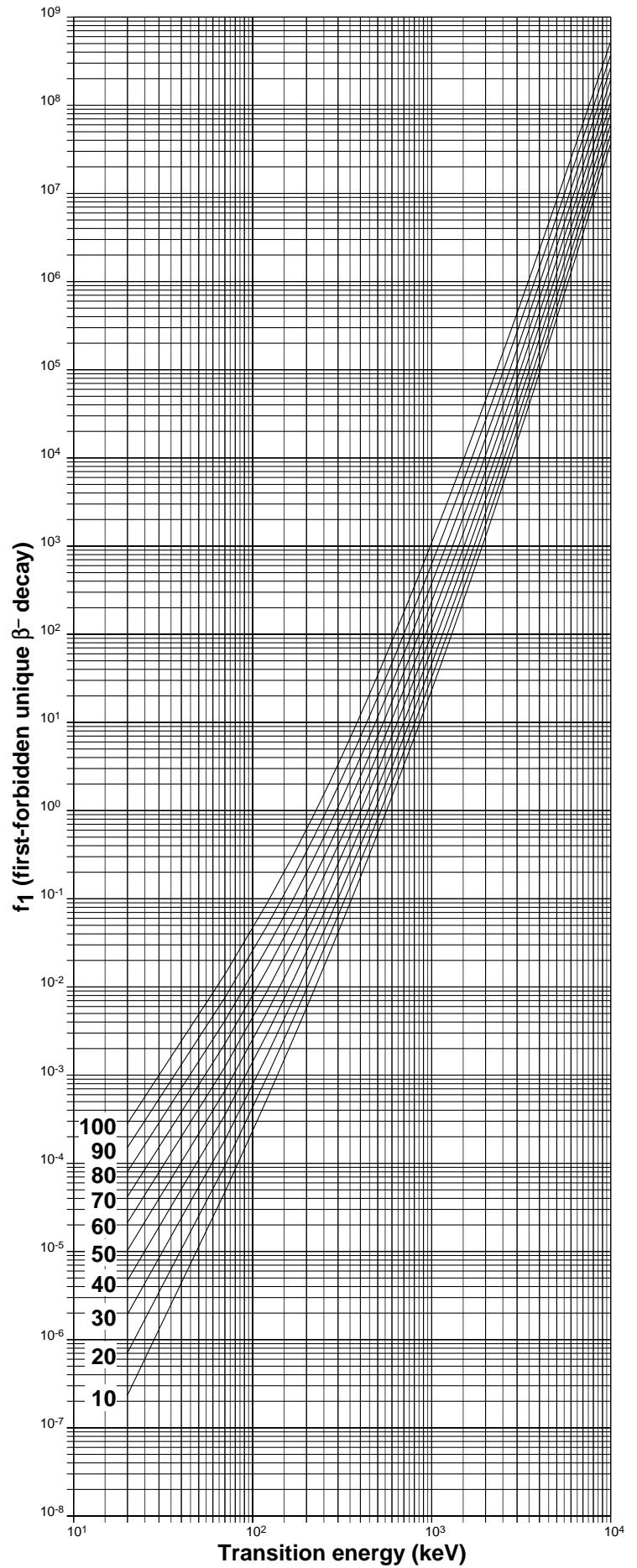
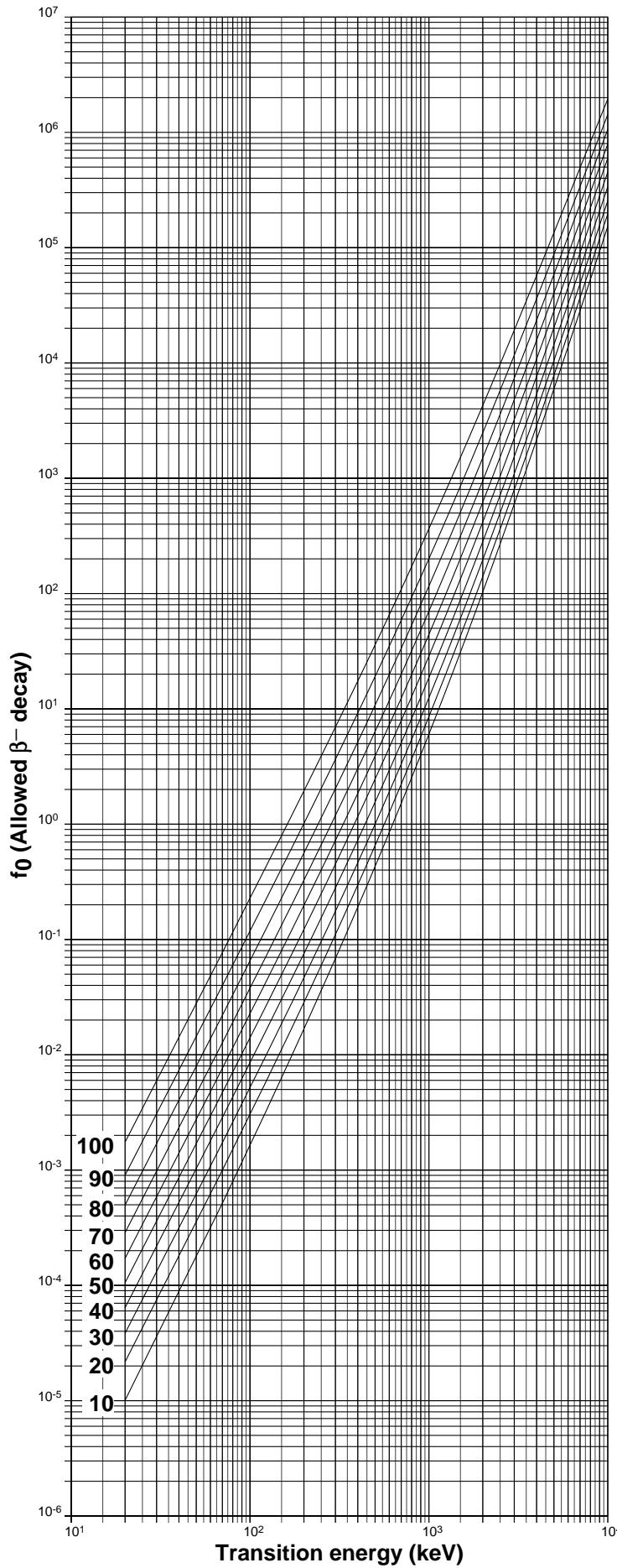


Figure 7. Allowed and first-forbidden unique f^{EC} function for Z=10-100

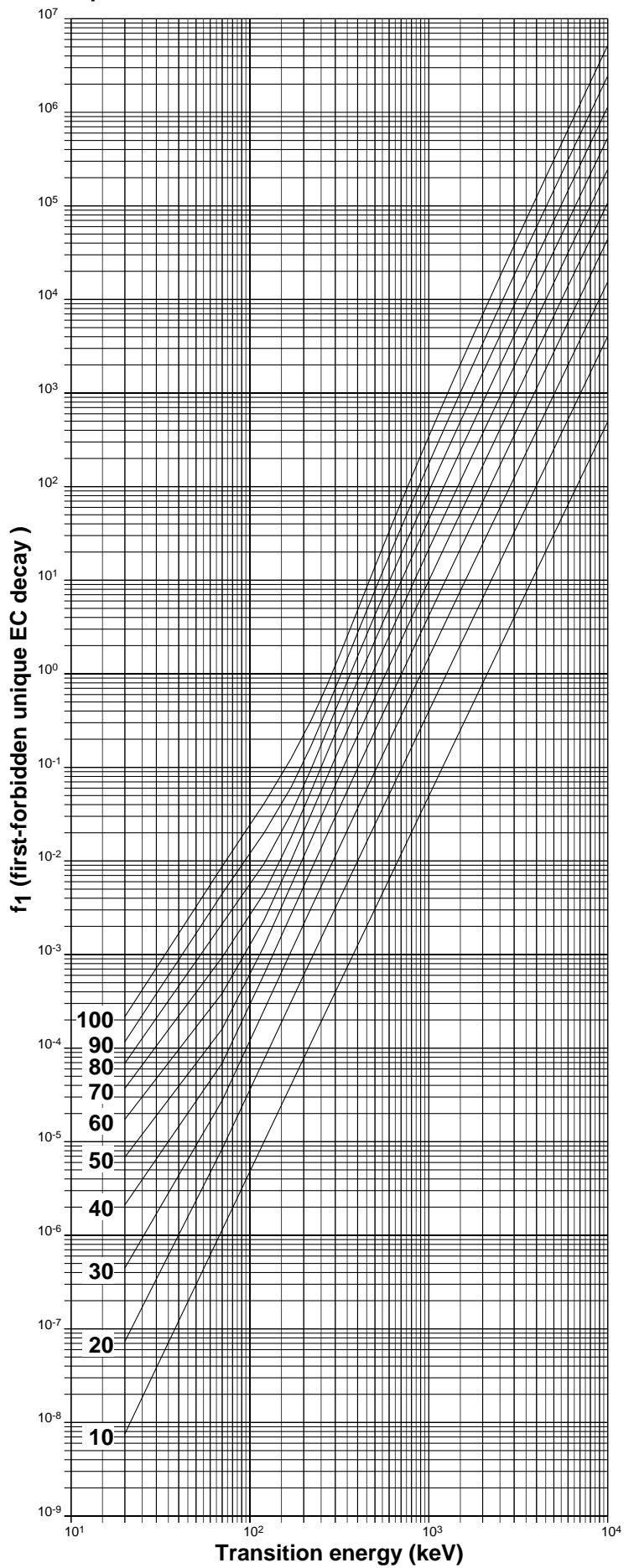
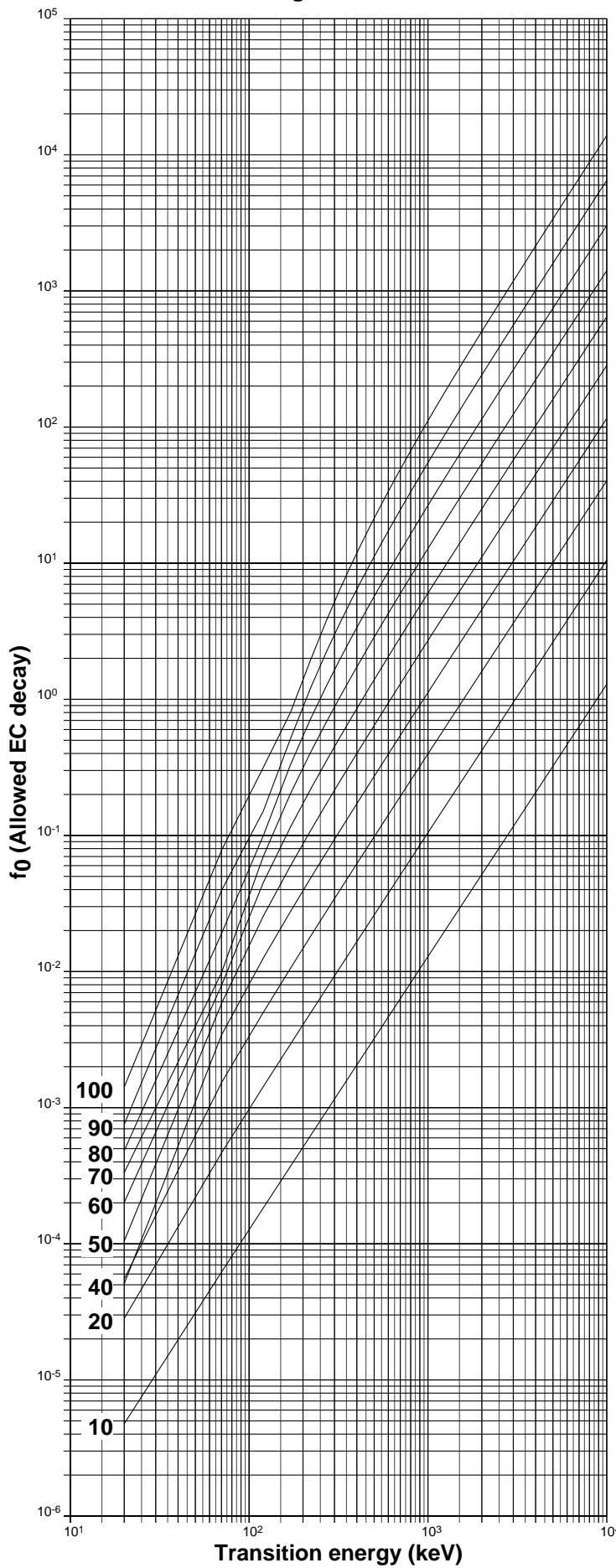


Figure 8. EC/ β^+ for allowed and first-forbidden unique decays with Z=10-100

