Eur. Phys. J. C 4, 711–715 (1998)
DOI 10.1007/s100529800778

THE EUROPEAN PHYSICAL JOURNAL C
© Springer-Verlag 1998

An estimate of the Higgs boson mass in two loop approximation in a noncommutative differential geometry

Yoshitaka Okumura

Department of Natural Science, Chubu University, Kasugai, 487, Japan (e-mail: okum@isc.chubu.ac.jp)

Received: 16 July 1997 / Published online: 23 February 1998

Abstract. An estimation of the Higgs boson mass is performed by numerically solving the renormalization group equations in the two loop approximation based on the condition $g^2 = (5/3)g'^2 = 4\lambda$ for $\mathrm{SU(2)_L}$, $\mathrm{U(1)_Y}$ gauge and the Higgs quartic coupling constants, respectively. This condition is introduced in the new scheme of our noncommutative differential geometry (NCG) for the reconstruction of the standard model. However, contrary to SU(5) GUT without supersymmetry, the grand unification of coupling constants is not realized in this scheme. The physical mass of the Higgs boson depends strongly on the top quark mass m_{top} through the Yukawa coupling of the top quark in the β functions. The two loop effect lowers the numerical value calculated within the one loop approximation by several GeV. The Higgs boson mass varies from 150.93GeV to 167.96GeV corresponding to $169.47\mathrm{GeV} \leq m_{\mathrm{top}} \leq 181.00\mathrm{GeV}$. We find $m_{\mathrm{H}} = 158.90\mathrm{GeV}$ for $m_{\mathrm{top}} = 175.01\mathrm{GeV}$ and $m_{\mathrm{H}} = 166.98\mathrm{GeV}$ for $m_{\mathrm{top}} = 180.37\mathrm{GeV}$.

1 Introduction

The Higgs mechanism is essential for any spontaneously broken gauge theory. Its presence ensures the renormalizability of the theory and makes the theory realistic by giving masses to particles, such as gauge and matter fields, through the vacuum expectation value of the Higgs boson field. The standard model in particle physics also involves the Higgs mechanism and shows remarkable agreement with existing data. After the discovery of the top quark in 1994, the only undetected particle in the standard model is the Higgs boson. Now, studies concerning the Higgs boson search are being conducted in both theoretical and experimental settings and it is expected that the Higgs boson will be discovered within a decade in future experiments at Fermi-Lab and CERN.

Many models (most notably, the technicolor model, the Kaluza-Klein model and recently the approach based on noncommutative differential geometry (NCG) on the discrete) have been constructed for the purpose of understanding the Higgs mechanism. Among these, the NCG approach, originally proposed by Connes [1], provides a unified picture of gauge and Higgs fields as a generalized connection on the principal bundle with the base space $M_4 \times Z_N$. It should be noted that the NCG approach does not demand any physical modes other than the usual one.

Many versions of the NCG approach have appeared since Connes's original presentation, and the standard model has been successfully reconstructed using these approaches. The characteristic feature of the reconstruction of the gauge theory in the NCG approach is to impose restrictions on the gauge and the Higgs quartic coupling constants. This is because the gauge and Higgs fields are represented together as a generalized gauge field. These restrictions yield numerical estimates of the Weinberg angle and mass relations involving the gauge boson and other particles, such as the Higgs boson and top quark in tree level. Several works have appeared [6–8] estimating the quantum effects of these relations by assuming them to hold at some renormalization point.

The present author has also proposed an unique formulation based on a NCG [3,5]. Our formulation using a NCG employs a generalization of the usual differential geometry on an ordinary manifold to the discrete manifold $M_4 \times Z_N$. The reconstruction of SO(10) GUT and the left-right symmetric gauge model [4] had already been performed using our NCG scheme. In a NCG on $M_4 \times Z_2$, the extra differential one-form χ in Z_2 is introduced in addition to the usual one-form dx^μ in M_4, and therefore our formulation is very similar to ordinary differential geometry, contrastingly, in Connes' original scheme the Dirac matrices γ_μ and γ_5 are used to describe the generalized gauge field. In a NCG, the gauge field and the Higgs boson field are given as coefficients of dx^μ and χ, respectively, in the generalized connection on $M_4 \times Z_2$. However, there is no symmetry to mix dx^μ and χ, and, therefore, the ordinary gauge field cannot be transformed into the Higgs boson field. In [5], the reconstruction of the standard model is successfully carried out based on a new scheme of our NCG. Three generations of fermions, including left and right-handed quarks and leptons, are incorporated. In addition, the strong interaction is nicely included in this scheme. The relations $g^2 = (5/3)g'^2 = 4\lambda$ are introduced in [5], where g, g' and λ are $\mathrm{SU(2)_L}$ and $\mathrm{U(1)_Y}$ gauge coupling constants and the Higgs quartic coupling constant, respectively. However, the grand unification for gauge cou-

pling constants is not achieved in this scheme. The former part of this relation leads to $\sin^2\theta_{\rm W}=3/8$, and the latter part leads to the mass relation $m_{\rm H}=\sqrt{2}m_{\rm W}$. We assume that the relations $g^2=(5/3)g'^2=4\lambda$ hold at one renormalization point. With this assumption, we can perform the renormalization group analysis of the running coupling constants g, g' and λ, and the physical Higgs boson mass can thus be estimated. Within the one loop approximation, this analysis was carried out in [8]. In that context, we found $m_{\rm H}=164.01$GeV for $m_{\rm top}=175$GeV. In this article, we will perform the two loop analysis of the Higgs boson mass using the same method.

2 The review of our NCG on $M_4\times Z_2$

We first briefly review our previous formulation [5], because it is not well-known among particle physicists. For a detailed description, we refer the reader to [5].

Let us start with the equation of the generalized gauge field $\mathcal{A}(x,y)$ on the principal bundle with the base space $M_4\times Z_2$,

$$\mathcal{A}(x,y)=\sum_i a_i^\dagger(x,y)\boldsymbol{d}a_i(x,y)+\sum_j b_j^\dagger(x,y)\boldsymbol{d}b_j(x,y), \tag{2.1}$$

where $a_i(x,y)$ and $b_j(x,y)$ are square-matrix-valued functions and are taken so as to commute with each other, because $\sum_i a_i^\dagger(x,y)\boldsymbol{d}a_i(x,y)$ is the flavor sector, including the flavor gauge and the Higgs fields, while $\sum_j b_j^\dagger(x,y)\boldsymbol{d}b_j(x,y)$ corresponds to the color sector. The indices i and j are variables of the extra internal space which we cannot identify. The operator $\boldsymbol{d}$ in (2.1) is the generalized exterior derivative defined as follows:

$$\boldsymbol{d}=d+d_\chi, \tag{2.2}$$
$$da_i(x,y)=\partial_\mu a_i(x,y)dx^\mu, \tag{2.3}$$
$$d_\chi a_i(x,y)=[-a_i(x,y)M(y)+M(y)a_i(x,-y)]\chi, \tag{2.4}$$
$$db_j(x,y)=\partial_\mu b_j(x,y)dx^\mu, \tag{2.5}$$
$$d_\chi b_j(x,y)=0. \tag{2.6}$$

Here, dx^μ is the ordinary one-form basis taken to be dimensionless in Minkowski space M_4, and χ is the one-form basis also assumed to be dimensionless in the discrete space Z_2. We have introduced the x-independent matrix $M(y)$ whose hermitian conjugation is given by $M(y)^\dagger=M(-y)$. The matrix $M(y)$ determines the scale and pattern of the spontaneous breakdown of the gauge symmetry. Thus, (2.6) implies that the color symmetry of the strong interaction does not break spontaneously.

Using the algebraic rules in (2.2)-(2.6) and the shifting rule invoked in [5], $\mathcal{A}(x,y)$ can be rewritten as

$$\mathcal{A}(x,y)=A_\mu(x,y)dx^\mu+\Phi(x,y)\chi+G_\mu(x)dx^\mu, \tag{2.7}$$

where

$$A_\mu(x,y)=\sum_i a_i^\dagger(x,y)\partial_\mu a_i(x,y), \tag{2.8}$$

$$\Phi(x,y)=\sum_i a_i^\dagger(x,y)\left(-a_i(x,y)M(y)+M(y)a_i(x,-y)\right)=a_i^\dagger(x,y)\,\partial_y a_i(x,y), \tag{2.9}$$

$$G_\mu(x)=\sum_j b_j^\dagger(x)\partial_\mu b_j(x). \tag{2.10}$$

The functions $A_\mu(x,y)$, $\Phi(x,y)$ and $G_\mu(x)$ here are identified with the gauge field in the flavor symmetry, Higgs fields, and the color gauge field responsible for the strong interaction, respectively. The gauge transformations of these fields are well defined in the usual manner and it follows that

$$H(x,y)=\Phi(x,y)+M(y) \tag{2.11}$$

is an un-shifted Higgs field whereas $\Phi(x,y)$ denotes a shifted Higgs field with vanishing vacuum expectation value. The nilpotency of $\boldsymbol{d}$ is proved using (2.2)-(2.6) along with another algebraic rule in [5].

With these considerations, we can construct the gauge covariant field strength,

$$\mathcal{F}(x,y)=F(x,y)+\mathcal{G}(x), \tag{2.12}$$

where $F(x,y)$ and $\mathcal{G}(x)$ are the field strengths of flavor and color gauge fields, respectively, and given as

$$F(x,y)=\boldsymbol{d}A(x,y)+A(x,y)\wedge A(x,y),$$
$$\mathcal{G}(x)=d\,G(x)+g_sG(x)\wedge G(x). \tag{2.13}$$

Using the algebraic rules defined in (2.2)-(2.6), we have

$$F(x,y)=\frac{1}{2}F_{\mu\nu}(x,y)dx^\mu\wedge dx^\nu+D_\mu\Phi(x,y)dx^\mu\wedge\chi+V(x,y)\chi\wedge\chi, \tag{2.14}$$

where

$$F_{\mu\nu}(x,y)=\partial_\mu A_\nu(x,y)-\partial_\nu A_\mu(x,y)+[A_\mu(x,y),A_\mu(x,y)], \tag{2.15}$$
$$D_\mu\Phi(x,y)=\partial_\mu\Phi(x,y)+A_\mu(x,y)(M(y)+\Phi(x,y))-(\Phi(x,y)+M(y))A_\mu(x,-y), \tag{2.16}$$
$$V(x,y)=(\Phi(x,y)+M(y))(\Phi(x,-y)+M(-y))-Y(x,y). \tag{2.17}$$

The quantity $Y(x,y)$ in (2.17) is auxiliary field and expressed as

$$Y(x,y)=\sum_i a_i^\dagger(x,y)M(y)M(-y)a_i(x,y). \tag{2.18}$$

This function may become a constant field in the present construction. In contrast to $F(x,y)$, $\mathcal{G}(x)$ is simply denoted as

$$\mathcal{G}(x)=\frac{1}{2}G_{\mu\nu}(x)dx^\mu\wedge dx^\nu=\frac{1}{2}\{\partial_\mu G_\nu(x)-\partial_\nu G_\mu(x)+g_s[G_\mu(x),G_\mu(x)]\}dx^\mu\wedge dx^\nu. \tag{2.19}$$

With the same metric structure as in [5] we can obtain the gauge invariant Yang-Mills-Higgs Lagrangian (YMH)

$$\begin{aligned}\mathcal{L}_{\rm YMH}(x) &= -{\rm Tr}\sum_{y=\pm}\frac{1}{\tilde{g}^2} < \mathcal{F}(x,y), \mathcal{F}(x,y) > \\ &= -{\rm Tr}\sum_{y=\pm}\frac{1}{2\tilde{g}^2}F^{\dagger}_{\mu\nu}(x,y)F^{\mu\nu}(x,y) \\ &\quad +{\rm Tr}\sum_{y=\pm}\frac{1}{\tilde{g}^2}(D_\mu\Phi(x,y))^{\dagger}D^{\mu}\Phi(x,y) \\ &\quad -{\rm Tr}\sum_{y=\pm}\frac{1}{\tilde{g}^2}V^{\dagger}(x,y)V(x,y) \\ &\quad -{\rm Tr}\sum_{y=\pm}\frac{1}{2\tilde{g}^2}G^{\dagger}_{\mu\nu}(x)G^{\mu\nu}(x). \qquad (2.20)\end{aligned}$$

Here, Tr denotes the trace over internal symmetry matrices including the color, flavor symmetries and generation space. The third term on the right-hand side is the potential term of the Higgs particle.

3 A numerical estimation of the Higgs boson mass

In reconstructing the standard model in the present scheme, three generations of left and right-handed quarks and leptons together with the strong interaction must be taken into account. A characteristic point of this formulation is to take the left and right-handed fermions $\psi(x,y)$ with arguments x and $y(=\pm)$ in M_4 and Z_2, respectively, as

$$\psi(x,+) = \begin{pmatrix} u^r_{\rm L} \\ u^g_{\rm L} \\ u^b_{\rm L} \\ \nu_{\rm L} \\ d^r_{\rm L} \\ d^g_{\rm L} \\ d^b_{\rm L} \\ e_{\rm L}\end{pmatrix}, \qquad \psi(x,-) = \begin{pmatrix} u^r_{\rm R} \\ u^g_{\rm R} \\ u^b_{\rm R} \\ 0 \\ d^r_{\rm R} \\ d^g_{\rm R} \\ d^b_{\rm R} \\ e_{\rm R}\end{pmatrix}, \qquad (3.1)$$

where the subscripts L and R denote the left-handed and right-handed fermions and the superscripts r, g and b represent the color indices. It should be noted that $\psi(x,y)$ has an index for the three generations, as do the explicit expressions for fermions on the right-hand sides of (3.1). Thus, $\psi(x,\pm)$ is a vector in the 24-dimensional space. In order to construct the Dirac Lagrangian of the standard model corresponding to $\psi(x,\pm)$ in (3.1), we need a 24-dimensional generalized covariant derivative composed of gauge and Higgs fields on $M_4 \times Z_2$. The gauge fields $A_\mu(x,y)$ and $G_\mu(x)$ in this covariant derivative must constitute the differential representation of the fermion fields in (3.1), and therefore they are expressed in 24×24 matrix forms. The Higgs field $\Phi(x,y)$ is also taken to give the correct Yukawa interaction in the Dirac Lagrangian and is expressed as a 24×24 matrix (see [5] for details). We find Yang-Mills-Higgs Lagrangian for the standard model as follows:

$$\begin{aligned}\mathcal{L}_{\rm YMH} &= -\frac{1}{4}\sum_{k=1}^{3}\left(F^k_{\mu\nu}\right)^2 - \frac{1}{4}B^2_{\mu\nu} \\ &\quad +|D_\mu h|^2 - \lambda(h^{\dagger}h-\mu^2)^2 \\ &\quad -\frac{1}{4}\sum_{a=1}^{8}G^a_{\mu\nu}{}^{\dagger}G^{a\mu\nu}, \qquad (3.2)\end{aligned}$$

where

$$F^k_{\mu\nu} = \partial_\mu A^k_\nu - \partial_\nu A^k_\mu + g\epsilon^{klm}A^l_\mu A^m_\nu, \qquad (3.3)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \qquad (3.4)$$

$$D^\mu h = [\,\partial_\mu - \frac{i}{2}\,(\sum_k \tau^k g A^k_{L\mu} + \tau^0\, g' B_\mu\,)\,]\,h,$$

$$h = \begin{pmatrix}\phi^+ \\ \phi_0+\mu\end{pmatrix}, \qquad (3.5)$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_c f^{abc}G^b_\mu G^c_\nu, \qquad (3.6)$$

with the following restrictions for coupling constants:

$$g^2 = \frac{\tilde{g}^2}{12},\ g'^2 = \frac{\tilde{g}^2}{20},\ \lambda = \frac{\tilde{g}^2}{48},\ g_c^2 = \frac{g_s^2\tilde{g}^2}{12}. \qquad (3.7)$$

Equation (3.7) leads to the relation between coupling constants:

$$g^2 = \frac{5}{3}g'^2 = 4\lambda \neq g_c^2, \qquad (3.8)$$

which implies that the weak, electromagnetic and Higgs quartic coupling constants become equal and also yields $\sin^2\theta_{\rm W} = 3/8$ for the Weinberg angle as well as the mass relations

$$m^2_{\rm W} = \frac{1}{4}g^2v^2,\ m^2_{\rm Z} = \frac{2}{5}g^2v^2,\ m^2_{\rm H} = \frac{1}{2}g^2v^2. \qquad (3.9)$$

with the vacuum expectation value of the Higgs boson v. These relations hold only at tree level. Here, we assume that these relations hold at a renormalization point and consider their quantum effects by use of the renormalization group (RG) equations.

With the notation

$$\alpha_3 = \frac{g_c^2}{4\pi},\ \alpha_2 = \frac{g^2}{4\pi},\ \alpha_1 = \frac{5}{3}\frac{g'^2}{4\pi},\ \alpha_{\rm H} = \frac{\lambda}{4\pi} \qquad (3.10)$$

for $\mathrm{SU(3)}_c$, $\mathrm{SU(2)}_{\rm L}$ and $\mathrm{U(1)}_{\rm Y}$ gauges and the Higgs quartic coupling constants, respectively, the RG equations for these coupling constants are expressed as

$$\mu\frac{\partial\alpha_i}{\partial\mu} = \beta_i,\ (i=1,2,3),\ \mu\frac{\partial\alpha_{\rm H}}{\partial\mu} = \beta_{\rm H}, \qquad (3.11)$$

where the β-functions in the right hand sides are given in [9] in the two loop approximation. The Yukawa coupling constants of quarks written in 3×3 matrix form in three generations are included in these β-functions. We now assume the top quark mass is dominant in the evaluation of the RG equations. Masses of all particles in the

standard model are introduced through the vacuum expectation value v of the Higgs field. In this context, the running masses of gauge and Higgs bosons are defined as

$$m_{\rm W} = \sqrt{\pi\alpha_2}v,\ m_{\rm H} = \sqrt{8\pi\alpha_{\rm H}}v. \tag{3.12}$$

The top quark mass is also expressed as

$$m_{\rm top} = \sqrt{2\pi\alpha_{\rm Y}}v, \tag{3.13}$$

with the Yukawa coupling constant $\alpha_{\rm Y}$ whose RG equation is also given in [10]. The RG equation for v is also given in [10] in the two loop approximation.

The RG equations for α_i $(i = 1, 2, 3)$, $\alpha_{\rm H}$, $\alpha_{\rm Y}$ and v are highly non-linear equations with complicated coupling. In order to solve these equations, we need six conditions. Three of these conditions, those for α_i $(i = 1, 2)$ and v, are given experimentally as [10]

$$\begin{aligned} &\alpha_1(0) = 0.01698, \quad \alpha_2(0) = 0.03364, \\ &v(0) = 246.22{\rm GeV}, \end{aligned} \tag{3.14}$$

and the initial condition for α_3 is given as [11]

$$\alpha_3(0) = 0.118 \pm 0.003, \tag{3.15}$$

at $\mu = m_{\rm Z}$ with the variable $t = \log(\mu/m_{\rm Z})$. That is, these conditions are given at the neutral gauge boson mass $m_{\rm Z} = 91.187{\rm GeV}$. According to (3.13), the physical top quark mass $m_{\rm top}$ satisfies the equation

$$m_{\rm top} = \sqrt{2\pi\alpha_{\rm Y}(t_{\rm top})}v(t_{\rm top}), \tag{3.16}$$

where $t_{\rm top} = \log(m_{\rm top}/m_{\rm Z})$. This equation constitutes one condition to solve the RG equations. The remaining condition is

$$\alpha_2(t_0) = \alpha_1(t_0) = 4\alpha_{\rm H}(t_0), \tag{3.17}$$

where t_0 is a constant, fixed in the numerical calculations. The value of t_0 determines the energy at which the weak, electromagnetic and Higgs quartic interactions are unified. With these considerations, we can find the running Higgs boson mass from (3.12) as

$$m_{\rm H}(t) = \sqrt{8\pi\alpha_{\rm H}(t)}v(t). \tag{3.18}$$

The physical Higgs boson mass $m_{\rm H}$ is determined by imposing the condition that

$$m_{\rm H} = \sqrt{8\pi\alpha_{\rm H}(t_{\rm H})}v(t_{\rm H}) \tag{3.19}$$

with $t_{\rm H} = \log(m_{\rm H}/m_{\rm Z})$.

The top quark mass has a considerable effect on the Higgs boson mass through (3.16). The world average value of the top quark mass $m_{\rm top}$ from the experiments of CDF and D0 was reported in review works [12] as

$$m_{\rm top} = 175 \pm 6{\rm GeV}. \tag{3.20}$$

We investigated the Higgs boson mass by varying the top quark mass in the range of (3.20). Table 1. shows the physical Higgs boson mass versus the top quark mass numerically calculated at $\alpha_3(0) = 0.118$ in (3.15). Compared with the one loop analysis of the Higgs boson mass in [8], two loop effects lower the numerical values by around 6GeV because, for example, $m_{\rm H} = 164.01{\rm GeV}$ for $m_{\rm top} = 175{\rm GeV}$ in the one loop analysis. This difference seems crucial since a Higgs boson with mass below 160GeV could not decay into $\rm W^+W^-$ and also experiments designed to search for the Higgs boson depend greatly upon the present analysis. It should be noted that owing to the unitarity requirement, $m_{\rm top}$ cannot exceed 190GeV because λ would become minus at a much higher value of t in such a case. It is important to investigate the dependence of $\alpha_3(0)$ on the Higgs boson mass because of its rather large error shown in (3.15). We obtain the numerical results that $m_{\rm H} = 157.80{\rm GeV}$ at $m_{\rm top} = 175.04{\rm GeV}$ and $\alpha_3(0) = 0.121$, and $m_{\rm H} = 160.09{\rm GeV}$ at $m_{\rm top} = 175.02{\rm GeV}$ and $\alpha_3(0) = 0.115$. Thus, these results lead to the conclusion that the experimental errors of $\alpha_3(0)$ in (3.15) do not affect the estimation of the Higgs boson mass significantly.

It is interesting to investigate the running of the gauge and Higgs quartic coupling constants, because these coupling constants are unified at a point t_0 as shown in (3.17). Figure 1 displays the running of the three coupling constants.

Table 1. Higgs boson mass versus top quark mass

top quark (GeV)	Higgs boson (GeV)
169.47	150.93
170.15	151.88
171.69	154.07
172.46	155.18
173.40	156.55
174.35	157.93
175.01	158.90
176.13	160.56
177.16	162.10
178.26	163.77
179.19	165.16
180.37	166.98
181.00	167.96

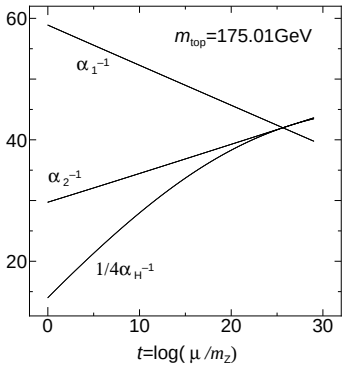

Fig. 1. The running of the three coupling constants

4 Conclusion

In this paper, we have determined the Higgs boson mass by numerically solving the renormalization group equations with the relation between coupling constants (3.8). It is introduced in the reconstruction of the standard model based on our new scheme of NCG [5]. We assumed that (3.8) holds at a renormalization point t_0. However, this leads to an interesting result that the weak, electromagnetic and Higgs quartic coupling constants become equal at t_0 as shown Fig. 1. In the case of $m_{\rm top} = 175.01$GeV, $t_0 = 25.636$ which amounts to $\mu = 1.24 \times 10^{13}$GeV.

We obtain $150.93\text{GeV} \leq m_{\rm H} \leq 167.96\text{GeV}$ in the range of top quark mass $169.47\text{GeV} \leq m_{\rm top} \leq 181.00\text{GeV}$. We hope that this result will be useful for experiments searching for the Higgs boson.

References

1. A. Connes, p.9 in The Interface of Mathematics and Particle Physics, ed. D. G. Quillen, G. B. Segal, and Tsou. S. T., Clarendon Press, Oxford, 1990. See also, Alain Connes and J. Lott, Nucl. Phys. **B** (Proc. Suppl.) **18B**, 29 (1990)
2. References in J. Madore and J. Mourad, hep-th/9601169
3. Y. Okumura, Prog. Theor. Phys., **96**, 1021 (1996)
4. Y. Okumura, Phys. Rev., **D50** (1994), 1026, Prog. Theor. Phys.,**94** (1995), 589, Nuovo Cim., **110A** (1997), 267
5. Y. Okumura, Prog. Theor. Phys. **95**, 969 (1996)
6. E. Alvarez, J.M. Gracia-Bondía and C.P. Martín, Phys. Lett. , **B306**, 55 (1993); ibit. **B329**, 259 (1994)
7. T. Shinohara, K. Nishida, H. Tanaka and I.S. Sogami, Prog. Theor. Phys. **96** , 1179 (1996)
8. Y. Okumura, Prog. Theor. Phys. **98**, 1333 (1997)
9. M.E. Machacek and M.T. Vaughn, Phys. Lett. **103B**, 427 (1981); Nucl. Phys. **B222**, 83 (1983); ibit. **B136**, 221 (1984); ibit. **B249**, 70 (1985)
10. H. Arason, D.J. Castaño, B. Kesthelyi, S. Mikaelian, E.J. Piard, P. Ramond, Phys. Rev. **D46**, 3945 (1992);
11. R.M. Barnett et al., Phys.Rev. **D54**, 1 (1996); G. Altarelli, hep-ph/9611239
12. S. Willenbrock, hep-ph/9611240; S. Dawson, hep-ph/9609340