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Complete control of gauge parameter dependence in the Abelian Higgs model

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Abstract. We examine the dependence on all gauge parameters in the example of the Abelian Higgs model by applying a general algebraic method which roots in an extension of the usual Slavnov-Taylor identity. This method automatically yields all information about the gauge parameter dependence of Green functions and therefore especially allows to control the range of "good" normalization conditions. In this context we show that the physical on-shell normalization conditions are in complete agreement with the restrictions dictated by the enlarged Slavnov-Taylor identity and that the coupling can be fixed in an easily handleable way on the Ward identity of local gauge invariance. As an application of the general method we also study the Callan-Symanzik equation and the renormalization group equation of the Abelian Higgs model.

1 Introduction

The need to fix the gauge when quantizing a gauge theory perturbatively introduces a set of arbitrary gauge parameters into the action. Therefore one unavoidably has the task to control the dependence of the theory on these gauge parameters. Especially, it has to be proven that *physical* quantities indeed are gauge parameter independent. For instance, the gauge parameter independence of the S-matrix, already suggested in [1], was proven in [2] for gauge theories that do not contain any massless particles due to a complete spontaneous breakdown of symmetry. This proof, however, relies on a special set of on-shell normalization conditions and also makes use of a rather complicated technical tool, namely the Wilson operator product expansion. On the other hand, looking at pure gauge theories with massless gauge bosons, where the S-matrix does not exist, the gauge parameter independence of the β-functions has been shown. This, however, solely has been achieved by explicitly refering to an invariant renormalization scheme [3].

In the standard model of electroweak interactions the prerequisites needed for the proofs of the examples mentioned above are not fulfilled due to the masslessness of the photon and parity violation in the fermion sector. Hence the state of the art concerning the control of gauge parameter dependence is quite unsatisfactory and the necessity for having at hand a general (i.e. model- and scheme-independent) and easily manageable tool arises. Such a tool is given by the algebraic method first proposed in [4] which also allows for the control of gauge parameter dependence of single Green functions. As a preparatory step for similar investigations in the standard model this general method has been applied to the Abelian Higgs model in [5]. But in [5] attention was restricted to the dependence of the theory on one gauge parameter only. Among other things results proven in [6] by explicitly using an invariant scheme and special properties of the model could be reproduced in a model- and scheme-independent way. The present paper, now, enlarges the considerations of [5] to the *full* control of gauge parameter dependence (i.e. the control of the dependence of the theory on *all* gauge parameters) and hence completes the treatment of [5] in this sense. Again in view of the application of the algebraic method to the rather complicated standard model, this model containing quite a lot of gauge parameters, it seems to be instructive and in fact necessary to *completely* work out this method, applied in its *full* extent, in the simpler case of the Abelian Higgs model as a preliminary. The necessary prerequisites for an analogous discussion of gauge parameter dependence in the standard model are in the meanwhile available due to [7].

The algebraic method essentially roots in a certain extension of the ordinary BRS transformations: All the gauge parameters of the model now are allowed to transform under BRS into Grassmann variables. It then follows that constructing the Green functions in accordance with this enlarged BRS invariance also automatically yields all information about the gauge parameter dependence of the original Green functions, some of which are also used in the normalization conditions. Because these normalization conditions have to be chosen in agreement with the gauge parameter dependence of the theory (in order not to ruin, for instance, the gauge parameter independence of the S-matrix) we hence have at hand a powerful tool for controlling the range of allowed normalization conditions. In this context it turns out that the conditions of [2] and [3] just

build special sets of adequate normalization conditions (cf. Sect. 5, [4], [8]).

The structure of the paper will be as follows: In a first part (including Sects. 2–7) we enlarge the results obtained in [5] to the case when all gauge parameters undergo BRS transformations. This first part therefore parallels the discussion of [5]. Whenever the treatment is *completely* analogous to the one in [5] we will skip calculational details and refer to [5], but nevertheless the present paper is fully self-contained as far as the line of argument is concerned: Hence the first part will contain a short recapitulation of the Abelian Higgs model (Sect. 2), the method of BRS transforming gauge parameters (Sect. 3), the general solution of the classical approximation (Sect. 4), the extension of the restrictions found classically for some of the parameters of the model to higher orders (Sect. 5) and global and local Ward identities. In order to illustrate how far one can get with algebraic considerations alone and to complete the algebraically abstract treatment we construct in a second part (Sects. 8, 9) parametric differential equations, namely the Callan-Symanzik equation, the renormalization group equation and the equation governing the dependence of the theory on the ghost mass. In this context we will also prove the gauge parameter independence of some of the coefficient functions to all orders of the perturbative expansion. Sect. 10 summarizes the results.

2 The Abelian Higgs model

We start with a short presentation of the Abelian Higgs model, thereby emphasizing some aspects which will become relevant in the following. The model consists of a doublet of scalar fields $\underline{\varphi} = (\varphi_1, \varphi_2)$ and a gauge field A_μ with an interaction, that breaks $U(1)$ gauge invariance spontaneously. In conventional normalization it can be described by the classical action

$$\Gamma_{inv} = \int \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (D_\mu \underline{\varphi})(D^\mu \underline{\varphi}) - \frac{1}{8} \frac{m_H^2}{m^2} e^2 \left(\varphi_1^2 + 2 \frac{m}{e} \varphi_1 + \varphi_2^2 \right)^2 \right\} \tag{2.1}$$

with:

$$\begin{aligned} F_{\mu\nu} &\equiv \partial_\mu A_\nu - \partial_\nu A_\mu \ , \\ D_\mu \varphi_1 &\equiv \partial_\mu \varphi_1 + e A_\mu \varphi_2 \ , \\ D_\mu \varphi_2 &\equiv \partial_\mu \varphi_2 - e A_\mu \left(\varphi_1 + \frac{m}{e} \right) \end{aligned} \tag{2.2}$$

Γ_{inv} respects the discrete symmetry of charge conjugation and $U(1)$ symmetry, i.e. it is invariant under the $U(1)$ transformations:

$$\begin{aligned} \delta_\omega A_\mu &= \partial_\mu \omega \ , \\ \delta_\omega \varphi_1 &= -e\omega \varphi_2 \ , \\ \delta_\omega \varphi_2 &= e\omega \left(\varphi_1 + \frac{m}{e} \right) \end{aligned} \tag{2.3}$$

The shift $\frac{m}{e}$ of the field φ_1 produces the mass m for the vector field A_μ and φ_1 is the physical Higgs field with mass m_H, whereas φ_2 takes the role of the would-be Goldstone boson eaten up by A_μ.

In order to quantize the model the gauge has to be fixed. To this end we add the gauge fixing term

$$\Gamma_{g.f.} = \int \left\{ \frac{1}{2} \xi B^2 + B \partial A - eB \times \left((\hat{\varphi}_1 - \xi_A \frac{m}{e}) \varphi_2 - \hat{\varphi}_2 (\varphi_1 - \hat{\xi}_A \frac{m}{e}) \right) \right\} \ . \tag{2.4}$$

B is an auxiliary field with $\delta_\omega B = 0$, ξ and ξ_A denote the gauge parameters[1], and $\hat{\xi}_A$ is an additional free parameter of the model which will be fixed on the global Ward identity, see Sect. 6. In (2.4) a further doublet of external fields $\underline{\hat{\varphi}} = (\hat{\varphi}_1, \hat{\varphi}_2)$ has been introduced in order to allow for a *proper* formulation of rigid and local symmetry in terms of Ward identities, see [9]. Under $U(1)$ the doublet $\underline{\hat{\varphi}}$ transforms according to

$$\delta_\omega \hat{\varphi}_1 = -e\omega \hat{\varphi}_2 \ , \ \delta_\omega \hat{\varphi}_2 = e\omega (\hat{\varphi}_1 - \xi_A \frac{m}{e}) \ . \tag{2.5}$$

Of course, the gauge fixing term (2.4) violates local gauge invariance non-trivially. To retain a symmetry one has to further enlarge the model by introducing the Faddeev-Popov ($\phi\pi$) fields $c, \bar{c}$ and to extend local gauge transformations to BRS transformations:

$$\begin{aligned} sA_\mu &= \partial_\mu c, & sc &= 0, \\ s\varphi_1 &= -ec\varphi_2, & s\varphi_2 &= ec\left(\varphi_1 + \tfrac{m}{e}\right), \\ s\bar{c} &= B, & sB &= 0, \\ s\hat{\varphi}_i &= q_i, & sq_i &= 0 \quad , \quad i = 1, 2, \end{aligned} \tag{2.6}$$

where $\underline{q} = (q_1, q_2)$ is another doublet of external fields. Adding the $\phi\pi$-action $\Gamma_{\phi\pi}$ such that

$$\Gamma_{g.f.} + \Gamma_{\phi\pi} = s \int \bar{c} \left\{ \frac{1}{2} \xi B + \partial A - e \times \left((\hat{\varphi}_1 - \xi_A \frac{m}{e}) \varphi_2 - \hat{\varphi}_2 (\varphi_1 - \hat{\xi}_A \frac{m}{e}) \right) \right\}, \tag{2.7}$$

$\Gamma_{inv} + \Gamma_{g.f.} + \Gamma_{\phi\pi}$ is BRS symmetric. The BRS symmetry is a powerful technical tool which is essential for the proof of renormalizability and unitarity of the S-matrix. It also defines the model in question in an implicit way (see below).

Finally, we have to care about the non-linear BRS transformations $s\varphi_i$ which are not well-defined in higher orders of perturbation theory due to their non-linearity. In order to circumvent this difficulty we couple these BRS variations to external fields Y_i with $sY_i = 0$ and add an external field part:

$$\Gamma_{e.f.} = \int \{ Y_1 (s\varphi_1) + Y_2 (s\varphi_2) \} \tag{2.8}$$

The complete BRS invariant classical action is now given by:

$$\Gamma_{cl} = \Gamma_{inv} + \Gamma_{g.f.} + \Gamma_{\phi\pi} + \Gamma_{e.f.} \tag{2.9}$$

[1] The t'Hooft gauge fixing term $\xi_A m \int B\varphi_2$ is necessary in order to avoid a non-integrable infrared singularity in the $\langle \varphi_2 \varphi_2 \rangle$ propagator

The BRS invariance of the theory is expressed by the Slavnov-Taylor (ST) identity

$$\mathcal{S}(\Gamma) \equiv \int \left\{ \partial_\mu c \frac{\delta\Gamma}{\delta A_\mu} + B\frac{\delta\Gamma}{\delta\bar{c}} + \frac{\delta\Gamma}{\delta\underline{Y}}\frac{\delta\Gamma}{\delta\underline{\varphi}} + \underline{q}\frac{\delta\Gamma}{\delta\underline{\hat{\varphi}}} \right\} = 0 \ . \tag{2.10}$$

At the classical level Γ is just the classical action Γ_{cl}, whereas at the quantum level Γ denotes the vertex functional $\Gamma = \Gamma_{cl} + O(\hbar)$. It can be proven that (2.10) together with appropriate normalization conditions, invariance under charge conjugation and the gauge condition (2.4) uniquely defines the model to all orders of perturbation theory. This is in contrast to the standard model where in addition to the ST identity also rigid invariance, a local Ward identity and some consistency relations are needed for a full algebraic characterization of the model [7].

In a first step one has to look for the most general, field polynomial (i.e. classical) solution Γ_{cl}^{gen} of the ST identity (2.10) and the gauge condition (2.4) which is invariant under charge conjugation (for quantum numbers see Table 1), and to prove that it coincides with Γ_{cl} after the application of appropriate normalization conditions. This procedure also yields information about all the free parameters of the theory. The most general solution was calculated in [9] and is presented in appendix A. The free parameters in Γ_{cl}^{gen} are the usual field and coupling renormalizations $z_1, z_2, z_A, z_m, z_{m_H}, z_e$,

$$\begin{aligned} \varphi_i \longrightarrow \sqrt{z_i}(\varphi_i - x_i\hat{\varphi}_i) \ , \ A_\mu \longrightarrow \sqrt{z_A}A_\mu \\ m \longrightarrow \sqrt{z_m}m \ , \ m_H \longrightarrow \sqrt{z_{m_H}}m_H \ , \ e \longrightarrow z_e e \quad , \end{aligned} \tag{2.11}$$

as well as the gauge parameters ξ, ξ_A, the parameter μ (see appendix A), $\hat{\xi}_A$ (which is prescribed by the global Ward identity, see (6.3)) and the two parameters x_1, x_2, which appear in the combination

$$\bar{\varphi}_i = \varphi_i - x_i\hat{\varphi}_i \tag{2.12}$$

that replaces φ_i in Γ_{inv}.

These parameters have to be fixed by normalization conditions in each order of the perturbative expansion. In the following we will choose (for reasons which will become clear later on) physical on-shell normalization conditions:

$$\begin{aligned} Re\ \Gamma_{\varphi_1\varphi_1}(p^2 = m_H^2) = 0 \text{ fixes } z_{m_H} \\ \Gamma^T(p^2 = m^2) = 0 \text{ fixes } z_m \\ \Gamma_{c\bar{c}}(p^2 = m^2_{\text{ghost}}) = 0 \ , \\ m^2_{\text{ghost}} = \xi_A m^2 \quad \text{fixes } \xi_A \end{aligned} \tag{2.13}$$

$$\begin{aligned} \partial_{p^2}\Gamma^T(p^2 = m^2) = 1 \text{ fixes } z_A \\ Re\ \partial_{p^2}\Gamma_{\varphi_1\varphi_1}(p^2 = m_H^2) = 1 \text{ fixes } z_1 \\ \partial_{p^2}\Gamma_{\varphi_2\varphi_2}(p^2 = \kappa^2) = 1 \text{ fixes } z_2 \end{aligned} \tag{2.14}$$

$$\begin{aligned} \Gamma_{Y_1 q_1}(p^2 = \kappa^2) = x_1^{(0)} \text{ fixes } x_1 \\ \Gamma_{Y_2 q_2}(p^2 = \kappa^2) = x_2^{(0)} \text{ fixes } x_2 \end{aligned} \tag{2.15}$$

$$\Gamma_{\varphi_1} = 0 \quad \text{fixes} \quad \mu \tag{2.16}$$

In (2.13), (2.14) the transversal part of the vector 2-point function is given by:

$$\begin{aligned} \Gamma_{A^\mu A^\nu}(p,-p) &\equiv \Gamma_{\mu\nu}(p,-p) \\ &= (\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})\Gamma^T(p^2) + \frac{p_\mu p_\nu}{p^2}\Gamma^L(p^2) \end{aligned} \tag{2.17}$$

It remains to give a normalization condition for the coupling e. Preliminary (see Sects. 5, 7), we fix the coupling on the 3-point vertex function $\Gamma_{A_\mu\varphi_1\varphi_2}$ at a normalization momentum p_{norm}:

$$\begin{aligned} \partial_{p_1^\nu}\Gamma_{A_\mu\varphi_1\varphi_2}(-p_1-p_2,p_1,p_2)\Big|_{\{p_i\}=p_{norm}} \\ = -ie\eta^{\mu\nu} \quad \text{fixes} \quad z_e \end{aligned} \tag{2.18}$$

It is easily checked that these normalization conditions when applied to the tree approximation Γ_{cl}^{gen} (see appendix A) exactly yield Γ_{cl} (2.9), if we set $x_i = 0$.

3 Algebraic control of gauge parameter dependence

We now want to turn to the proper subject of the present paper, namely the control of gauge parameter dependence. To this end we first observe that at the level of the classical action Γ_{cl} (2.9) the dependence on the two gauge parameters ξ and ξ_A is given by two BRS variations,

$$\begin{aligned} \partial_\xi\Gamma_{cl} = \tfrac{1}{2}\int B^2 = \tfrac{1}{2}s\int \bar{c}B \quad \text{and} \\ \partial_{\xi_A}\Gamma_{cl} = m\int\left\{B\varphi_2 - e\bar{c}(\varphi_1 + \tfrac{m}{e})c\right\} = m\ s\int\bar{c}\varphi_2 \quad , \end{aligned} \tag{3.1}$$

respectively. Therefore the right hand sides of (3.1) vanish between physical states and physical quantities (like the S-matrix) are ξ- and ξ_A-independent in the tree approximation.

The question now arises whether – and if yes, how – this statement can be extended to higher orders. In the affirmative case we furthermore would like to use a construction which is easily manageable and which does not rely on the specific model and/or a specific renormalization scheme. Of course, if the model in question permits a gauge-invariant regularization, such a general approach does not seem to be necessary at first sight. But because many models lack this property it is nevertheless desirable to have in hand such a model- and scheme-independent procedure for controlling gauge parameter dependence and to see how it works. In addition, it will turn out that some quite general results are *only* (or at least much more easily) accessible with the proposed method.

For this purpose let us therefore allow the gauge parameters ξ and ξ_A to transform under BRS into Grassmann variables χ and χ_A, respectively, with $\phi\pi$-charge $+1$ [4]:

$$s\xi = \chi \quad , \quad s\xi_A = \chi_A \quad , \quad s\chi = 0 = s\chi_A \tag{3.2}$$

Table 1. Quantum numbers of the fields ($\tilde{\varphi}_i = \varphi_i, \hat{\varphi}_i$)

fields	A_μ	B	$\tilde{\varphi}_1$	$\tilde{\varphi}_2$	c	$\bar{c}$	Y_1	Y_2	q_1	q_2
dim	1	2	1	1	0	2	3	3	1	1
charge conj.	-	-	+	-	-	-	+	-	+	-
$Q_{\phi\pi}$	0	0	0	0	+1	-1	-1	-1	+1	+1

Hence, the ST identity modifies into:

$$\mathcal{S}(\Gamma) + \chi\partial_\xi\Gamma + \chi_A\partial_{\xi_A}\Gamma = 0 \tag{3.3}$$

Differentiation of (3.3) with respect to χ or χ_A and evaluating the results at $\chi = 0 = \chi_A$ leads to

$$\begin{aligned} -\, s_\Gamma^{\chi=0=\chi_A}(\partial_\chi\Gamma)\Big|_{\chi=0=\chi_A} + \partial_\xi\Gamma|_{\chi=0=\chi_A} = 0\,, \\ -\, s_\Gamma^{\chi=0=\chi_A}(\partial_{\chi_A}\Gamma)\Big|_{\chi=0=\chi_A} + \partial_{\xi_A}\Gamma|_{\chi=0=\chi_A} = 0\,, \end{aligned} \tag{3.4}$$

where in the model under investigation s_Γ is given by:

$$\begin{aligned} s_\Gamma = \int \left\{ \partial c\frac{\delta}{\delta A} + B\frac{\delta}{\delta\bar{c}} + \frac{\delta\Gamma}{\delta\underline{Y}}\frac{\delta}{\delta\underline{\varphi}} + \frac{\delta\Gamma}{\delta\underline{\varphi}}\frac{\delta}{\delta\underline{Y}} + \underline{q}\frac{\delta}{\delta\underline{\hat{\varphi}}} \right\} \\ +\chi\partial_\xi + \chi_A\partial_{\xi_A} \end{aligned} \tag{3.5}$$

s_Γ being – roughly speaking – the functional generalization of s, (3.4) is nothing else but the functional analog of (3.1) which we were looking for and which can be easily controlled in higher orders. Therefore proving (3.3) to all orders of perturbation theory automatically yields all information about gauge parameter dependence of the 1-PI Green functions in an algebraic way.

4 Slavnov-Taylor identity for $\chi \neq 0$, $\chi_A \neq 0$

In accordance with the observations of the proceeding section gauge parameter dependence is completely governed by the χ- and χ_A-enlarged ST identity[2]:

$$\begin{aligned} \mathcal{S}(\Gamma) \equiv \int \left\{ \partial_\mu c\frac{\delta\Gamma}{\delta A_\mu} + B\frac{\delta\Gamma}{\delta\bar{c}} + \frac{\delta\Gamma}{\delta\underline{Y}}\frac{\delta\Gamma}{\delta\underline{\varphi}} + \underline{q}\frac{\delta\Gamma}{\delta\underline{\hat{\varphi}}} \right\} \\ +\chi\partial_\xi\Gamma + \chi_A\partial_{\xi_A}\Gamma \;=\; 0 \end{aligned} \tag{4.1}$$

First we have to look for the general *classical* solution $\Gamma = \Gamma_{cl}^{gen}$ of (4.1) in order to control the free parameters of the theory and to learn something about their gauge parameter dependence eventually. Because the ST identity does not prescribe the gauge fixing terms we can also postulate the gauge condition (2.4) to hold for the (χ- and χ_A-independent part of the) solution Γ of (4.1). The gauge condition (2.4) is linear in propagating fields and hence it can be held in this form to all orders of perturbation theory.

[2] From here on the symbol $\mathcal{S}$ collectively denotes all the differential operators on the r.h.s. of (3.3)

Using the fact that χ and χ_A are Grassmann variables, Γ can be split into four parts in the tree approximation:

$$\Gamma = \hat{\Gamma} + \chi Q + \chi_A Q_A + \chi\chi_A Q_{\chi\chi A} \tag{4.2}$$

Inserting (4.2) into the ST identity (4.1) and again making use of $\chi^2 = 0 = \chi_A^2$ one immediately finds that at the classical level (4.1) is equivalent to the following four equations:

$$\begin{aligned} \chi^0, \chi_A^0 \;:\; & \int \left\{ \partial_\mu c\frac{\delta\hat{\Gamma}}{\delta A_\mu} + B\frac{\delta\hat{\Gamma}}{\delta\bar{c}} + \frac{\delta\hat{\Gamma}}{\delta\underline{Y}}\frac{\delta\hat{\Gamma}}{\delta\underline{\varphi}} + \underline{q}\frac{\delta\hat{\Gamma}}{\delta\underline{\hat{\varphi}}} \right\} \\ & = \; 0 \end{aligned} \tag{4.3}$$

$$\chi^1, \chi_A^0 \;:\; \partial_\xi\hat{\Gamma} \;=\; s_{\hat{\Gamma}}^{\chi=0=\chi_A} Q \tag{4.4}$$

$$\chi^0, \chi_A^1 \;:\; \partial_{\xi_A}\hat{\Gamma} \;=\; s_{\hat{\Gamma}}^{\chi=0=\chi_A} Q_A \tag{4.5}$$

$$\begin{aligned} \chi^1, \chi_A^1 \;:\; & \int \left\{ \frac{\delta Q}{\delta\underline{Y}}\frac{\delta Q_A}{\delta\underline{\varphi}} - \frac{\delta Q_A}{\delta\underline{Y}}\frac{\delta Q}{\delta\underline{\varphi}} \right\} - \partial_\xi Q_A + \partial_{\xi_A} Q \\ & = \; s_{\hat{\Gamma}}^{\chi=0=\chi_A} Q_{\chi\chi A} \end{aligned} \tag{4.6}$$

$s_{\hat{\Gamma}}$ is given by (3.5) (with $\hat{\Gamma}$ replacing Γ).

The first of these equations is nothing else but the (ordinary) ST identity for $\chi = 0 = \chi_A$ which has been studied in [9] and the general solution of which – needed for the calculation of Q and Q_A – is presented in appendix A.

Furthermore, (4.2) implies that Q, Q_A and $Q_{\chi\chi A}$ have dimension less than or equal to four and are even under charge conjugation and that Q as well as Q_A carry $\phi\pi$-charge -1 whereas $Q_{\chi\chi A}$ has $\phi\pi$-charge -2. Due to the quantum numbers of $Q_{\chi\chi A}$ there are no terms contributing to $Q_{\chi\chi A}$ (see table of quantum numbers):

$$Q_{\chi\chi A} \equiv 0 \tag{4.7}$$

For Q, Q_A one has to choose the most general ansatz compatible with the quantum numbers, see [5], and to insert these expressions into (4.4), (4.5). After a straightforward calculation one finds ($Q_{(A)} = Q, Q_A$):

$$Q_{(A)} = Q_{e.f.(A)} + Q_{\phi\pi,1(A)} + Q_{\phi\pi,2(A)} \tag{4.8}$$

with ($x_1^{(0)} = x_2^{(0)} \equiv x$ (see (6.3)) and $\bar{\varphi}_i = \varphi_i - x\hat{\varphi}_i$):

$$\begin{aligned} Q_{e.f.(A)} = \int \Big\{ & \frac{1}{4}(\partial_{\xi_{(A)}}\ln z_1 + \partial_{\xi_{(A)}}\ln z_2)(Y_1\bar{\varphi}_1 + Y_2\bar{\varphi}_2) \\ & + \frac{1}{4}(\partial_{\xi_{(A)}}\ln z_1 - \partial_{\xi_{(A)}}\ln z_2)(Y_1\bar{\varphi}_1 - Y_2\bar{\varphi}_2) \\ & - \partial_{\xi_{(A)}} x(Y_1\hat{\varphi}_1 + Y_2\hat{\varphi}_2) \Big\} \end{aligned} \tag{4.9}$$

$$Q_{\phi\pi,1(A)} = \int \Big\{ -\frac{1}{4} e\bar{c}(\partial_{\xi_{(A)}} \ln z_1 + \partial_{\xi_{(A)}} \ln z_2) \times \Big((\bar{\varphi}_1 + \frac{\sqrt{z_m}}{\sqrt{z_1} z_e} \frac{m}{e}) \hat{\varphi}_2 - \bar{\varphi}_2 (\hat{\varphi}_1 - \xi_A \frac{m}{e}) \Big) - \frac{1}{4} e\bar{c}(\partial_{\xi_{(A)}} \ln z_1 - \partial_{\xi_{(A)}} \ln z_2) \times \Big((\bar{\varphi}_1 + \frac{\sqrt{z_m}}{\sqrt{z_1} z_e} \frac{m}{e}) \hat{\varphi}_2 + \bar{\varphi}_2 (\hat{\varphi}_1 - \xi_A \frac{m}{e}) \Big) \Big\} \tag{4.10}$$

$$Q_{\phi\pi,2} = \frac{1}{2} \int \bar{c} B \tag{4.11}$$

$$Q_{\phi\pi,2\ A} = m \int \bar{c} \bar{\varphi}_2 \tag{4.12}$$

Please note that with (4.8) (4.6) is fulfilled automatically.

Hence the coefficients in Q and Q_A are completely determined as functions of the parameters z_1, z_2, z_m, z_e and x which appear in the general solution of the ST identity for $\chi = 0 = \chi_A$. But the χ- and χ_A-enlarged ST identity does not only fully fix Q and Q_A; in addition (4.4), (4.5) force some of the free parameters to be both ξ- and ξ_A-independent:

$$\begin{aligned} &\partial_\xi z_e = 0 = \partial_{\xi_A} z_e \ , \ \partial_\xi z_A = 0 = \partial_{\xi_A} z_A \ , \\ &\partial_\xi z_m = 0 = \partial_{\xi_A} z_m \ , \ \partial_\xi z_{m_H} = 0 = \partial_{\xi_A} z_{m_H} \ , \\ &\partial_\xi \mu^2 = 0 = \partial_{\xi_A} \mu^2 \end{aligned} \tag{4.13}$$

In contrast to this the wave function renormalizations z_1, z_2 and x can be arbitrary functions of ξ and ξ_A.

Two remarks are of some relevance at this point:
The (physical) normalization conditions given in Sect. 2 trivially fulfil the constraints (4.13) in the tree approximation. In higher orders of perturbation theory, however, the constraints (4.13) will extend to restrictions of the ξ- and ξ_A-dependence of some non-local Green functions (the subject of the next section) which are also used in the normalization conditions and the splitting of which into $\xi(\xi_A)$-dependent and $\xi(\xi_A)$-independent parts is much less transparent. Hence some care is needed in order not to introduce wrong gauge parameter dependence into the theory, i.e. it has to be proven explicitly that the normalization conditions chosen are in agreement with the restrictions (4.13) extended to higher orders.

The second remark concerns the t'Hooft gauge

$$\xi_A = \xi \tag{4.14}$$

which seems to be excluded in the present treatment because ξ and ξ_A are viewed as being *independent* gauge parameters. But with the following recipe it is nevertheless possible to make a transition from the general to the t'Hooft case:

- Set ∂_{ξ_A} equal to zero in all places of occurence, this partial derivative having already been taken into account in the t'Hooft gauge via $s\xi = \chi$
- Take then $\chi_A = \chi$

It is easily seen that this procedure leads to the correct results.

5 Gauge parameter dependence of Green functions

The next step would be the proof of the χ- and χ_A-dependent ST identity (4.1) to all orders of perturbation theory. We will not present the detailed proof here but instead refer to [4] where it was shown that the proof of the enlarged ST identity ($\chi \neq 0, \chi_A \neq 0$) can be reduced to the proof of the ordinary ST identity ($\chi = 0 = \chi_A$): The only possible obstruction to the validity of the ST identity would be the presence of anomalies which, however, are absent in the Abelian Higgs model. Hence we can acchieve

$$\mathcal{S}(\Gamma) = 0 \tag{5.1}$$

also in the case of BRS transforming gauge parameters ξ and ξ_A, namely by an appropriate choice of counterterms. Γ now denotes the generating functional of 1-PI Green functions. Accordingly the validity of (5.1) will be assumed throughout the following.

We now want to comment on the extensions of the constraints (4.13) to higher orders[3]. The fundamental starting point for all considerations that follow are the equations (3.4) which have to be differentiated with respect to suitable fields and finally evaluated for all fields equal to zero. Because this discussion again parallels the analogous discussion of [5] for one BRS transforming gauge parameter we skip the details of the calculations here and only summarize the results.

The continuation of the ξ- and ξ_A-independence of μ^2 to higher orders is obtained by differentiating (3.4) once with respect to φ_1; the final answer is that Γ_{Y_1} has to be χ- and χ_A-independent to all orders of the perturbative expansion:

$$\partial_\chi \Gamma_{Y_1} = 0 \quad \text{and} \quad \partial_{\chi_A} \Gamma_{Y_1} = 0 \tag{5.2}$$

Classically, $\partial_\xi z_A = 0 = \partial_{\xi_A} z_A$ and $\partial_\xi z_m = 0 = \partial_{\xi_A} z_m$ imply that the transversal part of the vector 2-point function is completely gauge parameter independent. The algebraic method now allows to prove in a simple way that this statement holds true to all orders of perturbation theory,

$$\partial_\xi \Gamma^T_{A_\mu A_\nu} = 0 \quad \text{and} \quad \partial_{\xi_A} \Gamma^T_{A_\mu A_\nu} = 0 \ . \tag{5.3}$$

(To see this one has to differentiate (3.4) with respect to A_μ, A_ν and to use an argument concerning Lorentz invariance.)

The extension of $\partial_\xi z_{m_H} = 0 = \partial_{\xi_A} z_{m_H}$ to higher orders, resulting from differentiating (3.4) twice with respect to φ_1, leads to

$$\partial_{\chi_{(A)}} \Gamma_{Y_1 \varphi_1}(p^2) \ \ \Gamma_{\varphi_1 \varphi_1}(p^2) = -\partial_{\xi_{(A)}} \Gamma_{\varphi_1 \varphi_1}(p^2) \quad . \tag{5.4}$$

Equation (5.4) completely governs the ξ- and ξ_A-dependence of the Higgs self-energy, this dependence not being trivial at all due to the existence of non-trivial insertions of the vertices $\chi \bar{c} B$ and $\chi_A m \bar{c} \varphi_2$ into the vertex function $\Gamma_{Y_1 \varphi_1}$.

[3] In this context we will restrict ourselfes to the case of a stable Higgs particle, i.e. $m_H^2 < 4m^2$

In each of the cases above it is easy *but nevertheless necessary* to show that the on-shell normalization conditions are "good" normalization conditions, i.e. that they are in agreement with the constraints dictated by the χ- and χ_A-enlarged ST identity.

Finally, $\partial_\xi z_e = 0 = \partial_{\xi_A} z_e$ is extended to higher orders by differentiating (3.4) with respect to $A_\mu, \varphi_1, \varphi_2$. This leads to two highly non-trivial equations which completely determine the ξ- and ξ_A-dependence of $\Gamma_{A_\mu\varphi_1\varphi_2}$. If the coupling is to be fixed directly with the help of $\Gamma_{A_\mu\varphi_1\varphi_2}$ (see (2.18)) one has the quite cumbersome task to introduce two additional reference points ξ_0 and ξ_{A_0} in order to fix the ξ- and ξ_A-independent part of $\Gamma_{A_\mu\varphi_1\varphi_2}$ and to explicitly control the gauge parameter dependence of $\Gamma_{A_\mu\varphi_1\varphi_2}$ via the enlarged ST identity. However, in Sect. 7 we will propose a much easier possibility for fixing the coupling by using the local Ward identity.

6 Rigid invariance

In [9] it was proven that the χ- and χ_A-independent part of the generating functional of 1-PI Green functions obeys a Ward identity (WI) of rigid symmetry to all orders of perturbation theory,

$$\hat{W}^{gen}\,\Gamma|_{\chi=0=\chi_A} = 0 \quad , \tag{6.1}$$

where $\hat{W}^{gen}$ denotes the (deformed) Ward operator:

$$\begin{aligned}\hat{W}^{gen} \equiv \int \Big\{ & -z^{-1}\varphi_2 \frac{\delta}{\delta\varphi_1} + z(\varphi_1 - \hat{\xi}_A \frac{m}{e}) \\ & \times \frac{\delta}{\delta\varphi_2} - zY_2 \frac{\delta}{\delta Y_1} + z^{-1} Y_1 \frac{\delta}{\delta Y_2} \\ & -z^{-1}\hat{\varphi}_2 \frac{\delta}{\delta\hat{\varphi}_1} + z(\hat{\varphi}_1 - \xi_A \frac{m}{e}) \\ & \times \frac{\delta}{\delta\hat{\varphi}_2} - z^{-1} q_2 \frac{\delta}{\delta q_1} + z q_1 \frac{\delta}{\delta q_2} \Big\}\end{aligned} \tag{6.2}$$

The appearance of a *deformed* Ward operator is due to the fact that physical on-shell normalization conditions (which are "good" normalization conditions, see Sect. 5) have been used. In other words: The WI (6.1) does not prescribe the values of z and ξ_A, instead these parameters are fixed uniquely by explicit normalization conditions, namely the normalization conditions imposed on the residua of the Higgs and Goldstone field (2.14) and the mass normalization of the ghosts and the Higgs (2.13). Nevertheless, (6.1) restricts some other parameters *at the classical level*:

$$x_1^{(0)} = x_2^{(0)} \equiv x \;\; , \;\; \hat{\xi}_A = -1 + x\xi_A \tag{6.3}$$

Now we are going to study the modifications of (6.1) when BRS transforming gauge parameters ξ and ξ_A are included.

Just as in the case of one BRS transforming gauge parameter [5] the application of $\hat{W}^{gen}$ (6.2) to the general solution Γ^{gen}_{cl} (4.2) of the ST identity (4.1) leads to terms which are non-linear in the propagating fields and therefore not well-defined in higher orders. In order to overcome this difficulty these terms have to be absorbed into functional operators χV^{gen} and $\chi_A V^{gen}_A$ extending $\hat{W}^{gen}$ (see [5]). An easy calculation then proves that the χ- and χ_A-enlarged Ward-operator

$$W^{gen} = \hat{W}^{gen} + \chi V^{gen} + \chi_A V^{gen}_A \quad , \tag{6.4}$$

with

$$V^{gen}_{(A)} = \partial_{\xi_{(A)}} \int \Big\{ z(\hat{\varphi}_1 - \xi_A \frac{m}{e}) \frac{\delta}{\delta q_2} - z^{-1}\hat{\varphi}_2 \frac{\delta}{\delta q_1} \Big\} \quad , \tag{6.5}$$

when acting on Γ^{gen}_{cl}, only leads to terms linear in the propagating fields:

$$W^{gen}\Gamma^{gen}_{cl} = \chi\Delta_{br} + \chi_A \Delta_{br_A} \quad , \tag{6.6}$$

$$\Delta_{br_{(A)}} = \partial_{\xi_{(A)}} \int \Big\{ z^{-1} Y_1 \varphi_2 - z Y_2 (\varphi_1 - \hat{\xi}_A \frac{m}{e}) \Big\} \tag{6.7}$$

Next it has to be proven that the WI (6.6) is valid to all orders of perturbation theory:

$$W^{gen}\Gamma = \chi\Delta_{br} + \chi_A \Delta_{br_A} \quad , \tag{6.8}$$

where Γ now denotes the generating functional of 1-PI Green functions.

This proof only relies on the quantum action principle and BRS invariance and hence it is scheme-independent. But because this proof completely parallels the proof given in [5] for one BRS transforming gauge parameter, we skip it here and refer the interested reader to [5].

7 The local Ward identity

We conclude the first part of the present paper, which extends the results of [5] to the case when all gauge parameters of the model undergo BRS transformations by looking at the local Ward identity. This local WI governs the invariance of Green functions under (deformed) local gauge transformations and also yields information about the ξ- and ξ_A-dependence of these Green functions. We again start with the local WI as it was proven in [9] for $\chi = 0 = \chi_A$ to all orders of perturbation theory,

$$\left((e + \delta e) w^{gen}(x) - \partial_\mu \frac{\delta}{\delta A_\mu} \right) \Gamma \Big|_{\chi=0=\chi_A} = \Box B \quad , \tag{7.1}$$

and then generalize to $\chi \neq 0$ and $\chi_A \neq 0$. In (7.1) $w^{gen}(x)$ denotes the (χ- and χ_A-dependent) local Ward operator which is obtained from the global one (6.4) by taking away the integration,

$$W^{gen} = \int d^4x \; w^{gen}(x) \quad , \tag{7.2}$$

and δe – to be fixed by the normalization condition for the coupling – is of order $\hbar$.

In the classical approximation a straightforward calculation shows that the following local WI holds true:

$$(ew^{gen}(x)-\partial_\mu\frac{\delta}{\delta A_\mu})\Gamma^{gen}_{cl}=\Box B+e\chi D_{br}(x)+e\chi_A D_{br_A}(x) \tag{7.3}$$

$D_{br}(x)$ and $D_{br_A}(x)$ are the non-integrated breaking terms Δ_{br} and Δ_{br_A} (6.7), respectively:

$$\Delta_{br_{(A)}}=\int d^4x\; D_{br_{(A)}}(x) \tag{7.4}$$

In order to proceed to higher orders one has to make use of the same two, general, ingredients which allow for the proof of the global WI, namely the action principle and the transformation behaviour of the local Ward operator $w^{gen}(x)$ (7.2) under BRS transformations. Again, this discussion copies the one given in [5] for one BRS transforming gauge parameter and hence we refer to [5] for details.

The final result of that discussion is that the following χ- and χ_A-dependent local WI holds true to all orders of the perturbation theory:

$$\begin{aligned}\left((e+\delta e)w^{gen}(x)-\partial_\mu\frac{\delta}{\delta A_\mu}\right)\Gamma\\ =\Box B+(e+\delta e)\chi D_{br}(x)\\ +(e+\delta e)\chi_A D_{br_A}(x)\end{aligned} \tag{7.5}$$

In addition the detailed proof shows that the overall normalization factor of the matter transformations has to be ξ- and ξ_A-independent in all orders of the perturbative expansion:

$$\partial_\xi(e+\delta e)=0=\partial_{\xi_A}(e+\delta e) \tag{7.6}$$

This result is highly non-trivial and can be obtained in this generality only with the formalism of BRS transforming gauge parameters.

But, having proven (7.6) we have at hand a new possibility for fixing the coupling: Following the line of argument, the normalization condition for the coupling has to respect the ξ- and ξ_A-independence of the factor $e+\delta e$. This is trivially fulfilled if we demand $\delta e=0$, i.e. if we require the local WI to be exact to all orders of perturbation theory:

$$\left(ew^{gen}(x)-\partial_\mu\frac{\delta}{\delta A_\mu}\right)\Gamma\Big|_{\chi=0=\chi_A}=\Box B \tag{7.7}$$

The normalization condition (7.7) (replacing (2.18)) is much easier manageable in concrete calculations.

In summary, we have shown that the on-shell normalization conditions taken together with the requirement "local WI exact to all orders" are in agreement with the χ- and χ_A-enlarged ST identity and hence guarantee a correct treatment of full gauge parameter dependence in explicit calculations.

8 BRS-symmetric insertions

As an application of the general formalism developed so far we want to study parametric differential equations of the type

$$\lambda\partial_\lambda\Gamma=\Delta_\lambda\cdot\Gamma \tag{8.1}$$

in the next section, where λ denotes a (set of) parameter(s) of the theory. Due to the action principle Δ_λ is an insertion of dimension less than or equal to four, even under charge conjugation and BRS invariant. This last property holds because of

$$\begin{aligned}0&=\lambda\partial_\lambda\mathcal{S}(\Gamma)=s_\Gamma(\lambda\partial_\lambda\Gamma)=s_\Gamma(\Delta_\lambda\cdot\Gamma)\\ &=s_{\Gamma_{cl}}\Delta_\lambda+\mathcal{O}(\hbar)\end{aligned} \tag{8.2}$$

for λ being independent of ξ and ξ_A. Therefore, as a preparatory step we first have to classify all BRS-symmetric insertions, which carry the same quantum numbers as Γ. Because in the present paper we are mainly interested in questions concerning gauge parameter dependence we will pay special attention to the appearing of ξ- and ξ_A-dependence.

In order to solve the cohomological problem mentioned above we once more return to the classical level and write down all independent field polynomials fulfilling

$$s_{\Gamma_{cl}}\Delta_\lambda=0\quad . \tag{8.3}$$

Then we have to translate these polynomials to BRS-invariant operators, only this last representation being valid to all orders of perturbation theory. Because the solution of this problem for $\chi=0=\chi_A$ was already given in [9] we will make use of the following trick to handle the modifications for $\chi\neq0$, $\chi_A\neq0$:

First we decompose Δ_λ into three parts by explicitly separating χ- and χ_A-dependence:

$$\Delta_\lambda=\Delta^0_\lambda+\chi\Delta^-_\lambda+\chi_A\Delta^-_{A,\lambda} \tag{8.4}$$

(Please note that due to the quantum numbers of Δ_λ no term proportional to $\chi\chi_A$ can appear.)

Splitting $\Gamma_{cl}=\hat\Gamma_{cl}+\chi Q+\chi_A Q_A$ and $s_{\Gamma_{cl}}$ in the same way,

$$\begin{aligned}s_{\Gamma_{cl}}&=s^{\chi=0=\chi_A}_{\hat\Gamma_{cl}}+\chi(\partial_\xi+\mathcal{O})\\ &\quad+\chi_A(\partial_{\xi_A}+\mathcal{O}_A)\quad ,\\ \text{with}\quad \mathcal{O}&=\int\left\{\frac{\delta Q}{\delta\underline{\varphi}}\frac{\delta}{\delta\underline{Y}}-\frac{\delta Q}{\delta\underline{Y}}\frac{\delta}{\delta\underline{\varphi}}\right\}\quad ,\\ \mathcal{O}_A&=\int\left\{\frac{\delta Q_A}{\delta\underline{\varphi}}\frac{\delta}{\delta\underline{Y}}-\frac{\delta Q_A}{\delta\underline{Y}}\frac{\delta}{\delta\underline{\varphi}}\right\}\quad ,\end{aligned} \tag{8.5}$$

we find that (8.3) is equivalent to the following four equations:

$$s^{\chi=0=\chi_A}_{\hat\Gamma_{cl}}\Delta^0_\lambda=0 \tag{8.6}$$

$$s^{\chi=0=\chi_A}_{\hat\Gamma_{cl}}\Delta^-_\lambda=(\partial_\xi+\mathcal{O})\Delta^0_\lambda \tag{8.7}$$

$$s^{\chi=0=\chi_A}_{\hat\Gamma_{cl}}\Delta^-_{A,\lambda}=(\partial_{\xi_A}+\mathcal{O}_A)\Delta^0_\lambda \tag{8.8}$$

$$(\partial_\xi+\mathcal{O})\Delta^-_{A,\lambda}=(\partial_{\xi_A}+\mathcal{O}_A)\Delta^-_\lambda \tag{8.9}$$

Now it is easy to see that it is always possible to find a $\hat\Delta^-_\lambda$ such that:

$$\Delta^-_\lambda=(\partial_\xi+\mathcal{O})\hat\Delta^-_\lambda\quad\text{and}\quad\Delta^-_{A,\lambda}=(\partial_{\xi_A}+\mathcal{O}_A)\hat\Delta^-_\lambda \tag{8.10}$$

We remark that due to (8.10) (8.9) is fulfilled automatically.

With these preparations we have

$$\begin{aligned}(\partial_\xi + \mathcal{O})(\Delta^0_\lambda - s^{\chi=0=\chi_A}_{\hat\Gamma_{cl}}\hat\Delta^-_\lambda)\\ = s^{\chi=0=\chi_A}_{\hat\Gamma_{cl}}\Delta^-_\lambda - (\partial_\xi+\mathcal{O})s^{\chi=0=\chi_A}_{\hat\Gamma_{cl}}\hat\Delta^-_\lambda\\ = s^{\chi=0=\chi_A}_{\hat\Gamma_{cl}}\Delta^-_\lambda - s^{\chi=0=\chi_A}_{\hat\Gamma_{cl}}(\partial_\xi+\mathcal{O})\hat\Delta^-_\lambda\\ = 0\end{aligned} \tag{8.11}$$

and in the same way

$$(\partial_{\xi_A} + \mathcal{O}_A)(\Delta^0_\lambda - s^{\chi=0=\chi_A}_{\hat\Gamma_{cl}}\hat\Delta^-_\lambda) = 0 \quad . \tag{8.12}$$

But that means

$$\Delta^0_\lambda = s^{\chi=0=\chi_A}_{\hat\Gamma_{cl}}\hat\Delta^-_\lambda + \hat\Delta^0_\lambda \tag{8.13}$$

with

$$(\partial_\xi+\mathcal{O})\hat\Delta^0_\lambda = 0 = (\partial_{\xi_A}+\mathcal{O}_A)\hat\Delta^0_\lambda \tag{8.14}$$

and

$$s^{\chi=0=\chi_A}_{\hat\Gamma_{cl}}\hat\Delta^0_\lambda = 0 \quad . \tag{8.15}$$

Hence:

$$\begin{aligned}\Delta_\lambda &= \Delta^0_\lambda + \chi\Delta^-_\lambda + \chi_A\Delta^-_{A,\lambda}\\ &= s^{\chi=0=\chi_A}_{\hat\Gamma_{cl}}\hat\Delta^-_\lambda + \hat\Delta^0_\lambda + \chi(\partial_\xi+\mathcal{O})\hat\Delta^-_\lambda\\ &\quad +\chi_A(\partial_{\xi_A}+\mathcal{O}_A)\hat\Delta^-_\lambda\\ &= \hat\Delta^0_\lambda + s_{\Gamma_{cl}}\hat\Delta^-_\lambda\end{aligned} \tag{8.16}$$

As already mentioned, the solution of (8.15) was presented in [9] and we just give the list of terms contributing to $\hat\Delta^0_\lambda$ in appendix B. The crucial point in this context is, however: A short calculation starting from (8.14) shows that all terms in $\hat\Delta^0_\lambda$ which are *no* BRS variations, namely[4],

$$\int\frac{\delta\hat\Gamma_{cl}}{\delta\hat\varphi_0}\ ,\ \int\left\{A\frac{\delta}{\delta A}+c\frac{\delta}{\delta c}\right\}\hat\Gamma_{cl}\ ,\ m_H\partial_{m_H}\hat\Gamma_{cl}\ ,\ e\partial_e\hat\Gamma_{cl}\ , \tag{8.17}$$

have to appear with coefficients which are *independent of* ξ *and* ξ_A.

For $\hat\Delta^-_\lambda$ we choose the most general ansatz compatible with the quantum numbers of $\hat\Delta^-_\lambda$ ($\phi\pi$-charge: -1, C: +, dim: ≤ 4); in view of the generalization to higher orders, this ansatz can be brought into the form:

$$\begin{aligned}\hat\Delta^-_\lambda : &\int Y_1, \frac{\delta\hat\Gamma_{cl}}{\delta q_1}, \bar c\hat\varphi_2,\\ &Y_1\varphi_1, Y_1\hat\varphi_1, Y_2\varphi_2, Y_2\hat\varphi_2, \bar c\frac{\delta\hat\Gamma_{cl}}{\delta B}, \hat\varphi_i\frac{\delta\hat\Gamma_{cl}}{\delta q_i}, \bar c\hat\varphi_1\hat\varphi_2, \bar cB\end{aligned} \tag{8.18}$$

Looking to (B.1) – (B.4) and (8.16), we find that all terms in $\hat\Delta^0_\lambda$ which are BRS variations have to be modified when BRS transforming gauge parameters are included. Therefore we finally end up with the following basis of BRS invariant insertions which we directly give in the form of BRS invariant operators ($i = 1, 2$):

$$\begin{aligned}&f_{s,0}\int\frac{\delta\Gamma}{\delta\varphi_1} + (\chi\partial_\xi+\chi_A\partial_{\xi_A})f_{s,0}\int Y_1\\ &\quad = s_\Gamma(f_{s,0}\int Y_1)\\ &\hat f_{s,0}\int\frac{\delta\Gamma}{\delta\hat\varphi_1} + [(\chi\partial_\xi+\chi_A\partial_{\xi_A})\hat f_{s,0}]\int\frac{\delta\Gamma}{\delta q_1}\\ &\quad = s_\Gamma(\hat f_{s,0}\int\frac{\delta\Gamma}{\delta q_1})\\ &\tilde f_3\int\{B\hat\varphi_2 - \bar cq_2\} + (\chi\partial_\xi+\chi_A\partial_{\xi_A})\tilde f_3\int\bar c\hat\varphi_2\\ &\quad = s_\Gamma(\tilde f_3\int\bar c\hat\varphi_2)\end{aligned} \tag{8.19}$$

$$\begin{aligned}\mathcal{N}^{(\chi,\chi_A)}_{s,i}\Gamma &\equiv f_{s,i}\int\left\{\varphi_i\frac{\delta}{\delta\varphi_i} - Y_i\frac{\delta}{\delta Y_i}\right\}\Gamma\\ &\quad +(\chi\partial_\xi+\chi_A\partial_{\xi_A})f_{s,i}\int Y_i\varphi_i\\ &\quad = s_\Gamma(f_{s,i}\int Y_i\varphi_i)\\ \hat{\mathcal{N}}^{(\chi,\chi_A)}_{s,i}\Gamma &\equiv \hat f_{s,i}\int\left\{q_i\frac{\delta}{\delta q_i} + \hat\varphi_i\frac{\delta}{\delta\hat\varphi_i}\right\}\Gamma\\ &\quad +[(\chi\partial_\xi+\chi_A\partial_{\xi_A})\hat f_{s,i}]\int\hat\varphi_i\frac{\delta\Gamma}{\delta q_i}\\ &\quad = s_\Gamma(\hat f_{s,i}\int\hat\varphi_i\frac{\delta\Gamma}{\delta q_i})\\ \bar{\mathcal{N}}^{(\chi,\chi_A)}_{s,i}\Gamma &\equiv \bar f_{s,i}\int\left\{\hat\varphi_i\frac{\delta\Gamma}{\delta\varphi_i} - Y_iq_i\right\}\\ &\quad +(\chi\partial_\xi+\chi_A\partial_{\xi_A})\bar f_{s,i}\int Y_i\hat\varphi_i\\ &\quad = s_\Gamma(\bar f_{s,i}\int Y_i\hat\varphi_i)\\ \mathcal{N}^{(\chi,\chi_A)}_{B}\Gamma &\equiv f_B\int\left\{B\frac{\delta}{\delta B} + \bar c\frac{\delta}{\delta\bar c}\right\}\Gamma\\ &\quad +[(\chi\partial_\xi+\chi_A\partial_{\xi_A})f_B]\int\bar c\frac{\delta\Gamma}{\delta B}\\ &\quad = s_\Gamma(f_B\int\bar c\frac{\delta\Gamma}{\delta B})\\ \tilde f_4\int\{B\hat\varphi_1\hat\varphi_2 - \bar cq_1\hat\varphi_2 - \bar c\hat\varphi_1q_2\}&\\ &\quad +(\chi\partial_\xi+\chi_A\partial_{\xi_A})\tilde f_4\int\bar c\hat\varphi_1\hat\varphi_2\\ &\quad = s_\Gamma(\tilde f_4\int\bar c\hat\varphi_1\hat\varphi_2)\\ f_\xi\partial_\xi\Gamma + [(\chi\partial_\xi+\chi_A\partial_{\xi_A})f_\xi]\partial_\chi\Gamma&\\ &\quad = s_\Gamma(f_\xi\partial_\chi\Gamma)\end{aligned} \tag{8.20}$$

In addition there are the BRS symmetric operators from (8.17) (with $\hat\Gamma_{cl}$ replaced by Γ).

[4] The definition of the additional external field $\hat\varphi_0$ is also given in Appendix B

9 Parametric differential equations

Having finished the preparatory considerations dealing with the BRS symmetric insertions we now can turn to the derivation of some partial differential equations, namely the Callan-Symanzik (CS) equation and the renormalization group (RG) equation. We will also comment about the dependence of the theory on the ghost mass which is governed by the differential operator $\xi_A\partial_{\xi_A}$ due to the normalization condition (2.13).

9.1 CS equation

The CS equation describes the response of the system to the scaling of all *independent* parameters carrying dimension of mass. In the model under investigation the CS operator is hence given by

$$\underline{m}\partial_{\underline{m}} \equiv m\partial_m + m_H\partial_{m_H} + \kappa\partial_\kappa \tag{9.1}$$

and we have the task to construct the r.h.s. of $\underline{m}\partial_{\underline{m}}\Gamma = \ ?$ which according to the action principle

$$\underline{m}\partial_{\underline{m}}\Gamma = \Delta_m\cdot\Gamma \tag{9.2}$$

has to be an insertion with dimension less than or equal to four, even under charge conjugation and also BRS invariant. (I.e. $\underline{m}\partial_{\underline{m}}$ is an operator of the type $\lambda\partial_\lambda$ discussed in the previous section.) In [9] it was shown that in order to construct a *unique* r.h.s. of the CS equation rigid invariance has to be used, too. Therefore we next calculate the commutator of the χ- and χ_A-enlarged global Ward operator (6.4) and $\underline{m}\partial_{\underline{m}}$:

$$\begin{aligned}[W^{gen},\underline{m}\partial_{\underline{m}}] = z\int\left\{\hat\xi_A\frac{m}{e}\frac{\delta}{\delta\varphi_2}+\xi_A\frac{m}{e}\frac{\delta}{\delta\hat\varphi_2}\right\}\\ +(\chi\partial_\xi+\chi_A\partial_{\xi_A})z\xi_A\frac{m}{e}\int\frac{\delta}{\delta q_2}\end{aligned} \tag{9.3}$$

In order to make the line of argument as transparent as possible and to explicitly work out what is needed in the following we introduce the W^{gen}-symmetric extension $\underline{m}\tilde\partial_{\underline{m}}$ of $\underline{m}\partial_{\underline{m}}$ originating from (9.3),

$$\begin{aligned}\underline{m}\tilde\partial_{\underline{m}} \equiv \underline{m}\partial_{\underline{m}} + \hat\xi_A\frac{m}{e}\int\frac{\delta}{\delta\varphi_1}+\xi_A\frac{m}{e}\int\frac{\delta}{\delta\hat\varphi_1}\\ +(\chi\partial_\xi+\chi_A\partial_{\xi_A})\xi_A\frac{m}{e}\int\frac{\delta}{\delta q_1}\ ,\end{aligned} \tag{9.4}$$

$$[W^{gen},\underline{m}\tilde\partial_{\underline{m}}] = 0\ , \tag{9.5}$$

and consider the insertion $\underline{m}\tilde\partial_{\underline{m}}\Gamma = \tilde\Delta_m\cdot\Gamma$ instead of $\underline{m}\partial_{\underline{m}}\Gamma = \Delta_m\cdot\Gamma$. Due to (9.5) we have:

$$\begin{aligned}W^{gen}(\tilde\Delta_m\cdot\Gamma) &= \underline{m}\tilde\partial_{\underline{m}}W^{gen}\Gamma\\ &= \underline{m}\tilde\partial_{\underline{m}}(\chi\Delta_{br}+\chi_A\Delta_{br_A})\\ &= z(\chi\partial_\xi+\chi_A\partial_{\xi_A})\hat\xi_A\frac{m}{e}\int Y_2\end{aligned} \tag{9.6}$$

Please note that the application of W^{gen} to the term

$$-(\chi\partial_\xi+\chi_A\partial_{\xi_A})\hat\xi_A\frac{m}{e}\int Y_1\quad , \tag{9.7}$$

this term being part of the first insertion in (8.19), exactly cancels the r.h.s. of (9.6). Therefore, all other BRS symmetric[5] insertions building up $\tilde\Delta_m\cdot\Gamma$ have to be symmetrized with respect to W^{gen}: Only in this W^{gen}-symmetrized form they can contribute to the r.h.s. of the CS equation. For some of the operators in (8.17), (8.19), (8.20) (namely the (χ- and χ_A-enlarged) leg counting operators) this symmetrization can be achieved easily:

$$\begin{aligned}\mathcal{N}_s^{(\chi,\chi_A)}\Gamma &\equiv f_sN_s\Gamma - f_s\hat\xi_A\frac{m}{e}\int\frac{\delta\Gamma}{\delta\varphi_1}\\ &\quad+(\chi\partial_\xi+\chi_A\partial_{\xi_A})f_s\\ &\quad\times\int\left\{Y_1(\varphi_1-\hat\xi_A\frac{m}{e})+Y_2\varphi_2\right\}\ ,\\ \hat{\mathcal{N}}_s^{(\chi,\chi_A)} &\equiv \hat f_s\hat N_s - \hat f_s\xi_A\frac{m}{e}\int\frac{\delta}{\delta\hat\varphi_1}\\ &\quad+(\chi\partial_\xi+\chi_A\partial_{\xi_A})\hat f_s\\ &\quad\times\int\left\{(\hat\varphi_1-\xi_A\frac{m}{e})\frac{\delta}{\delta q_1}+\hat\varphi_2\frac{\delta}{\delta q_2}\right\}\ ,\\ N_A &\equiv \int\left\{A\frac{\delta}{\delta A}+c\frac{\delta}{\delta c}\right\}\quad ,\\ \mathcal{N}_B^{(\chi,\chi_A)} &\equiv f_BN_B+(\chi\partial_\xi+\chi_A\partial_{\xi_A})f_B\int\bar c\frac{\delta}{\delta B}\end{aligned} \tag{9.8}$$

The mixed operators containing $\hat\varphi_i\frac{\delta\Gamma}{\delta\varphi_i}$ are symmetrized like the leg counting operators:

$$\begin{aligned}&\bar{\mathcal{N}}_s^{(\chi,\chi_A)}\Gamma\\ &\equiv \bar f_s\bar N_s\Gamma - \bar f_s\xi_A\frac{m}{e}\int\frac{\delta\Gamma}{\delta\varphi_1}+\bar f_s\int\{q_1Y_1+q_2Y_2\}\\ &\quad+(\chi\partial_\xi+\chi_A\partial_{\xi_A})\bar f_s\int\left\{Y_1(\hat\varphi_1-\xi_A\frac{m}{e})+Y_2\hat\varphi_2\right\}\end{aligned} \tag{9.9}$$

In (9.8), (9.9) we have introduced the usual leg counting operators:

$$\begin{aligned}N_s &\equiv \int\left\{\varphi_1\frac{\delta}{\delta\varphi_1}+\varphi_2\frac{\delta}{\delta\varphi_2}-Y_1\frac{\delta}{\delta Y_1}-Y_2\frac{\delta}{\delta Y_2}\right\}\quad ,\\ \hat N_s &\equiv \int\left\{\hat\varphi_1\frac{\delta}{\delta\hat\varphi_1}+\hat\varphi_2\frac{\delta}{\delta\hat\varphi_2}+q_1\frac{\delta}{\delta q_1}+q_2\frac{\delta}{\delta q_2}\right\}\quad ,\\ \bar N_s &\equiv \int\left\{\hat\varphi_1\frac{\delta}{\delta\varphi_1}+\hat\varphi_2\frac{\delta}{\delta\varphi_2}\right\}\quad ,\\ N_B &\equiv \int\left\{B\frac{\delta}{\delta B}+\bar c\frac{\delta}{\delta\bar c}\right\}\end{aligned} \tag{9.10}$$

To find the W^{gen}-symmetric extensions of the differential operators $m_H\partial_{m_H}$, $e\partial_e$ and the operator containing ∂_ξ

[5] The operators extending $\underline{m}\partial_{\underline{m}}$ in (9.4) taken together with (9.7) (times -1) just constitute the first two BRS symmetric insertions in (8.19); hence the remaining contributions to $\tilde\Delta_m\cdot\Gamma$ have to be BRS symmetric

(see last line of (8.20)),

$$m_H\partial_{m_H} \to m_H\tilde{\partial}_{m_H} \ , \ e\partial_e \to e\tilde{\partial}_e \ ,$$
$$f_\xi\partial_\xi + (\chi\partial_\xi + \chi_A\partial_{\xi_A})f_\xi\partial_\chi \to \tilde{\partial}_\xi \ , \qquad (9.11)$$

indeed requires some calculation. The final expressions being rather lengthy we present the explicit results of the symmetrization in Appendix C.

Finally we observe that the insertion $\frac{\delta}{\delta\hat{\varphi}_0}$ already is W^{gen}-symmetric and that the remaining two insertions in (8.19), (8.20) cannot be extended in a W^{gen}-symmetric way.

Thus the final answer is: (9.8), (9.9), (C.5) and $\frac{\delta}{\delta\hat{\varphi}_0}$ provide a basis of BRS symmetric and rigidly invariant operators which are even under charge conjugation and have dimension less than or equal to four. Hence the insertion $\tilde{\Delta}_m \cdot \Gamma$ can be decomposed as follows:

$$\begin{aligned}\tilde{\mathcal{C}}\Gamma \equiv \Big(&\underline{m}\tilde{\partial}_{\underline{m}} + \beta_e e\tilde{\partial}_e + \beta_{m_H} m_H\tilde{\partial}_{m_H} + \tilde{\beta}_\xi\tilde{\partial}_\xi \\ &-\tilde{\gamma}_s\mathcal{N}_s^{(\chi,\chi_A)} - \hat{\tilde{\gamma}}_s\hat{\mathcal{N}}_s^{(\chi,\chi_A)} - \bar{\tilde{\gamma}}_s\bar{\mathcal{N}}_s^{(\chi,\chi_A)} \\ &-\gamma_A N_A - \tilde{\gamma}_B\mathcal{N}_B^{(\chi,\chi_A)} - \alpha_{inv}\int\delta_{\hat{\varphi}_0}\Big)\Gamma \\ = &-(\chi\partial_\xi + \chi_A\partial_{\xi_A})\hat{\xi}_A\frac{m}{e}\int Y_1 \end{aligned} \qquad (9.12)$$

Equation (9.12) is the CS equation in the manifestly W^{gen}-symmetric form. The important result in our context is that the β-functions β_e and β_{m_H} as well as the anomalous dimension γ_A and α_{inv} are independent of both the gauge parameters ξ and ξ_A to all orders of perturbation theory. The coefficient functions $\tilde{\beta}_\xi, \tilde{\gamma}_s, \hat{\tilde{\gamma}}_s, \bar{\tilde{\gamma}}_s, \tilde{\gamma}_B$ also are ξ- and ξ_A-independent, but the usual (i.e. complete) β- and γ-functions

$$\beta_\xi\xi = \tilde{\beta}_\xi f_\xi \ , \ \gamma_s = \tilde{\gamma}_s f_s \ , \ \hat{\gamma}_s = \hat{\tilde{\gamma}}_s\hat{f}_s \ ,$$
$$\bar{\gamma}_s = \bar{\tilde{\gamma}}_s\bar{f}_s \ , \ \gamma_B = \tilde{\gamma}_B f_B \qquad (9.13)$$

a priori may depend on both the gauge parameters ξ and ξ_A through the factors $f_{(s,\xi)}(\xi,\xi_A)$ appearing in the leg counting operators $\mathcal{N}_s^{(\chi,\chi_A)}$ (9.8), (9.9) and in $\tilde{\partial}_\xi$ (C.5).

So far one can get with symmetry considerations alone. If additional information about the coefficient functions is requested one has to test (9.12) on the gauge condition (2.4), to make use of the local WI (7.5) and/or to carry out explicit calculations: Testing (9.12) on the gauge condition (2.4) we find

$$\begin{aligned}\gamma_B &= -\gamma_A \qquad (9.14)\\ \beta_\xi \ &= \ 2\gamma_B = -2\gamma_A \\ \beta_e + \gamma_A - \gamma_s - \hat{\gamma}_s &= (\beta_e e\partial_e + \beta_{m_H} m_H\partial_{m_H} - 2\gamma_A\xi\partial_\xi)\ln z\end{aligned}$$

and hence also γ_B and β_ξ are completely gauge parameter-independent to all orders. Furthermore, using the validity of the local WI (7.5) and the normalization condition for the coupling (7.7) yields (see [9] for details):

$$\gamma_A = \beta_e \qquad (9.15)$$

We want to conclude this subsection by rewriting the CS equation in its much more convenient form which separates the hard and soft breaking on the left and right hand side of the CS equation:

$$\begin{aligned}&\Big(\underline{m}\partial_{\underline{m}} + \beta_e e\partial_e + \beta_{m_H} m_H\partial_{m_H} - \gamma_s N_s \\ &\quad -\hat{\gamma}_s\hat{N}_s - \bar{\gamma}_s\bar{N}_s - \beta_e(N_A - N_B + 2\xi\partial_\xi) \\ &\quad -\gamma_1\int\Big\{\varphi_1\frac{\delta}{\delta\varphi_1} - Y_1\frac{\delta}{\delta Y_1} + \hat{\varphi}_1\frac{\delta}{\delta\hat{\varphi}_1} + q_1\frac{\delta}{\delta q_1}\Big\} \\ &\quad -(\chi\partial_\xi + \chi_A\partial_{\xi_A})\int\Big\{-\gamma_s\hat{\varphi}_1\frac{\delta}{\delta q_1} + \hat{\gamma}_s\hat{\varphi}_2\frac{\delta}{\delta q_2}\Big\} \\ &\quad -2\beta_e\chi\partial_\chi\Big)\Gamma \\ = &-\frac{m}{e}\int\Big\{(\hat{\xi}_A + \alpha_1)\frac{\delta}{\delta\varphi_1} + (\xi_A + \hat{\alpha}_1)\frac{\delta}{\delta\hat{\varphi}_1} \\ &\quad -\alpha_{inv}\frac{e}{m}\frac{\delta}{\delta\hat{\varphi}_0} + (\chi\partial_\xi + \chi_A\partial_{\xi_A})(\xi_A + \hat{\alpha}_1)\frac{\delta}{\delta q_1}\Big\}\Gamma \\ &\quad +\bar{\gamma}_s\int\{q_1Y_1 + q_2Y_2\} \\ &\quad +(\chi\partial_\xi + \chi_A\partial_{\xi_A})\int\Big\{(\gamma_s + \gamma_1)Y_1\varphi_1 + \gamma_s Y_2\varphi_2 \\ &\quad -\frac{m}{e}(\hat{\xi}_A + \alpha_1)Y_1 + \bar{\gamma}_s(Y_1\hat{\varphi}_1 + Y_2\hat{\varphi}_2)\Big\}\end{aligned} \qquad (9.16)$$

with

$$\begin{aligned}\gamma_1 &= (\beta_e e\partial_e + \beta_{m_H} m_H\partial_{m_H} - 2\beta_e\xi\partial_\xi)\ln z \\ &= \mathcal{O}(\hbar^2) \\ \hat{\gamma}_s &= 2\beta_e - \gamma_s - \gamma_1 \\ \alpha_1 &= (\gamma_1 + \gamma_s - \beta_e)\hat{\xi}_A + \bar{\gamma}_s\xi_A \\ &\quad +(\beta_e e\partial_e + \beta_{m_H} m_H\partial_{m_H} - 2\beta_e\xi\partial_\xi)\hat{\xi}_A \\ \hat{\alpha}_1 &= (\gamma_1 + \hat{\gamma}_s - \beta_e)\xi_A\end{aligned} \qquad (9.17)$$

In (9.16), (9.17) we have already incorporated the relations (9.14) and (9.15). Therefore only the coefficient functions $\beta_e, \beta_{m_H}, \gamma_s, \bar{\gamma}_s$ and the coefficient $\alpha_{inv} = \frac{1}{2}m_H^2 + \mathcal{O}(\hbar)$ of the soft insertion $\int\delta_{\hat{\varphi}_0}$ turn out to be independent and have to be determined by explicit calculations (see [9]).

9.2 Dependence on the ghost mass

Due to the normalization condition (2.13) the dependence of the theory on the ghost mass is encoded in the differential operator $\xi_A\partial_{\xi_A}$ and we have to analyse the r.h.s. of $\xi_A\partial_{\xi_A}\Gamma = \ ?$. This analysis almost completely parallels the analysis of the CS equation, but with one minor change: Instead of (8.2) we now have:

$$0 = \xi_A\partial_{\xi_A}\mathcal{S}(\Gamma) = s_\Gamma(\xi_A\partial_{\xi_A}\Gamma) - \chi_A\partial_{\xi_A}\Gamma \qquad (9.18)$$

However, differentiating the ST identity with respect to χ_A we find

$$s_\Gamma(\partial_{\chi_A}\Gamma) = \partial_{\xi_A}\Gamma \qquad (9.19)$$

Hence the action principle, (9.18) and (9.19) imply that

$$(\xi_A\partial_{\xi_A}+\chi_A\partial_{\chi_A})\Gamma=\Delta_{\xi_A}\cdot\Gamma \quad , \tag{9.20}$$

where $\Delta_{\xi_A}\cdot\Gamma$ is a BRS symmetric insertion. But due to

$$\begin{aligned} s_\Gamma(\xi_A\partial_{\chi_A}\Gamma) &= (s_\Gamma\xi_A)\partial_{\chi_A}\Gamma+\xi_A s_\Gamma(\partial_{\chi_A}\Gamma)\\ &= \chi_A\partial_{\chi_A}\Gamma+\xi_A\partial_{\xi_A}\Gamma \end{aligned} \tag{9.21}$$

$\Delta_{\xi_A}\cdot\Gamma$ also has to be a BRS variation and hence only BRS variations can contribute to $\Delta_{\xi_A}\cdot\Gamma$. From here on the discussion is completely analogous to the discussion of the CS equation; we skip the details and just present the result:

$$\begin{aligned} &\Big(\xi_A\tilde\partial_{\xi_A}+\chi_A\partial_{\chi_A}+\tilde\beta_\xi^{\xi_A}\tilde\partial_\xi-\tilde\gamma_s^{\xi_A}\mathcal{N}_s^{(\chi,\chi_A)}\\ &\quad-\tilde{\hat\gamma}_s^{\xi_A}\hat{\mathcal{N}}_s^{(\chi,\chi_A)}-\tilde{\bar\gamma}_s^{\xi_A}\bar{\mathcal{N}}_s^{(\chi,\chi_A)}-\tilde\gamma_B^{\xi_A}\mathcal{N}_B^{(\chi,\chi_A)}\Big)\Gamma\\ &=(\chi\partial_\xi+\chi_A\partial_{\xi_A})\,z^{-1}\,\xi_A\partial_{\xi_A}\int zY_1(\varphi_1-\hat\xi_A\frac{m}{e}) \end{aligned} \tag{9.22}$$

In (9.22) $\xi_A\tilde\partial_{\xi_A}$ is the W^{gen}-symmetric extension of $\xi_A\partial_{\xi_A}$ and given by:

$$\begin{aligned} \xi_A\tilde\partial_{\xi_A}\equiv\xi_A\partial_{\xi_A}&-z^{-1}\,\xi_A\partial_{\xi_A}\int z\Big\{(\varphi_1-\hat\xi_A\frac{m}{e})\frac{\delta}{\delta\varphi_1}\\ &+(\hat\varphi_1-\xi_A\frac{m}{e})\frac{\delta}{\delta\hat\varphi_1}-Y_1\frac{\delta}{\delta Y_1}+q_1\frac{\delta}{\delta q_1}\Big\}\\ &-(\chi\partial_\xi+\chi_A\partial_{\xi_A})\,z^{-1}\,\xi_A\partial_{\xi_A}\int z(\hat\varphi_1-\xi_A\frac{m}{e})\frac{\delta}{\delta q_1} \end{aligned} \tag{9.23}$$

Introducing the "real" β- and γ-functions like in (9.13) the test of (9.22) on the gauge condition (2.4) yields:

$$\begin{aligned} &\gamma_B^{\xi_A}=0\\ &\beta_\xi^{\xi_A}=2\gamma_B^{\xi_A}=0\\ &-\gamma_s^{\xi_A}-\hat\gamma_s^{\xi_A}-\gamma_B^{\xi_A}=(\xi_A\partial_{\xi_A}+\beta_\xi^{\xi_A}\xi\partial_\xi)\ln z\\ &\Leftrightarrow\hat\gamma_s^{\xi_A}=-\gamma_s^{\xi_A}-\xi_A\partial_{\xi_A}\ln z \end{aligned} \tag{9.24}$$

Again, we can separate in (9.22) the hard and soft breaking on the left and right hand side; thereby using (9.24) we end up with the following form, which for brevity we only give for all external fields set equal to zero:

$$\begin{aligned} &\Big(\xi_A\partial_{\xi_A}+\chi_A\partial_{\chi_A}-\gamma_s^{\xi_A}N_s-\hat\gamma_s^{\xi_A}\hat N_s\\ &\quad-\bar\gamma_s^{\xi_A}\bar N_s-\xi_A\partial_{\xi_A}\ln z\int\varphi_1\frac{\delta}{\delta\varphi_1}\Big)\Gamma\Big|_{ext.f.\equiv0}\\ &=-\frac{m}{e}\int\Big\{\alpha\frac{\delta}{\delta\varphi_1}+\xi_A(1-\gamma_s^{\xi_A})\frac{\delta}{\delta\hat\varphi_1}\\ &\quad+(\chi\partial_\xi+\chi_A\partial_{\xi_A})\xi_A(1-\gamma_s^{\xi_A})\frac{\delta}{\delta q_1}\Big\}\Gamma\Big|_{ext.f.\equiv0} \end{aligned} \tag{9.25}$$

with

$$\alpha=-\hat\gamma_s^{\xi_A}\hat\xi_A+\bar\gamma_s^{\xi_A}\xi_A+\xi_A\partial_{\xi_A}\hat\xi_A=x\xi_A+\mathcal{O}(\hbar) \tag{9.26}$$

9.3 RG equation

The derivation of the RG equation once more starts with the action principle

$$\kappa\partial_\kappa\Gamma=\Delta_\kappa\cdot\Gamma \tag{9.27}$$

which tells us that $\Delta_\kappa\cdot\Gamma$ has to be an insertion of dimension less than or equal to four, invariant under charge conjugation and in addition BRS symmetric due to (8.2). In order to arrive at a more convenient form of the RG equation we now introduce a new set of BRS symmetric operators (see also [10]) representing the two- and three-dimensional BRS symmetric classical field polynomials, i.e. instead of (8.19) and $\int\delta_{\hat\varphi_0}$ we are going to use:

$$\begin{aligned} m\partial_m\;,\;&f_A\partial_{\xi_A}\Gamma+[(\chi\partial_\xi+\chi_A\partial_{\xi_A})f_A]\partial_{\chi_A}\Gamma\\ &=s_\Gamma(f_A\partial_{\chi_A}\Gamma)\;,\\ \int\frac{\delta}{\delta\hat\varphi_0}\;,\;\tilde f\int\{B\hat\varphi_2-\bar cq_2\}&+(\chi\partial_\xi+\chi_A\partial_{\xi_A})\tilde f\int\bar c\hat\varphi_2\\ &=s_\Gamma(\tilde f\int\bar c\hat\varphi_2) \end{aligned} \tag{9.28}$$

Hence according to BRS invariance alone, $\Delta_\kappa\cdot\Gamma$ can be decomposed into a sum of the BRS symmetric operators (9.28), (8.20) and (the remaining four-dimensional operators in) (8.17):

$$\begin{aligned} \kappa\partial_\kappa\Gamma=\Big(&-\beta_m^\kappa m\partial_m-\tilde\beta_{\xi_A}^\kappa(f_A\partial_{\xi_A}\\ &+[(\chi\partial_\xi+\chi_A\partial_{\xi_A})f_A]\partial_{\chi_A}\Big)\\ &+\alpha_{inv}^\kappa\int\delta_{\hat\varphi_0}-\beta_{m_H}^\kappa m_H\partial_{m_H}\\ &-\beta_e^\kappa e\partial_e+\gamma_A^\kappa N_A+\tilde\gamma_B^\kappa\mathcal{N}_B^{(\chi,\chi_A)}\\ &+\sum_{i=1}^{2}\Big\{\tilde\gamma_{s,i}^\kappa\mathcal{N}_{s,i}^{(\chi,\chi_A)}+\tilde{\bar\gamma}_{s,i}^\kappa\bar{\mathcal{N}}_{s,i}^{(\chi,\chi_A)}+\tilde{\hat\gamma}_{s,i}^\kappa\hat{\mathcal{N}}_{s,i}^{(\chi,\chi_A)}\Big\}\\ &-\tilde\beta_\xi^\kappa(f_\xi\partial_\xi+[(\chi\partial_\xi+\chi_A\partial_{\xi_A})f_\xi]\partial_\chi)\Big)\Gamma\\ &+\tilde\gamma^\kappa s_\Gamma(\tilde f\int\bar c\hat\varphi_2)+\tilde{\bar\gamma}^\kappa s_\Gamma(\tilde f_4\int\bar c\hat\varphi_1\hat\varphi_2) \end{aligned} \tag{9.29}$$

Differentiating (9.29) with respect to φ_1, setting all fields equal to zero and making use of the normalization condition $\Gamma_{\varphi_1}=0$ (2.16) it immeadiately follows that:

$$\alpha_{inv}^\kappa\equiv0 \tag{9.30}$$

With this result in mind three further tests of (9.29) on the physical normalization conditions (2.13) concerning the mass normalizations of the Higgs, the vector and the ghost directly imply

$$\beta_{m_H}^\kappa\equiv0\quad,\quad\beta_m^\kappa\equiv0\quad,\quad\tilde\beta_{\xi_A}^\kappa\equiv0 \tag{9.31}$$

to all orders of perturbation theory. Therefore due to the physical normalization conditions the first four terms on

the r.h.s. of (9.29) are absent and no β-function in connection with a (physical) mass appears in the RG equation.

In order to conclude the derivation of the RG equation we now have to exploit rigid invariance of the theory: To this end we first apply W^{gen} (6.4) to the RG equation (9.29) and then also use the rigid WI (6.8):

$$\begin{aligned} W^{gen}\kappa\partial_\kappa\Gamma &= [W^{gen},\kappa\partial_\kappa]\Gamma + \kappa\partial_\kappa W^{gen}\Gamma \\ &= -(\kappa\partial_\kappa W^{gen})\Gamma + \kappa\partial_\kappa(\chi\Delta_{br} + \chi_A\Delta_{br_A}) \end{aligned} \tag{9.32}$$

This leads after some calculation to the final form of the RG equation:

$$\begin{aligned} &\Bigg(\kappa\partial_\kappa + \beta_e^\kappa e\partial_e + \beta_\xi^\kappa\xi\partial_\xi - \gamma_A^\kappa N_A \\ &-\gamma_B^\kappa N_B - \gamma_s^\kappa N_s - \hat\gamma_s^\kappa\hat N_s - \bar\gamma_s^\kappa\bar N_s \\ &-\gamma_1^\kappa\int\left\{\varphi_1\frac{\delta}{\delta\varphi_1} - Y_1\frac{\delta}{\delta Y_1} + \hat\varphi_1\frac{\delta}{\delta\hat\varphi_1} + q_1\frac{\delta}{\delta q_1}\right\} \\ &-(\chi\partial_\xi + \chi_A\partial_{\xi_A})\left[\int\left\{(\hat\gamma_s^\kappa + \gamma_1^\kappa)\hat\varphi_1\frac{\delta}{\delta q_1}\right.\right. \\ &\left.\left.+\hat\gamma_s^\kappa\hat\varphi_2\frac{\delta}{\delta q_2} + \gamma_B^\kappa\bar c\frac{\delta}{\delta B}\right\} - \beta_\xi^\kappa\xi\partial_\chi\right]\Bigg)\Gamma \\ &= \bar\gamma_s^\kappa\int\{q_1Y_1 + q_2Y_2\} \\ &+(\chi\partial_\xi + \chi_A\partial_{\xi_A})\int\{(\gamma_s^\kappa + \gamma_1^\kappa)Y_1\varphi_1 \\ &+\gamma_s^\kappa Y_2\varphi_2 + \bar\gamma_s^\kappa(Y_1\hat\varphi_1 + Y_2\hat\varphi_2)\} \end{aligned} \tag{9.33}$$

with

$$\gamma_1^\kappa = (\kappa\partial_\kappa + \beta_e^\kappa e\partial_e + \beta_\xi^\kappa\xi\partial_\xi)\ln z \tag{9.34}$$

In (9.33) we have already introduced the full β- and γ-functions of the RG equation like in (9.13). Again, our analysis shows that the β-function β_e^κ and the anomalous dimension γ_A^κ have to be ξ- and ξ_A-independent to all orders of the loop expansion.

Additionally, rigid invariance (9.32) also imposes two restrictions for the coefficient functions of the RG equation:

$$\begin{aligned} &(\kappa\partial_\kappa + \beta_\xi^\kappa\xi\partial_\xi + \beta_e^\kappa e\partial_e)(z\hat\xi_A\frac{m}{e}) \\ &\quad = -z\xi_A\frac{m}{e}\bar\gamma_s^\kappa - z\hat\xi_A\frac{m}{e}\gamma_s^\kappa \\ &\beta_e^\kappa - \hat\gamma_s^\kappa \\ &\quad = (\kappa\partial_\kappa + \beta_\xi^\kappa\xi\partial_\xi + \beta_e^\kappa e\partial_e)\ln z \end{aligned} \tag{9.35}$$

Some further information about the coefficient functions results from testing the RG equation on the gauge condition (2.4)[6]:

$$\begin{aligned} \gamma_B^\kappa &= -\gamma_A^\kappa \\ \beta_\xi^\kappa &= 2\gamma_B^\kappa = -2\gamma_A^\kappa \\ \gamma_s^\kappa &= -\gamma_B^\kappa = \gamma_A^\kappa \end{aligned} \tag{9.36}$$

Hence also $\gamma_B^\kappa, \gamma_s^\kappa$ and β_ξ^κ are fully gauge parameter independent.

[6] When deriving (9.36) we make use of (9.35)

Finally, one further relation emerges from the validity of the local WI (7.5) and the normalization condition for the coupling (7.7):

$$\gamma_A^\kappa = \beta_e^\kappa \tag{9.37}$$

Therefore, there is only one independent coefficient function appearing in the RG equation, namely the β-function β_e^κ, which has to be determined by an explicit calculation.

10 Conclusions

In the present paper we have examined the renormalization of the Abelian Higgs model including BRS variations of all the gauge parameters. The advantage of such an extended procedure (when compared to the usual one) is due to the fact that this procedure also yields full information about the gauge parameter dependence of 1-PI Green functions automatically and in an easily manageable way and therefore prohibits (just by construction) a wrong adjustment of counterterms which in turn would spoil the gauge parameter independence of the S-matrix. In the usual construction (i.e. without introducing BR transforming gauge parameters) such a simple guiding principle is missing and it is a quite troublesome and heavily controllable task to adjust the counterterms correctly.

In this context we have shown that the normalization conditions needed in order to fix the free parameters of the theory cannot be chosen arbitrarily but instead have to respect the restrictions dictated by the enlarged ST identity. Especially we have proven that the physical on-shell normalization conditions are in complete agreement with those restrictions. Furthermore, the method of BRS varying gauge parameters yields a well handleable tool for controlling the range of "good" normalization conditions, i.e. normalization conditions, which are not in contradiction with the enlarged ST identity.

Some further results of the algebraic method we find interesting, too:
The enlarged ST identity also allowed us to show that the transversal part of the vector 2-point function has to be completely gauge parameter-independent to all orders of perturbation theory.
In the course of proving the local WI we found the ξ- and ξ_A-independence of the overall normalization factor of the matter transformations, a result, which gave rise to an alternative and elegant possibility for fixing the coupling, namely by requiring the local WI to be exact to all orders.

Finally, we derived the Callan-Symanzik and the renormalization group equation of the Abelian Higgs model thereby showing among other things that the β-functions $\beta_e^{(\kappa)}, \beta_{m_H}$ and $\beta_\xi^{(\kappa)}$ as well as the anomalous dimensions $\gamma_A^{(\kappa)}, \gamma_B^{(\kappa)}$ and γ_s^κ have to be fully gauge parameter-independent to all orders of the perturbative expansion. In the same way we studied the ghost mass equation in order to complete the analysis for the soft t'Hooft gauge parameter.

The examination of the Abelian Higgs model, chosen as the simplest example of a gauge theory with sponta-

neous breakdown of symmetry, thus clearly shows of what kind the considerations have to be and yields a hint what kind of results could possibly be expecxted when the general algebraic method will be applied to more complicated, physical, models, especially to the standard model of electroweak interactions.

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Appendix A

In the course of looking for the most general classical solution of the χ- and χ_A-enlarged ST identity (4.1) the most general solution of the gauge condition (2.4) and the ordinary (that is χ- and χ_A-independent) ST identity

$$\mathcal{S}(\hat{\Gamma}) = \int \left\{ \partial_\mu c \, \frac{\delta \hat{\Gamma}}{\delta A_\mu} + B \frac{\delta \hat{\Gamma}}{\delta \bar{c}} + \frac{\delta \hat{\Gamma}}{\delta \underline{Y}} \frac{\delta \hat{\Gamma}}{\delta \underline{\varphi}} + \underline{q} \frac{\delta \hat{\Gamma}}{\delta \underline{\hat{\varphi}}} \right\} = 0 \tag{A.1}$$

is needed. This solution was constructed in [9], and we just present the result here:

$$\hat{\Gamma}^{gen}_{cl} = \Lambda(A_\mu, \bar{\varphi}_1, \bar{\varphi}_2) \; + \; \Gamma_{g.f.} \; + \; \Gamma_{\phi\pi} \; + \; \Gamma_{e.f.} \; , \tag{A.2}$$

with

$$\bar{\varphi}_i = \varphi_i - x_i \hat{\varphi}_i \; , \; i = 1, 2 \; . \tag{A.3}$$

The part $\Lambda = \Lambda(A_\mu, \bar{\varphi}_1, \bar{\varphi}_2)$ describing the gauge field A_μ and matter fields φ_i is given by:

$$\begin{aligned}
\Lambda = \int \Big\{ & -\frac{z_A}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} z_1 (\partial_\mu \bar{\varphi}_1)(\partial^\mu \bar{\varphi}_1) \\
& + \frac{1}{2} z_2 (\partial_\mu \bar{\varphi}_2)(\partial^\mu \bar{\varphi}_2) \\
& + z_e e \sqrt{z_1} \sqrt{z_2} \sqrt{z_A} \left((\partial_\mu \bar{\varphi}_1) \bar{\varphi}_2 - \bar{\varphi}_1 (\partial_\mu \bar{\varphi}_2) \right) A^\mu \\
& + \frac{1}{2} z_e^2 e^2 z_A (z_1 \bar{\varphi}_1^2 + z_2 \bar{\varphi}_2^2) A_\mu A^\mu \\
& + \frac{1}{2} z_m m^2 z_A A_\mu A^\mu - \sqrt{z_2} \sqrt{z_m} m \sqrt{z_A} (\partial_\mu \bar{\varphi}_2) A^\mu \\
& + z_e e \sqrt{z_m} m \sqrt{z_1} z_A \bar{\varphi}_1 A_\mu A^\mu \\
& + \frac{1}{2} \mu^2 (z_1 \bar{\varphi}_1^2 + 2 \sqrt{z_1} \frac{\sqrt{z_m} m}{z_e e} \bar{\varphi}_1 + z_2 \bar{\varphi}_2^2) \\
& - \frac{1}{8} \frac{z_{m_H} m_H^2}{z_m m^2} z_e^2 e^2 (z_1 \bar{\varphi}_1^2 + 2 \sqrt{z_1} \frac{\sqrt{z_m} m}{z_e e} \bar{\varphi}_1 + z_2 \bar{\varphi}_2^2)^2 \Big\}
\end{aligned} \tag{A.4}$$

The gauge fixing part $\Gamma_{g.f.}$ is of course nothing else but (2.4):

$$\begin{aligned}
\Gamma_{g.f.} = \int \Big\{ & \frac{1}{2} \xi B^2 + B \partial A - eB \\
& \times \Big[(\hat{\varphi}_1 - \xi_A \frac{m}{e}) \varphi_2 - \hat{\varphi}_2 (\varphi_1 - \hat{\xi}_A \frac{m}{e}) \Big] \Big\}
\end{aligned} \tag{A.5}$$

For the remaining two parts, the external field part $\Gamma_{e.f.}$ and the $\phi\pi$-part $\Gamma_{\phi\pi}$, one gets

$$\begin{aligned}
\Gamma_{e.f.} = \int \{ & Y_1 (-e z_e \sqrt{\frac{z_2}{z_1}} \sqrt{z_A} \bar{\varphi}_2 c + x_1 q_1) \\
& + Y_2 (e z_e \sqrt{\frac{z_1}{z_2}} \sqrt{z_A} (\bar{\varphi}_1 + \frac{\sqrt{z_m} m}{\sqrt{z_1} z_e e}) c + x_2 q_2) \}
\end{aligned} \tag{A.6}$$

and

$$\begin{aligned}
\Gamma_{\phi\pi} = \int \{ & -\bar{c} \Box c + e \bar{c} (q_1 \varphi_2 - q_2 (\varphi_1 - \hat{\xi}_A \frac{m}{e})) \\
& + e \bar{c} (\hat{\varphi}_1 - \xi_A \frac{m}{e}) (z_e e \sqrt{\frac{z_1}{z_2}} \sqrt{z_A} (\bar{\varphi}_1 \\
& + \frac{\sqrt{z_m} m}{\sqrt{z_1} z_e e}) c + x_2 q_2) \\
& - e \bar{c} \hat{\varphi}_2 (-z_e e \sqrt{\frac{z_2}{z_1}} \sqrt{z_A} \bar{\varphi}_2 c + x_1 q_1) \}.
\end{aligned} \tag{A.7}$$

The free parameters in the general solution of the ST identity (A.1) are the wave function normalizations z_1, z_2 and z_A, the mass renormalizations of the vector and the Higgs-particle, i.e. z_m, z_{m_H}, the coupling renormalization z_e, the parameters x_1, x_2, the parameter μ, the gauge parameters ξ, ξ_A and the parameter $\hat{\xi}_A$. These parameters are not prescribed by the ST identity (A.1) and therefore have to be fixed by appropriate normalization conditions to all orders (see Sect. 2).

Appendix B

The solution of (8.15) was given in [9]; first we present a list of all terms of dimension less than or equal to *three* which contribute to $\hat{\Delta}^0_\lambda$:

$$\begin{aligned}
\int \frac{\hat{\Gamma}_{cl}}{\delta \varphi_1} &= s^{\chi=0=\chi_A}_{\hat{\Gamma}_{cl}} \int Y_1 \quad , \\
\int \frac{\delta \hat{\Gamma}_{cl}}{\delta \hat{\varphi}_1} &= s^{\chi=0=\chi_A}_{\hat{\Gamma}_{cl}} \int \{ -x Y_1 - e \bar{c} \bar{\varphi}_2 \} \quad , \\
\int \{ B \hat{\varphi}_2 - \bar{c} q_2 \} &= s^{\chi=0=\chi_A}_{\hat{\Gamma}_{cl}} \int \bar{c} \hat{\varphi}_2
\end{aligned} \tag{B.1}$$

and

$$\int \left\{ z_1 \bar{\varphi}_1^2 + 2 z_1 v \bar{\varphi}_1 + z_2 \bar{\varphi}_2^2 \right\}$$

In order to have a proper definition of this last invariant in higher orders we are forced to introduce a further external field $\hat{\varphi}_0$ of dimension two, even under charge conjugation and invariant under BRS and rigid transformations, which couples to this invariant. Therefore the above BRS symmetric term is replaced by:

$$\int \frac{\delta \hat{\Gamma}_{cl}}{\delta \hat{\varphi}_0} \tag{B.2}$$

The *four*-dimensional BRS symmetric terms contributing to $\hat{\Delta}^0_\lambda$ are given by ($i = 1, 2$):

$$\begin{aligned}
\int \left\{ \varphi_i \frac{\delta}{\delta \varphi_i} - Y_i \frac{\delta}{\delta Y_i} \right\} \hat{\Gamma}_{cl} &= s^{\chi=0=\chi_A}_{\hat{\Gamma}_{cl}} \int Y_i \varphi_i \quad , \\
\int \left\{ \hat{\varphi}_i \frac{\delta \hat{\Gamma}_{cl}}{\delta \varphi_i} - Y_i q_i \right\} &= s^{\chi=0=\chi_A}_{\hat{\Gamma}_{cl}} \int Y_i \hat{\varphi}_i \quad , \\
\int \left\{ B \frac{\delta}{\delta B} + \bar{c} \frac{\delta}{\delta \bar{c}} \right\} \hat{\Gamma}_{cl} &= s^{\chi=0=\chi_A}_{\hat{\Gamma}_{cl}} \int \bar{c} \frac{\delta \hat{\Gamma}_{cl}}{\delta B} \quad , \\
\int \left\{ \hat{\varphi}_i \frac{\delta}{\delta \hat{\varphi}_i} + q_i \frac{\delta}{\delta q_i} \right\} \hat{\Gamma}_{cl} &= s^{\chi=0=\chi_A}_{\hat{\Gamma}_{cl}} \int \hat{\varphi}_i \frac{\delta \hat{\Gamma}_{cl}}{\delta q_i} \quad , \\
\int \{ B \hat{\varphi}_1 \hat{\varphi}_2 - \bar{c} q_1 \hat{\varphi}_2 - \bar{c} \hat{\varphi}_1 q_2 \} &= s^{\chi=0=\chi_A}_{\hat{\Gamma}_{cl}} \int \bar{c} \hat{\varphi}_1 \hat{\varphi}_2 \quad , \\
\xi \partial_\xi \hat{\Gamma}_{cl} &= s^{\chi=0=\chi_A}_{\hat{\Gamma}_{cl}} \xi Q \quad , \qquad \text{(B.3)}
\end{aligned}$$

and

$$\int \left\{ A \frac{\delta}{\delta A} + c \frac{\delta}{\delta c} \right\} \hat{\Gamma}_{cl} \quad , \quad m_H \partial_{m_H} \hat{\Gamma}_{cl} \quad , \quad e \partial_e \hat{\Gamma}_{cl} \qquad \text{(B.4)}$$

Please note that due to (8.14) the coefficients with which the terms in (B.2) and (B.4) appear in $\hat{\Delta}^0_\lambda$ are independent of both ξ and ξ_A.

Appendix C

In this appendix we present the W^{gen}-symmetric extensions of the BRS invariant insertions

$$m_H \partial_{m_H} \ , \ e \partial_e \ , \ f_\xi \partial_\xi + (\chi \partial_\xi + \chi_A \partial_{\xi_A}) f_\xi \partial_\chi \ . \qquad \text{(C.1)}$$

Just in order to compactify the notation in the formulae below we introduce two ξ- and ξ_A-independent factors f_H and f_e multiplying $m_H \partial_{m_H}$ and $e \partial_e$, respectively. (These factors have to be independent of ξ and ξ_A due to the results of Sect. 8.) Next we define ($i = H, e, \xi$):

$$\begin{aligned}
\nabla_i &= m_H \partial_{m_H}, e \partial_e, \partial_\xi \quad , \qquad \text{(C.2)} \\
\hat{\nabla}_i &= -f_i \frac{1}{z} \nabla_i \int z \left\{ (\varphi_1 - \hat{\xi}_A \frac{m}{e}) \frac{\delta}{\delta \varphi_1} \right. \\
&\quad \left. - Y_1 \frac{\delta}{\delta Y_1} + (\hat{\varphi}_1 - \xi_A \frac{m}{e}) \frac{\delta}{\delta \hat{\varphi}_1} + q_1 \frac{\delta}{\delta q_1} \right\} \\
&\quad - (\chi \partial_\xi + \chi_A \partial_{\xi_A}) f_i \frac{1}{z} \nabla_i \\
&\quad \times \int z (\hat{\varphi}_1 - \xi_A \frac{m}{e}) \frac{\delta}{\delta q_1} \ , \qquad \text{(C.3)} \\
\hat{\hat{\nabla}}_i \Gamma &= -(\chi \partial_\xi + \chi_A \partial_{\xi_A}) f_i \frac{1}{z} \nabla_i \\
&\quad \times \int z Y_1 (\varphi_1 - \hat{\xi}_A \frac{m}{e}) \qquad \text{(C.4)}
\end{aligned}$$

The W^{gen}-symmetric extensions of the operators in (C.1) then are given by:

$$\begin{aligned}
& f_H m_H \partial_{m_H} \Gamma \to f_H m_H \tilde{\partial}_{m_H} \Gamma \\
& \quad \equiv f_H m_H \partial_{m_H} \Gamma + \hat{\nabla}_H \Gamma + \hat{\hat{\nabla}}_H \Gamma \ , \\
& f_e e \partial_e \Gamma \to f_e e \tilde{\partial}_e \Gamma \\
& \quad \equiv f_e e \partial_e \Gamma + \hat{\nabla}_e \Gamma + \hat{\hat{\nabla}}_e \Gamma \ , \qquad \text{(C.5)} \\
& f_\xi \partial_\xi \Gamma + [(\chi \partial_\xi + \chi_A \partial_{\xi_A}) f_\xi] \partial_\chi \Gamma \to \tilde{\partial}_\xi \Gamma \\
& \quad \equiv f_\xi \partial_\xi \Gamma + [(\chi \partial_\xi + \chi_A \partial_{\xi_A}) f_\xi] \partial_\chi \Gamma + \hat{\nabla}_\xi \Gamma + \hat{\hat{\nabla}}_\xi \Gamma
\end{aligned}$$

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