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High precision tests of QED and physics beyond the standard model

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Abstract. We study the four most significant high precision observables of QED —the anomalous electron and muon magnetic moments, the hydrogen Lamb shift and muonium hyperfine splitting— in the context of $SU(2) \otimes U(1)$ gauge-invariant effective Lagrangians. The agreement between the theoretical predictions for these observables and the experimental data places bounds on the lowest dimension operators of the effective Lagrangians. We also place bounds on such effective operators using other experimental data. Comparison of the two types of bounds allows us to discuss the potential of each one of the four high precision observables in the search for physics beyond the Standard Model. We find that the anomalous electron and muon magnetic moments are sensitive to new physics while the hydrogen Lamb shift and muonium hyperfine splitting are not.

1 Introduction and motivation

QED is the textbook example of the triumph of quantum field theory: it is a consistent and predictive theory that agrees with experiment to a very high accuracy [1]. We know, of course, that QED is a low energy remnant of the more complete Standard Model (SM) of electroweak interactions. Still, QED is usually treated as a self-contained theory, into which one may incorporate the corrections from electroweak and strong interactions. Also, and this is important for the present article, any non-standard deviations from QED are assumed to come from extensions of QED that respect the $U(1)$ electromagnetic gauge invariance.

This last point is illustrated by a classical example. Consider the anomalous magnetic moment of the electron $a_e \equiv (g-2)_e/2$. To parameterise deviations from QED one introduces the $U(1)$ invariant effective Lagrangian:

$$\mathcal{L} = \frac{\alpha_1}{\Lambda}\, \overline{\psi_e}\, \sigma^{\mu\nu}\, \psi_e\, F_{\mu\nu} \tag{1}$$

Here α_1 is a coupling constant and Λ is a energy scale. The (tree-level) contribution to a_e is

$$\delta a_e = 2\, \frac{\alpha_1}{\Lambda}\, \frac{2\, m_e}{e} \tag{2}$$

The agreement between the experimental measure [2] and the theoretical prediction [3] for a_e sets the stringent limit

$$-6.9 \times 10^{-11} \leq \delta a_e \leq 4.3 \times 10^{-11} \tag{3}$$

This limit on δa_e (and on all other observables in the article) is obtained at the 95% C. L.

In this fashion one can obtain an upper bound on the coefficients of the effective Lagrangian:

$$\frac{\alpha_1}{\Lambda} \lesssim 2 \times 10^{-5}\ \text{TeV}^{-1} \tag{4}$$

Although there is nothing wrong with this type of analysis, we think one can and should go beyond it. One of the reasons is due to the well-known success of the standard $SU(2) \otimes U(1)$ model in describing the electroweak data. Deviations from the SM have been parameterised in terms of effective Lagrangians that respect $SU(2) \otimes U(1)$ gauge invariance [4]. Here, we will follow the same prescription, namely we will describe the effects of physics beyond the SM by a set of $SU(2) \otimes U(1)$ gauge-invariant effective Lagrangians that modify the high precision QED observables. In fact, in the example we have presented, where the Lagrangian (1) is used, the allowed values of Λ are much greater than the Fermi scale[1] and thus it should be regarded as natural to use the full $SU(2) \otimes U(1)$ invariance instead of the electromagnetic $U(1)$ invariance. We further remark that by using the full $SU(2) \otimes U(1)$ gauge group we are sensitive to physics beyond the SM rather than just to QED. Since the SM includes QED we have widened the scope of the effective Lagrangian approach; going from the framework where (1) and (4) hold to the analysis performed here.

In this article we will study the four high precision QED observables that are known with the greatest precision [1]: $(g-2)_e$, $(g-2)_\mu$, the Lamb shift and muonium hyperfine splitting. Experimental data on such observables

[1] One can estimate [5] the coupling constant α_1 in (4) to be of order $\alpha_1 \approx e/16\pi^2 \simeq 10^{-3}$

(we call these experiments "QED experiments") restrict the coefficients of the lowest-dimension operators in the effective Lagrangian approach. As we will see, the operators that lead to modifications of QED observables will also alter other quantities measured in other experiments like LEP (we call these experiments "non-QED"). This fact can be used to compare the ability of different experiments to push the search for new physics. Both QED and non-QED experiments restrict the coefficients of the effective operators. Which experiments lead to the most restrictive limits will tell us whether, for a particular QED observable, the high precision QED tests are or are not competitive with non-QED experiments in the search for physics beyond the Standard Model. In the article, we will first calculate for each QED observable the bounds on all the effective operators from QED experiments and afterwards from non-QED experiments. At the end, we will discuss our results and compare various bounds. Some of our conclusions may be relevant in the light of the upcoming experiment [6] at the Brookhaven Alternative Gradient Synchroton (AGS) to measure the anomalous magnetic moment of the muon with a precision $\Delta a_\mu = \pm 4 \times 10^{-10}$.

2 The electron anomalous magnetic moment

The leading contributions to a_e come from the following two dimension six operators

$$\mathcal{O}_{eB} \equiv \overline{L_e}\, \sigma^{\mu\nu}\, e_R\, \Phi\, B_{\mu\nu} \tag{5}$$

and

$$\mathcal{O}_{eW} \equiv \overline{L_e}\, \sigma^{\mu\nu}\, \tau\, e_R\, \Phi\, \mathbf{W}_{\mu\nu} \tag{6}$$

where L_e is the left-handed isodoublet containing e_L, e_R is its right-handed partner, $\mathbf{W}_{\mu\nu}$ and $B_{\mu\nu}$ are the $SU(2)$ and $U(1)$ field strengths, Φ is the scalar doublet, and τ are the Pauli matrices.

Let us now in turn analyse the effects of these two operators. The effective Lagrangian corresponding to the operator (5) is

$$\mathcal{L} = \frac{\alpha_{eB}}{\Lambda^2}\, \mathcal{O}_{eB} \tag{7}$$

where α_{eB} is a coupling constant and Λ is a high energy scale. After electroweak symmetry breaking, the shift in a_e is

$$\delta a_e = \frac{\sqrt{2}}{v}\, \frac{2\, m_e}{e}\, c_W\, \epsilon_{eB} \tag{8}$$

where $\epsilon_{eB} \equiv \alpha_{eB}\, v^2/\Lambda^2$ ($v \simeq 246$ GeV is the Fermi scale). Hereafter we use $c_W \equiv \cos\theta_W$ and $s_W \equiv \sin\theta_W$. The limit (3) sets a bound on the parameter ϵ_{eB}:

$$-5 \times 10^{-6} \leq \epsilon_{eB}(a_e) \leq 3 \times 10^{-6} \tag{9}$$

Here, a_e inside the parentheses indicates that the limit on ϵ_{eB} is obtained from the consideration of the high precision QED observable a_e.

The Lagrangian (7) also leads to a modification of the standard Ze^+e^- coupling. The shift in the $\Gamma_e = \Gamma(Z \longrightarrow e^+e^-)$ width is

$$\frac{\delta\Gamma_e}{\Gamma_e} = \frac{s_W^2}{g_V^2 + g_A^2}\, \epsilon_{eB}^2 \tag{10}$$

where $g_V = -1/2 + 2s_W^2$ and $g_A = -1/2$. Γ_e is measured at the Z-peak at LEP [7], and it agrees well with the standard model prediction. One finds the restriction

$$|\epsilon_{eB}(\text{non-QED})| \leq 9 \times 10^{-2} \tag{11}$$

where now "non-QED" inside the parentheses signifies that we obtain the limit using experiments other than high precision QED observations.

The operator (6) also contributes to a_e. Writing

$$\mathcal{L} = \frac{\alpha_{eW}}{\Lambda^2}\, \mathcal{O}_{eW} \tag{12}$$

we find

$$\delta a_e = -\frac{\sqrt{2}}{v}\, \frac{2\, m_e}{e}\, s_W\, \epsilon_{eW} \tag{13}$$

with $\epsilon_{eW} \equiv \alpha_{eW}\, v^2/\Lambda^2$. Using (3), we get

$$-5 \times 10^{-6} \leq \epsilon_{eW}(a_e) \leq 8 \times 10^{-6} \tag{14}$$

The operator $\mathcal{O}_{eW}$ leads to couplings Ze^+e^- and $We\nu$ that would modify the standard model predictions. We find, however, that the possible shift in $Z \longrightarrow e^+e^-$ decay leads to the most restrictive limits of all the "non-QED" experiments. We obtain

$$|\epsilon_{eW}(\text{non-QED})| \leq 5 \times 10^{-2} \tag{15}$$

We should now comment on the question of cancellations among different effective contributions. The effective Lagrangian is a linear combination of both operators in (5) and (6), and the total contribution to a_e is the sum of both contributions in (8) and (13). A strong cancellation in the two contributions either to a_e (or to Γ_e) would be unnatural. Still, a partial cancellation could occur and thus the limits could be relaxed but presumably only by a factor of order one. Fortunately, our main conclusions depend only on the order of magnitude of the limit and not on such details. Consequently, we will assume that there are no fine-tuned cancellations among contributions to the observables.

3 The muon anomalous magnetic moment

There are two operators, similar to (5) and (6), that contribute to a_μ:

$$\begin{aligned} \mathcal{O}_{\mu B} &\equiv \overline{L_\mu}\, \sigma^{\mu\nu}\, \mu_R\, \Phi\, B_{\mu\nu} \\ \mathcal{O}_{\mu W} &\equiv \overline{L_\mu}\, \sigma^{\mu\nu}\, \tau\, \mu_R\, \Phi\, \mathbf{W}_{\mu\nu} \end{aligned} \tag{16}$$

The analysis is very similar to the case of a_e. The agreement between theory and experiment [8,9] restricts any contribution to a_μ as follows:

$$-1.4 \times 10^{-8} \leq \delta a_\mu \leq 2.2 \times 10^{-8} \tag{17}$$

which implies

$$\begin{aligned} -4\times 10^{-6} \le \epsilon_{\mu B}(a_\mu) \le 7\times 10^{-6} \\ -2\times 10^{-5} \le \epsilon_{\mu W}(a_\mu) \le 7\times 10^{-6} \end{aligned} \tag{18}$$

(The parameters $\epsilon_{\mu B}$ and $\epsilon_{\mu W}$ are defined in analogy to ϵ_{eB} and ϵ_{eW}).

The operators (16) modify $\Gamma(Z \longrightarrow \mu^+\mu^-)$. The LEP data imply

$$|\epsilon_{\mu B}(\text{non-QED})| \le 9\times 10^{-2} \tag{19}$$

$$|\epsilon_{\mu W}(\text{non-QED})| \le 5\times 10^{-2} \tag{20}$$

$\mathcal{O}_{\mu W}$ contains vertices like $W\mu\nu$ that modify for instance $\mu \longrightarrow e\nu\nu$. However, the corresponding limit on $\epsilon_{\mu W}$ is much less stringent than (20).

4 The Lamb shift

The splitting of the hydrogen levels $2S_{1/2}$ and $2P_{1/2}$, $\Delta E_H(2S_{1/2} - 2P_{1/2}) \equiv E_{\rm LS}$, known as the Lamb shift, is an important observable to test QED. The agreement between experiment [10] and theory [11] requires that other contributions to the Lamb shift respect the stringent limit

$$-38 \le \delta E_{\rm LS} \le 10 \text{ kHz} \tag{21}$$

There is a long list of dimension six operators that could contribute to the Lamb shift. However, after discarding the effective operators that induce redefinitions of the physical parameters and using the equations of motion in a rigorous way, one can select the following independent basis [12]:

$$\{\mathcal{O}_{eB}, \mathcal{O}_{eW}, \mathcal{O}_{\partial B}, \mathcal{O}_{DW}\} \tag{22}$$

where $\mathcal{O}_{eB}$, $\mathcal{O}_{eW}$ are defined in (5) and (6), and

$$\begin{aligned} \mathcal{O}_{\partial B} &\equiv \partial_\lambda B^{\mu\nu}\, \partial^\lambda B_{\mu\nu} \\ \mathcal{O}_{DW} &\equiv [D_\lambda \mathbf{W}^{\mu\nu}]^\dagger \left[D^\lambda \mathbf{W}_{\mu\nu}\right] \end{aligned} \tag{23}$$

Let us start with the first operator, $\mathcal{O}_{eB}$. Its effects are expressed via the Lagrangian (7), that arose earlier. Its contribution to the Lamb shift is given by

$$\delta E_{\rm LS} = \frac{(m_e\,\alpha)^3}{6\,\pi}\,\frac{e}{2\,m_e}\,\frac{\sqrt{2}}{v}\,c_W\,\epsilon_{eB} \tag{24}$$

The experimental limit (21) leads to

$$-7\times 10^{-3} \le \epsilon_{eB}(E_{\rm LS}) \le 2\times 10^{-3} \tag{25}$$

The Lagrangian (12), containing $\mathcal{O}_{eW}$, has a contribution similar to (24), with $c_W\,\epsilon_{eB} \to -s_W\,\epsilon_{eW}$. The corresponding restriction is

$$-3\times 10^{-3} \le \epsilon_{eW}(E_{\rm LS}) \le 2\times 10^{-2} \tag{26}$$

where $E_{\rm LS}$ inside the parentheses indicates that the limit is obtained using $E_{\rm LS}$.

While these two operators affect the $ee\gamma$ vertex, the operators (23) contribute to the Lamb shift through the photon self-energy. We find

$$\delta E_{\rm LS} = m_e\,\alpha^4\,\frac{m_e^2}{v^2}\,\left(c_W^2\,\epsilon_{\partial B} + s_W^2\,\epsilon_{DW}\right) \tag{27}$$

where $\epsilon_{\partial B}$ and ϵ_{DW} are defined in analogy to ϵ_{eB} and ϵ_{eW}. Assuming that there are no cancellations among the contributions of $\mathcal{O}_{\partial B}$ and $\mathcal{O}_{DW}$, yields

$$\begin{aligned} -6\times 10^3 \le \epsilon_{\partial B}(E_{\rm LS}) \le 2\times 10^3 \\ -2\times 10^4 \le \epsilon_{DW}(E_{\rm LS}) \le 5\times 10^3 \end{aligned} \tag{28}$$

Following our general strategy we now calculate the limits to the different ϵ's using other experimental data. The limits on ϵ_{eB} and ϵ_{eW} have already been quoted in (11) and (15). The best bounds on $\epsilon_{\partial B}$ and ϵ_{DW} come from the LEP measurements on Z widths. They are

$$\begin{aligned} -2\times 10^{-2} \le \epsilon_{\partial B}(\text{non-QED}) \le 2\times 10^{-2} \\ -6\times 10^{-3} \le \epsilon_{DW}(\text{non-QED}) \le 4\times 10^{-3} \end{aligned} \tag{29}$$

5 Muonium hyperfine splitting

Muonium is a system which displays many of the hydrogen properties but does not contain constituent hadrons. It is in this respect a good testing ground for QED. Its ground state hyperfine splitting, $\nu_{\mu\text{-hfs}}$, corresponds to the energy difference among states with parallel or antiparallel alignment of the e^- and μ^+ magnetic moments. It has been measured very accurately [13] and there are precise theoretical calculations [14]. Additional contributions to this observable are limited by

$$-2.5 \le \delta\nu_{\mu\text{-hfs}} \le 3.0 \text{ kHz} \tag{30}$$

The independent dimension six operators contributing to $\nu_{\mu\text{-hfs}}$ can be classified into two types. We have, first, $\mathcal{O}_{eB}$ and $\mathcal{O}_{eW}$ that affect the $ee\gamma$ vertex and $\mathcal{O}_{\mu B}$ and $\mathcal{O}_{\mu W}$ that affect the $\mu\mu\gamma$ vertex. These four operators have already appeared in our analysis. The second type are four-fermion operators. Using Fierz shuffling, one can select the following complete set of effective operators that we call $\mathcal{O}_{4f}$:

$$\mathcal{O}_{4f} = \left\{\mathcal{O}_{\ell\ell}^{(1)}, \mathcal{O}_{\ell\ell}^{(3)}, \mathcal{O}_{e\mu}, \mathcal{O}_{\ell\mu}, \mathcal{O}_{e\ell}\right\} \tag{31}$$

where

$$\begin{aligned} \mathcal{O}_{\ell\ell}^{(1)} &\equiv (\overline{L_e}\,\gamma^\mu\,L_e)(\overline{L_\mu}\,\gamma_\mu\,L_\mu) \\ \mathcal{O}_{\ell\ell}^{(3)} &\equiv (\overline{L_e}\,\gamma^\mu\,\tau\,L_e)(\overline{L_\mu}\,\gamma_\mu\,\tau\,L_\mu) \\ \mathcal{O}_{e\mu} &\equiv (\overline{e_R}\,\gamma^\mu\,e_R)(\overline{\mu_R}\,\gamma_\mu\,\mu_R) \\ \mathcal{O}_{\ell\mu} &\equiv (\overline{L_e}\,\gamma^\mu\,L_e)(\overline{\mu_R}\,\gamma_\mu\,\mu_R) \\ \mathcal{O}_{e\ell} &\equiv (\overline{e_R}\,\gamma^\mu\,e_R)(\overline{L_\mu}\,\gamma_\mu\,L_\mu) \end{aligned} \tag{32}$$

The contribution of $\mathcal{O}_{eB}$ and $\mathcal{O}_{eW}$ is calculated to be

$$\delta\nu_{\mu\text{-hfs}} = \frac{8}{3\,\pi}\,\alpha^2\,R_\infty\,\frac{m_e}{m_\mu}\,\left[\frac{2m_e}{e}\,\frac{\sqrt{2}}{v}\,(c_W\,\epsilon_{eB} - s_W\,\epsilon_{eW})\right] \tag{33}$$

and that from $\mathcal{O}_{\mu B}$ and $\mathcal{O}_{\mu W}$ is similar to (33), with $m_e \to m_\mu$ inside the brackets. The four-fermion effective operators contribute as

$$\delta\nu_{\mu\text{-hfs}} = \frac{\alpha}{\pi^2}\, R_\infty\, \frac{m_e^2}{v^2}\, \left(\epsilon_{\ell\ell}^{(1)} + \epsilon_{\ell\ell}^{(3)} + \epsilon_{e\mu} - \epsilon_{\ell\mu} - \epsilon_{e\ell}\right) \quad (34)$$

where R_∞ is the Rydberg constant.

Again excluding fortuitous cancellations, we use (30) to find

$$\begin{aligned} -4\times 10^{-2} \le\ & \epsilon_{eB}(\nu_{\mu\text{-hfs}}) \le 4\times 10^{-2} \\ -8\times 10^{-2} \le\ & \epsilon_{eW}(\nu_{\mu\text{-hfs}}) \le 6\times 10^{-2} \\ -2\times 10^{-4} \le\ & \epsilon_{\mu B}(\nu_{\mu\text{-hfs}}) \le 2\times 10^{-4} \\ -4\times 10^{-4} \le\ & \epsilon_{\mu W}(\nu_{\mu\text{-hfs}}) \le 3\times 10^{-4} \end{aligned} \quad (35)$$

and

$$\begin{aligned} -40 \le\ & \epsilon_{\ell\ell}^{(1)}(\nu_{\mu\text{-hfs}}) \le 50 \\ -40 \le\ & \epsilon_{\ell\ell}^{(3)}(\nu_{\mu\text{-hfs}}) \le 50 \\ -40 \le\ & \epsilon_{e\mu}(\nu_{\mu\text{-hfs}}) \le 50 \\ -50 \le\ & \epsilon_{\ell\mu}(\nu_{\mu\text{-hfs}}) \le 40 \\ -50 \le\ & \epsilon_{e\ell}(\nu_{\mu\text{-hfs}}) \le 40 \end{aligned} \quad (36)$$

where we have defined $\epsilon_i \equiv \alpha_i\, v^2/\Lambda^2$ and the α_i's are the corresponding coefficients of the operators (5), (6), (16), and (32) in the effective Lagrangian. Now $\nu_{\mu\text{-hfs}}$ inside the parentheses indicates that the limit is obtained using $\nu_{\mu\text{-hfs}}$.

As before, we now use other experimental data to limit the ϵ parameters. The best limits on $\epsilon_{eB}, \epsilon_{eW}, \epsilon_{\mu B}$, and $\epsilon_{\mu W}$ are extracted from the LEP data and have been already quoted in (11), (15), (19), and (20).

The new operators (32) have vertices that modify the standard prediction for $e^+e^- \longrightarrow \mu^+\mu^-$ and $\mathcal{O}_{\ell\ell}^{(3)}$ also modifies the Z-widths. LEP data restrict all these operators. Additionally, the operator $\mathcal{O}_{\ell\ell}^{(3)}$ alters the μ-decay prediction but the restriction is less severe. Finally, we obtain

$$\begin{aligned} |\epsilon_{\ell\ell}^{(1)}(\text{non-QED})| \le\ & 9\times 10^{-1} \\ -3\times 10^{-3} \le\ \epsilon_{\ell\ell}^{(3)}(\text{non-QED}) \le\ & 2\times 10^{-3} \\ |\epsilon_{e\mu}(\text{non-QED})| \le\ & 9\times 10^{-1} \\ |\epsilon_{\ell\mu}(\text{non-QED})| \le\ & 9\times 10^{-1} \\ |\epsilon_{e\ell}(\text{non-QED})| \le\ & 9\times 10^{-1} \end{aligned} \quad (37)$$

6 Summary and discussion

We have studied four high precision observables that provide excellent tests of QED. For each observable we have identified the effective Lagrangians that can contribute to it. The Lagrangians are composed of the (lowest-dimension) independent operators that are $SU(2)\otimes U(1)$ gauge-invariant.

There are two steps in our calculations. We have, first, bounded all the ϵ coefficients of the effective Lagrangian using QED experiments and theoretical predictions. The best bounds are given in (9), (14), (18), (28), and (36). Second, we have noticed that the operators in the effective Lagrangian contain terms that lead to new effects in observables other than the above four. We can thus use further data to bound the same coefficients, but now the data is not from the "QED" observables but rather from "non-QED" observables. In fact, the most restrictive "non-QED" data turns out to be LEP data.

We have bounds from two experimental sources. As we said in the introduction a comparison between them is enlightening since it is clear that the experiment placing the strongest bounds on the effective coefficients ϵ's is the one most sensitive to new physics. For a given ϵ, the comparison is done in the effective Lagrangian approach. Thus, our conclusions are expected to be model independent.

The anomalous magnetic moments of the electron and the muon restrict the coefficients of the operators $\mathcal{O}_{eB}$, $\mathcal{O}_{eW}$, and $\mathcal{O}_{\mu B}, \mathcal{O}_{\mu W}$ much more severely that any "non-QED" data. As a first conclusion, this suggests that by improving the high precision measurements of $(g-2)_e$ and $(g-2)_\mu$ one could be sensitive to physics beyond the standard electroweak model. In the light of this remark, we think it is interesting that $(g-2)_\mu$ will be measured with unprecedent precision at the AGS [6].

The conclusion we reach for the other two observables, the hydrogen Lamb shift and the muonium hyperfine splitting, is the opposite. Looking at the numerical limits obtained in this article, we see that the limits on the operators $\mathcal{O}_{eB}, \mathcal{O}_{eW}, \mathcal{O}_{\mu B}$, and $\mathcal{O}_{\mu W}$ obtained from the hydrogen Lamb shift and muonium hyperfine splitting are weaker than the limits using $(g-2)_e$ and $(g-2)_\mu$. For the remaining operators, namely $\mathcal{O}_{\partial B}, \mathcal{O}_{DW}$, and $\mathcal{O}_{4f}$, bounds from LEP data are more stringent. This suggests than these two observables are far from being sensitive to new physics.

Note added: After we finished the writing, we became aware of a related work published in [15]. In this reference, the authors have computed non-standard contributions to a_μ arising from composite fermions and gauge bosons, and have compared with constraints from LEP-2 when available. Our work differs from theirs in the following aspects. We use gauge-invariant effective Lagrangians and calculate at tree-level, while they calculate loops with form factors, excited leptons, etc. Also, they do a very exhaustive study but restricted to a_μ while we have included all the relevant QED observables.

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