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Some remarks on the baryon-meson couplings in the $1/N_c$ expansion

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Abstract. The original results for the baryon-pion couplings in the large N_c QCD can be understood in a simpler way in the Hartree-Fock picture. The large N_c relation and its $1/N_c$ correction between the heavy baryon-meson coupling and the light baryon-meson coupling are emphasized. Application to the baryon-ρ meson interactions is straightforward. The implications of recent experimental result for the strong coupling constants of the heavy baryon chiral Lagrangian are discussed.

Baryons provide us testing ground to Quantum Chromodynamics (QCD). Experiments have collected rich data for light baryons, and are accumulating more and more data for heavy baryons. For the theoretical calculation, the task is to apply model-independent methods of the nonperturbative QCD. The tools which fit the task are like lattice QCD, $1/N_c$ expansion, chiral Lagrangian, heavy quark effective theory (HQET), QCD sum rules and so on. In this paper, we discuss some aspects in the $1/N_c$ expansion of baryon-meson strong coupling constants. In the large N_c limit [1], the meson properties can be obtained by analyzing planar diagram, and baryon properties by considering Hartree-Fock picture. Hadrons can be understood qualitatively quite well.

Recently there are renewed interests in the $1/N_c$ expansion due to the work of Dashen et al. [2]. By combining the large N_c counting rules and the chiral Lagrangian, they pointed out that there is a contracted SU($2N_f$) light quark spin-flavor symmetry in the baryon sector in the large N_c limit. This symmetry can be also derived in the Hartree-Fock picture [3,4]. (Similar result was obtained before [5].) The observation of the light quark spin-flavor symmetry results in many quantitative applications of $1/N_c$ expansion to both light and heavy baryons [2, 6-10]. Within the framework of large N_c HQET, we discussed the heavy baryon masses [9], and emphasized in the Hartree-Fock picture that the heavy baryon mass equals to the heavy quark mass plus the proton mass in the large N_c limit. Actually this point was first pointed out in [11] by Chow and Wise. In the Hartree-Fock picture, by observing the light quark dominance in the large N_c limit, we further deduced that the heavy baryon-pion coupling constant equals to the light baryon-pion coupling constant, which is a result of Jenkins in [2]. This paper will develop this deduction. Our discussion will be not restricted to heavy baryons. It applies to both light and heavy baryons. And it will not be limited to the baryon interaction with pion only, the interaction can be with ρ meson. Actually the discussion considers baryon interactions with any light meson. However, it emphasizes the relation between the heavy baryon-meson coupling and the light baryon-meson coupling.

The baryons of the most general interests are in ground state, that means the quarks have no orbital angular momentum excitations in the constituent picture. For simplicity, only two flavors of light quarks are considered. In terms of their quantum numbers spin J and isospin I, they are $(I.J) = (\frac{1}{2},\frac{1}{2}),(\frac{3}{2},\frac{3}{2}),...,(\frac{N_c}{2},\frac{N_c}{2})$ for light baryons and $(I,J) = (0,0),(1,1),...,(\frac{N_c-1}{2},\frac{N_c-1}{2})$ for heavy baryons. In the large N_c limit, because of the light quark spin-flavor symmetry, baryons belonging to the same tower of (I,J) are degenerate. Each tower is an irreducible representation of this symmetry. As in [9], we have been working in the Hartree-Fock picture for baryons. Many interesting results can be obtained in a simple way. In the following, we just consider the ground state baryons. However, all the arguments can be generalized to the excited baryon cases.

The first result is that **in the large N_c limit, all the coupling constants of the baryon interactions with fixed light meson are equal to each other.** When we talk about the coupling constant, the possibly explicit factors of N_c and the Clebsch-Gordon coefficient have been factored out. Generally, every vertex of the baryon-meson interactions in a given tower of (I,J) can be expressed by the fields combination of the initial baryon, the final baryon and the light meson times some coupling coefficient. The coupling coefficient can be parameterized into the product of certain constant and Clebsch-Gordon coefficients due to the symmetry group of the interaction. The constant is then called coupling constant (or coupling for short). Note that both the initial and the final states of baryons in the interaction belong to same tower of (I,J). Consider the light baryon case. The quark spin-flavor sym-

metry tells us that all the light baryon-meson couplings are equal. And because the interaction is determined by the light quarks inside the baryons, the heavy baryon-meson couplings are also equal to each other due to this symmetry. In the $N_c \to \infty$ limit, the heavy baryons are dominated by the light quark systems that also dominate the light baryons. In this case, the heavy baryon-meson coupling constant equals to the light baryon-meson one. Therefore all the ground state baryon and light meson interaction couplings are the same in the large N_c limit. Of course, for different light mesons, the couplings should be different.

Note that it is the observation of the light quark dominance in large N_c baryons that establishes the large N_c equal relation between the heavy baryon-meson coupling and the light baryon-meson coupling in the Hartree-Fock picture. The light quark spin independence [3], or the light quark spin-flavor symmetry, only gives the coupling equal relation within one given tower of (I, J). The "heavy-light" equal relation, however, subjects to the $1/N_c$ correction.

The second result concerns the $1/N_c$ corrections of the above conclusion. $1/N_c$ corrections violate the spin-flavor symmetry. And we note the baryon spectrum has the relation $I = J$. Therefore the coupling of the baryon-meson interaction has the following $1/N_c$ expansion,

$$g = g_0[1 + c_1\frac{L^2}{N_c} + c_2\frac{J_1^2 + J_2^2}{N_c^2} + c_3\frac{L^4}{N_c^2} + O(\frac{1}{N_c^3})] \ , \quad (1)$$

where g_0 is the coupling constant in the large N_c limit, which is the same for both heavy and light baryons. c_i $(i = 1, 2, 3)$ are unknown coefficients. J_1 and J_2 denote the baryon spins of the initial and final states, respectively, and L stands for the light meson total angular momentum in the rest frame of the initial baryon, $\mathbf{L} = \mathbf{J}_1 - \mathbf{J}_2$, which counts the orbital angular momentum and the spin of the meson. When the $1/N_c$ corrections are considered, the coupling constant is function of J_1^2, J_2^2 and $J_1 \cdot J_2$. The quantity $J_1 \cdot J_2$, however, can be re-expressed in terms of J_1^2, J_2^2 and L^2. The factor N_c should appear so as to keep the N_c scaling for g. In the extreme case while in the baryon all the quark spins align in the same direction, J_1^2 and J_2^2 scale as $\frac{N_c^2}{4}$. Only by dividing a factor N_c^2, have the terms proportional to J_1^2 or J_2^2 in above equation the right N_c scaling. However, L^2 is always a fixed quantity which does not depend on N_c. So generally this term is suppressed by N_c. Because strong interaction is CP invariant, J_1^2 and J_2^2 terms have the same coefficient. For different towers of (I, J), that means for the light and heavy baryons, the coefficients c_i are not necessarily the same. From (1) and above discussion, we see the following points. (a) **Within a given tower of (I, J), the equal relation of the coupling constants receives a correction actually only at the order of $1/N_c^2$.** This is because within the tower of (I, J), the term $c_1\frac{L^2}{N_c}$ is a constant. The couplings can be redefined by absorbing this constant, so that there is an explicit equal relation among the redefined couplings, which gets the corrections from $1/N_c^2$. Furthermore, the redefinition can be made to absorb all the terms of $(L^2)^n$ $(n = 1, 2, 3, ...)$, which are constants within the tower of (I, J). In the mixing terms like $L^2(J_1^2 + J_2^2)/N_c^3$, the factor L^2/N_c can be absorbed into the coefficients. Therefore, the coupling constant can be expanded only by the powers of J_1^2/N_c^2 and J_2^2/N_c^2 in a given tower of (I, J). (b) **Between the different towers of (I, J), the equal relation receives correction at the order of $1/N_c$.** This is simply because the value of the coefficient c_1 in the light baryon case is generally different from that in the heavy baryon case. So their difference gives an order of $1/N_c$ correction to the coupling constant equal relation. Furthermore in the extreme situation while J_1^2 and $J_2^2 \sim N_c^2/4$, in (1) the sub-leading terms which are proportional to $(\frac{J_1^2}{N_c^2})^{m_1}(\frac{J_2^2}{N_c^2})^{m_2}$ (m_1 and m_2 are non-negative integers) have the leading behavior. Because of the light quark dominance, the large N_c equal relation of the couplings now gives that the summation of the coefficients of these sub-leading terms for the light baryons equals to that for the heavy baryons. Under the specific assumption that the large N_c limit and the heavy quark limit are commutative, we may also have the equal relation of the coefficient c_1 for the light baryons and the heavy baryons.

The first result of $1/N_c$ corrections (a) can be obtained directly in view of [4] in the Hartree-Fock picture. However because we also have had the large N_c coupling equal relation between different towers of (I, J), it is necessary to ask its $1/N_c$ correction. The discussion on the second result (b) has made the origin of the $1/N_c$ correction to the "heavy-light" coupling equal relation clear.

For the light meson being a pion, the main conclusions for the baryon-meson interactions we have obtained are the same as that originally obtained in [2]. However, working in the Hartree-Fock picture, the way of understanding is simpler, and is not subject to the soft pion limit. All the results can be immediately generalized to the baryon interactions with other light meson cases, like the baryon-ρ meson interactions. Similar to baryon-pion interaction vertex described in [2], the baryon-ρ vertex is

$$\bar{B}_2 G^{\rho}_{ai} B_1 \rho^{ai}, \quad (2)$$

where a denotes the isospin of ρ meson and i labels the spin component of the ρ meson in the baryon rest frame. The coupling coefficient can be written as

$$\langle I_2 I_{2z}, J_2 J_{2z}| \, G^{\rho}_{ai} \, |I_1 I_{1z}, J_1 J_{1z}\rangle \quad (3)$$
$$= N_c g^{\rho}(J_1, J_2)\sqrt{\frac{2J_1+1}{2J_2+1}} \begin{pmatrix} I_1 & 1| & I_2 \\ I_{1z} & a| & I_{2z} \end{pmatrix} \begin{pmatrix} J_1 & 1| & J_2 \\ J_{1z} & i| & J_{2z} \end{pmatrix},$$

where $g^{\rho}(J_1, J_2)$ is the coupling constant which is of order 1 because the N_c dependence has been factored out. The Clebsch-Gordon coefficient is determined from angular-momentum and isospin conservation. The coupling constant $g^{\rho}(J_1, J_2)$, which is the main topic of this paper, is given by (1). In this way, the $NN\rho$ and $N\Delta\rho$ interaction couplings are related to each other, where N and Δ denote nucleon and Δ-baryon. It is interesting to note that

the p-wave $NN\pi$ and $N\Delta\pi$ interaction vertices [2] have the same form as that of the s-wave $NN\rho$ and $N\Delta\rho$ interactions, therefore the ratios of the couplings, in which the Clebsch-Gordon coefficients are canceled, have the relation

$$\frac{g_{N\Delta\rho}}{g_{N\Delta\pi}} = \frac{g_{NN\rho}}{g_{NN\pi}} \ , \tag{4}$$

which is valid up to $1/N_c^2$. Actually this relation has been suggested by phenomenological models [12]. Correspondingly, we also have the same relation for heavy baryon case,

$$\frac{g_{\Lambda_Q\Sigma_Q\rho}}{g_{\Lambda_Q\Sigma_Q\pi}} = \frac{g_{NN\rho}}{g_{NN\pi}} \ . \tag{5}$$

However, this relation will receive correction at the order of $1/N_c$.

Finally let us make a comment on the recent experimental result of the strong couplings in heavy baryon chiral Lagrangians [13,14]. There are two coupling constants, g_1 and g_2, where the notation of [13] is adopted. They can be determined from the transitions $\Sigma_c^* \to \Sigma_c\pi$ and $\Sigma_c \to \Lambda_c\pi$, respectively. In the large N_c limit, the equal coupling relation gives

$$|g_1| = \sqrt{2}|g_2| = g_A^N \ , \tag{6}$$

where $g_A^N \simeq 1.25$ is the light baryon-pion coupling constant. From the CLEO data [15], it was obtained that [16]

$$|g_2| = 0.57 \pm 0.10 \ . \tag{7}$$

It deviates from the large N_c expectation remarkably. But such a deviation is not surprising, because the relation $|g_2| = \frac{1}{\sqrt{2}} g_A^N$ will accept correction at the order of $1/N_c$. This means that the coefficient c_1 of (1) for heavy baryon case is different from that for light baryon case. In other words, the large N_c limit and the heavy quark limit are not commutative. Another large N_c relation $|g_1| = \sqrt{2}|g_2|$, on the other hand, does not receive corrections until to the order of $1/N_c^2$. Therefore $|g_1| \simeq 0.81 \pm 0.14$ is expected quite accurate. It will be checked in the experiments in the near future.

In summary, the original results for the baryon-pion couplings in the large N_c QCD can be understood in a simpler way in the Hartree-Fock picture. The large N_c relation and its $1/N_c$ correction between the heavy baryon-meson coupling and the light baryon meson coupling are stressed. While the simplification maybe inspiring, we have made a simple application to the baryon-ρ meson interactions. We have also discussed implications of the recent experimental result for the strong coupling constants of the heavy baryon chiral Lagrangians.

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