Eur. Phys. J. C 2, 545–556 (1998)
DOI 10.1007/s100529800702

THE EUROPEAN PHYSICAL JOURNAL C
© Springer-Verlag 1998

Exclusive J/ψ and ψ' decays into baryon-antibaryon pairs

Jan Bolz[a], Peter Kroll

Fachbereich Physik, University of Wuppertal, D-42097 Wuppertal, Germany (e-mail: kroll@theorie.physik.uni-wuppertal.de)

Received: 10 March 1997 / Revised version: 18 June 1997 / Published online: 20 February 1998

Abstract. Within a perturbative approach we investigate decays of charmonium states into baryon-antibaryon pairs. Using a recently proposed wave function for the nucleon and suitable generalizations of it to the hyperons and decouplet baryons, we obtain the decay widths for the $B\overline{B}$ channels in reasonable agreement with data. An important difference to previous work is the use of the c-quark mass in the perturbative calculation instead of the charmonium mass. As a consequence of this feature our approach possesses the property that the J/ψ and the ψ' decay widths do not scale with a high power of the ratio of their masses.

1 Introduction

The decay $J/\psi \to p\overline{p}$ has been investigated within perturbative QCD by Brodsky and Lepage first [1]. Later on this analysis has been repeated several times, e.g. [2, 3], and even extended to the $\Delta\overline{\Delta}$ decay channel. It has been argued that the dominant dynamical mechanism is $c\bar{c}$ annihilation into three gluons and subsequent creation of light quark-antiquark pairs forming in turn the final state baryons. Three is the minimal number of gluons allowed in J/ψ decays; $c\bar{c}$ annihilations through one or two gluons are forbidden by colour and C-parity, respectively. Contributions from annihilations through more than three gluons constitute higher order corrections. The dominance of annihilation through gluons is most strikingly reflected in the narrow width for hadronic channels in a mass region where strong decays have typically widths of hundreds of MeV [4]. The dynamical suppression at work here is perturbative QCD of higher orders (the total hadronic J/ψ decay, for instance, is an $\mathcal{O}(\alpha_s^3)$ process) and is customarily regarded as evidence for the Zweig rule. Since the c and the $\bar{c}$ quarks only annihilate if they are separated by distances less than about $1/m_c$ and since the average virtuality of the gluons is about 1 GeV^2 one may expect perturbative QCD to be at work although corrections are presumably substantial. Indeed as the previous perturbative analyses, performed in the standard hard scattering approach, i.e. in collinear approximation, showed the J/ψ decay into $p\overline{p}$ seems to be fairly well described. This is in marked contrast to the case of the nucleon form factor where soft physics seems to dominate in the experimentally accessible region of momentum transfer [5–7]. A point to criticize in these studies of the J/ψ decays is the treatment of the strong coupling constant α_s. Since, as we mentioned, the average virtuality of the gluons is about 1 GeV^2 one would expect α_s to be of the order of 0.4 to 0.5 rather than 0.2 to 0.3 as is customarily chosen [2,3]. Since α_s enters to the sixth power into the expression for the width a variation of α_s from, say, 0.2 to 0.3 would lead to a change by a factor of 11 for the width. Thus, a large factor of uncertainty is hidden in these calculations preventing any severe test of the wave function utilized.

In constrast to previous works [1–3] we will not use the collinear approximation but rather the modified perturbative approach of Sterman et al. [8] in which transverse degrees of freedom are retained and Sudakov suppressions, comprising those gluonic radiative corrections not included in the evolution of the wave function, are taken into account. An important advantage of the modified perturbative approach is that the strong coupling constant can be used with a renormalization scale depending on the momentum fractions the quarks carry und thus large logs from higher orders of perturbative QCD are avoided. This choice of the renormalizaton scale entails singularities of α_s which are, however, compensated by the Sudakov factor. Hence, there is no uncertainty in the use of α_s. This is to be contrasted with the standard perturbative approach where either α_s is evaluated at a renormalization scale that is a constant fraction of M_ψ^2, or at a momentum fraction dependent scale in which case α_s is to be "frozen" at a certain value (typically 0.5) in order to avoid uncompensated α_s singularities in the end-point regions. The modified perturbative approach possesses another interesting feature: the soft end-point regions are strongly suppressed. Therefore, the bulk of the perturbative contribution comes from regions where the internal quarks and gluons are far off-shell. In contrast to the nucleon form factor, the $J/\psi \to B\overline{B}$ amplitude is not end-point sensitive. The suppression of the end-point regions does not, therefore, lead to a substantial reduction of the $J/\psi \to B\overline{B}$ amplitude. For the same reason, the size of that amplitude does not exhibit an extreme sensitivity to

[a] Supported by Deutsche Forschungsgemeinschaft

the baryon wave function utilized in the calculation as is, for instance, the case for the baryon form factor.

In the calculation of the decay widths we will make use of a (valence Fock state) wave function for the nucleon that we proposed recently [6]. That wave function was constructed in the following way: In accordance with the findings reported in [5] where it was shown that a reliably calculated perturbative contribution to the nucleon form factor is very small, the nucleon wave function is demanded to describe the form factor via the Drell-Yan overlap contribution for momentum transfers around $10\,\mathrm{GeV}^2$ and to be compatible with the available valence quark distribution functions of the nucleon. As a third constraint on the nucleon wave function the decay $J/\psi \to p\overline{p}$ was employed. The nucleon wave function proposed in [6] will be suitably generalized to the case of hyperons and decuplet baryons. The modified perturbative approach is then used to calculate the widths for the J/ψ decays into pairs of octet, B_8, and decuplet, B_{10}, baryons. We are also going to calculate the baryonic decays of the ψ' for which recently the first, still preliminary, experimental results for channels other than $p\overline{p}$ have been reported [9].

The paper is organized as follows: In Sect. 2 we introduce the octet baryon wave functions used in the analysis described in detail in a subsequent section and briefly recapitulate a few properties of light-cone wave functions. In Sect. 3 we present the wave functions for the decuplet baryons. Section 4 is devoted to the calculation of the $J/\psi \to B_8\overline{B}_8$ decay widths. This analysis is extended to the decays into decuplet baryons (Sect. 5) and to decays of other quarkonia into $B\overline{B}$ pairs (Sect. 6). Finally, Sect. 7 contains our conclusions.

2 The wave functions of the octet baryons

Generalizing the ansatz made for the nucleon in [5,6,10], we write the valence Fock states of the lowest lying octet baryons as (the plane waves are omitted for convenience)

$$
\begin{aligned}
|\, B_8\,,+\rangle \;=\; & \frac{\varepsilon_{a_1a_2a_3}}{\sqrt{3!}}\int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \\
& \Big\{\Psi^{B_8}_{123}\,|\, f^{a_1}_{1+} f^{a_2}_{1-} f^{a_3}_{2+}\rangle + \Psi^{B_8}_{213}\,|\, f^{a_1}_{1-} f^{a_2}_{1+} f^{a_3}_{2+}\rangle \\
& -\Big(\Psi^{B_8}_{132} + \Psi^{B_8}_{231}\Big)\,|\, f^{a_1}_{1+} f^{a_2}_{1+} f^{a_3}_{2-}\rangle\Big\} \qquad (2.1\mathrm{a}) \\
|\,\Lambda\,,+\rangle \;=\; & \frac{\varepsilon_{a_1a_2a_3}}{\sqrt{2}}\int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \\
& \big\{\Psi^{\Lambda}_{123}\,|\, u^{a_1}_{+}\, d^{a_2}_{-}\, s^{a_3}_{+}\rangle - \Psi^{\Lambda}_{213}\,|\, u^{a_1}_{-}\, d^{a_2}_{+}\, s^{a_3}_{+}\rangle \\
& + \big(\Psi^{\Lambda}_{132} - \Psi^{\Lambda}_{231}\big)\,|\, u^{a_1}_{+}\, d^{a_2}_{+}\, s^{a_3}_{-}\rangle\big\} \qquad (2.1\mathrm{b})
\end{aligned}
$$

where (2.1a) holds for all octet baryons except the Λ and the Σ^0. Obviously, for the proton and the Σ^+, f_1 represents an u quark and f_2 either a d or a s quark, respectively. For the Ξ^-, f_1 represents a s quark and f_2 a d one. The states of the neutron, Σ^- and Ξ^0 are obtained from those of the proton, Σ^+ and Ξ^- states, by exchanging $u \leftrightarrow d$, respectively. The baryon is assumed to be moving rapidly in the 3-direction. Hence the ratio of transverse, $\mathbf{k}_{\perp i}$, to longitudinal momenta, $x_i\mathbf{p}$, of the quarks is small and one may still use a spinor basis on the light cone. The integration measures are defined by

$$
\begin{aligned}
[\mathrm{d}x] &\equiv \prod_{i=1}^{3}\mathrm{d}x_i\,\delta(1-\sum_i x_i) \qquad (2.2)\\
\big[\mathrm{d}^2\mathbf{k}_\perp\big] &\equiv \frac{1}{(16\pi^3)^2}\prod_{i=1}^{3}\mathrm{d}^2\mathbf{k}_{\perp i}\,\delta^{(2)}(\sum_i \mathbf{k}_{\perp i})\,.
\end{aligned}
$$

The quark f_i is characterized by the momentum fraction x_i, by the transverse momentum $\mathbf{k}_{\perp i}$ as well as by its helicity λ_i and colour a_i. A three-quark state is then given by

$$
\begin{aligned}
|\, f^{a_1}_{1\lambda_1} f^{a_2}_{2\lambda_2} f^{a_3}_{3\lambda_3}\rangle &= \frac{1}{\sqrt{x_1x_2x_3}}\,|\, f^{a_1}_1; x_1, \mathbf{k}_{\perp 1}, \lambda_1\rangle \qquad (2.3)\\
&\times|\, f^{a_2}_2; x_2, \mathbf{k}_{\perp 2}, \lambda_2\rangle\,|\, f^{a_3}_3; x_3, \mathbf{k}_{\perp 3}, \lambda_3\rangle.
\end{aligned}
$$

The single-quark states are normalized as

$$
\begin{aligned}
&\langle f^{\prime\, a'_i}_i; x'_i, \mathbf{k}_{\perp i}{}', \lambda'_i\,|\, f^{a_i}_i; x_i, \mathbf{k}_{\perp i}, \lambda_i\rangle = \qquad (2.4)\\
&\quad 2x_i(2\pi)^3\delta_{a'_ia_i}\delta_{f'_if_i}\delta_{\lambda'_i\lambda_i}\delta(x'_i-x_i)\delta^{(2)}(\mathbf{k}'_{\perp i}-\mathbf{k}_{\perp i}).
\end{aligned}
$$

Since the 3-component of the orbital angular momentum, L_3, is assumed to be zero the quark helicities sum up to the baryon's helicity. (2.1a) is the most general ansatz for the $L_3 = 0$ projection of the three-quark nucleon wave function [11]. From the permutation symmetry between the two u quarks and from the requirement that the three quarks have to be coupled in an isospin 1/2 state it follows that there is only one independent scalar wave function. If the $L_3 \neq 0$ projections are included the entire nucleon state is described by three independent functions [11]. In general there are more than one scalar wave function for the other octet baryons if $\mathrm{SU(3)_F}$-symmetry breaking is taken into account. As already expressed in (2.1) we nevertheless assume that each octet baryon is described by a single scalar wave function which, for convenience, we write as

$$
\Psi^{B_8}_{123}(x,\mathbf{k}_\perp) = \frac{1}{8\sqrt{3!}}\,f_{(8)}(\mu_F)\phi^{B_8}_{123}(x,\mu_F)\,\Omega_{(8)}(x,\mathbf{k}_\perp)\,. \qquad (2.5)
$$

We assume that $\mathrm{SU(3)_F}$ symmetry is only broken by a quark mass dependence of ϕ^{B_8} (see below). $f_{(8)}$, being related to the wave function at the origin of the configuration space, is identified with the nucleon parameter f_N whose value was determined in [6] to amount to $6.64\cdot 10^{-3}$ GeV^2 at the scale of reference $\mu_0 = 1$ GeV.

The transverse momentum dependence of the baryon wave function is parameterized by a simple symmetric Gaussian

$$
\Omega_{(8)}(x,\mathbf{k}_\perp) = (16\pi^2)^2\frac{a^4_{(8)}}{x_1x_2x_3}\exp\left[-a^2_{(8)}\sum_{i=1}^{3}k^2_{\perp i}/x_i\right] \qquad (2.6)
$$

Table 1. Eigenfunctions and reduced anomalous dimensions for helicity 1/2 baryons

n	$\tilde{\phi}^n_{123}(x)$	$\tilde{\gamma}_n$
1	$x_1 - x_3$	20/9
2	$-2 + 3(x_1 + x_3)$	8/3
3	$2 - 7(x_1 + x_3) + 8(x_1^2 + x_3^2) + 4x_1x_3$	32/9
4	$(x_1 - x_3)(1 - 4/3(x_1 + x_3)$	40/9
5	$2 - 7(x_1 + x_3) + 14/3(x_1^2 + x_3^2) + 14x_1x_3$	14/3

and the transverse size parameter $a_{(8)}$ is assumed to be the same for all octet baryons. A value of 0.75 GeV^{-1} is used for that parameter (see [6]).

The last item to be specified in (2.5) is the distribution amplitude, $\phi^{B_8}_{ijk}(x,\mu_F) \equiv \phi^{B_8}(x_i,x_j,x_k,\mu_F)$, of an octet baryon B_8, which is conventionally normalized to unity

$$\int [\mathrm{d}x] \phi^{B_8}_{123}(x,\mu_F) = 1\,. \tag{2.7}$$

The distribution amplitude representing the wave function integrated over transverse momenta up to the factorization scale, μ_F, can be expanded upon the eigenfunctions of the evolution kernel being linear combinations of Appell polynomials (see [12,13])

$$\phi^{B_8}_{123}(x,\mu_F) = \phi_{\mathrm{AS}}(x)\left[1 + \sum_{n=1}^{\infty} B_n^{B_8}(\mu_F)\,\tilde{\phi}^n_{123}(x)\right] \tag{2.8}$$

where $\phi_{\mathrm{AS}}(x) \equiv 120\,x_1x_2x_3$ is the asymptotic distribution amplitude [12]. Evolution is incorporated by the factorization scale dependences of $f_{(8)}$ and the expansion coefficients B_n :

$$\begin{aligned} f_{(8)}(\mu_F) &= f_{(8)}(\mu_0)\,\left(\frac{\ln(\mu_0/\Lambda_{\mathrm{QCD}})}{\ln(\mu_F/\Lambda_{\mathrm{QCD}})}\right)^{2/3\beta_0}\,, \\ B_n^{B_8}(\mu_F) &= B_n^{B_8}(\mu_0)\,\left(\frac{\ln(\mu_0/\Lambda_{\mathrm{QCD}})}{\ln(\mu_F/\Lambda_{\mathrm{QCD}})}\right)^{\tilde{\gamma}_n/\beta_0} \end{aligned} \tag{2.9}$$

where $\beta_0 \equiv 11 - 2/3\,n_f$. The exponents $\tilde{\gamma}_n$ are the reduced anomalous dimensions. Because they are positive fractional numbers increasing with n [12], higher order terms in (2.8) are gradually suppressed. The reduced anomalous dimensions and the eigenfunctions $\tilde{\phi}^n_{123}$ are listed in Table 1 where the notation of [14] is adopted.

In [6] the nucleon distribution amplitude was found to have the simple form

$$\begin{aligned} \phi^N_{123}(x,\mu_0) &= \phi_{\mathrm{AS}}(x)\left[1 + \frac{3}{4}\tilde{\phi}^1_{123}(x) + \frac{1}{4}\tilde{\phi}^2_{123}(x)\right] \\ &= 60x_1x_2x_3\,[1 + 3x_1]\,. \end{aligned} \tag{2.10}$$

The nucleon wave function is fully specified now and, before turning to the discussion of the hyperon distribution amplitudes, we note in passing that the probability of the nucleon's valence Fock state is 0.17.

A suitable hyperon distribution amplitude is constructed by taking (2.10) and, in order to incorporate the empirically known breaking of SU(3)$_{\mathrm{F}}$ symmetry, multiplying it with a factor

$$\exp\left(-\frac{a^2_{(8)}m_s^2}{x_j}\right) \tag{2.11}$$

whenever the quark j is a strange one. That factor bears resemblance to the BHL exponential [15]. $a_{(8)}$ is the transverse size parameter already introduced in (2.6) and m_s is a still to be adjusted parameter related to the strange quark mass. Explicitly our hyperon distribution amplitudes read

$$\phi^{\Sigma}_{123}(x,\mu_0) = N_{\Sigma}\,\phi^N_{123}(x)\,\exp\left(-\frac{a^2_{(8)}m_s^2}{x_3}\right) \tag{2.12a}$$

$$\begin{aligned} &\phi^{\Lambda}_{123}(x,\mu_0) \\ &= \frac{1}{3}N_{\Lambda}\,\left(\phi^N_{123}(x) + 2\,\phi^N_{321}(x)\right)\,\exp\left(-\frac{a^2_{(8)}m_s^2}{x_3}\right) \end{aligned} \tag{2.12b}$$

$$\begin{aligned} &\phi^{\Xi}_{123}(x,\mu_0) \\ &= N_{\Xi}\,\phi^N_{123}(x)\,\exp\left(-a^2_{(8)}m_s^2\left[\frac{1}{x_1} + \frac{1}{x_2}\right]\right)\,. \end{aligned} \tag{2.12c}$$

The constants N_{B_8} ensure the correct normalizations (see (2.7)) of the hyperon distribution amplitudes ($N_{B_8} = 1$ for $m_s = 0$). The particular combination of ϕ_N's appearing in the Λ case (2.12b) is required by SU(3)$_{\mathrm{F}}$ symmetry.

In order to take into account evolution properly we expand the distribution amplitudes (2.12a)-(2.12c) upon the eigenfunctions of the evolution kernel (see (2.8)) up to terms of order $n = 5$. In Table 2 we quote four sets of expansion coefficients B_n corresponding to the following scenarios: Set 1 is obtained from $m_s = 0$, set 2 from $m_s = 150$ MeV (current strange quark mass) and set 4 from $m_s = 480$ MeV (constituent strange quark mass). The intermediate set 3 corresponds to $m_s = 350\,\mathrm{MeV} \approx (480^2 - 330^2)^{1/2}$ MeV (difference between the squares of strange and light constituent quark masses). This value appears if (2.11) is interpreted as the ratio of the BHL exponentials for a hyperon and the nucleon distribution amplitude.

In Fig. 1 we show contour plots of the four octet baryon distribution amplitudes for $m_s = 480$ MeV in order to illustrate the effect of the mass exponential (2.11). It can be seen that, compared to the nucleon case, the maxima of the Σ and Λ distribution amplitudes are shifted to the right, i.e. to larger x_3 values, whereas that of the Ξ distribution amplitude is shifted to the left. On the average, s quarks carry larger momentum fractions than d quarks if $m_s > 0$.

Distribution amplitudes for the octet baryons are also discussed in [16]. As compared to our ones these QCD sum rule based distribution amplitudes are strongly concentrated in the end-point regions and exhibit a more significant breaking of SU(3)$_{\mathrm{F}}$ symmetry. Moreover, they reveal

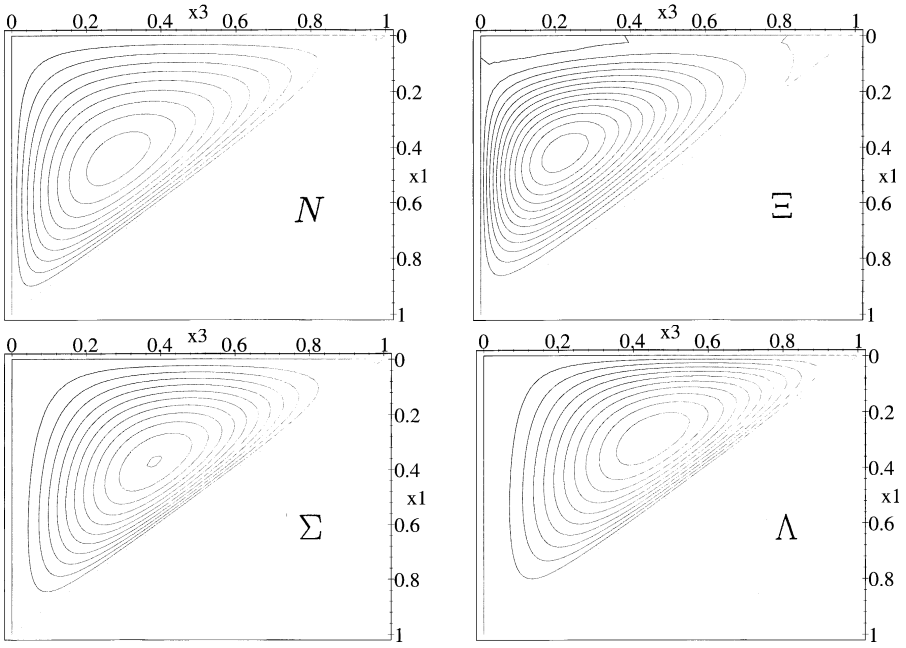

Fig. 1. Contour plots of octet baryon distribution amplitudes for $m_s = 480$ MeV and at a scale of 1 GeV

Table 2. Expansion coefficients $B_n(\mu_0)$ of octet baryon distribution amplitudes for various values of the parameter m_s

	Set 1 ($m_s = 0$)					Set 3 ($m_s = 350$ MeV)				
	B_1	B_2	B_3	B_4	B_5	B_1	B_2	B_3	B_4	B_5
Σ	0.750	0.250	0.000	0.000	0.000	0.216	0.394	-0.293	-0.914	0.241
Λ	-0.250	0.250	0.000	0.000	0.000	-0.721	0.389	-0.150	-0.574	0.093
Ξ	0.750	0.250	0.000	0.000	0.000	1.106	0.050	-0.282	1.717	-0.498
	Set 2 ($m_s = 150$ MeV)					Set 4 ($m_s = 480$ MeV)				
	B_1	B_2	B_3	B_4	B_5	B_1	B_2	B_3	B_4	B_5
Σ	0.623	0.284	-0.085	-0.285	0.065	-0.118	0.484	-0.404	-1.173	0.358
Λ	-0.360	0.282	-0.048	-0.195	0.027	-1.022	0.478	-0.182	-0.650	0.127
Ξ	0.831	0.201	-0.083	0.389	-0.129	1.338	-0.068	-0.384	2.943	-0.775

three pronounced maxima near the end-points. In summary, the distribution amplitudes proposed in [16] are very different from ours and also from the asymptotic distribution amplitude at experimentally accessible scales. As our distribution amplitudes but to a much greater extend, they possess the property that, on the average, a u (s) quark in the proton or the Σ^+ (Ξ^-, Λ) with the same helicity as its parent baryon carries the largest fraction of the baryon momentum.

3 The wave functions of the decuplet baryons

In analogy to (2.1) we write the valence Fock states of the decuplet baryons Δ and $\Sigma^\star$ with helicity $\lambda_{B_{10}} = 1/2$ as[1]

$$|\Delta^{++},+\rangle = \frac{\varepsilon_{a_1a_2a_3}}{\sqrt{2}}\int[\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp]\Psi^{\Delta}_{123}\,|\,u^{a_1}_+\,u^{a_2}_-\,u^{a_3}_+\,\rangle \quad (3.1a)$$

$$|\,B_{10}\,,+\,\rangle = \frac{\varepsilon_{a_1a_2a_3}}{\sqrt{3!}}\int[\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp]\,\Big\{\Psi^{B_{10}}_{123}\,|\,f^{a_1}_{1+}f^{a_2}_{1-}f^{a_3}_{2+}\,\rangle + \Psi^{B_{10}}_{213}\,|\,f^{a_1}_{1-}f^{a_2}_{1+}f^{a_3}_{2+}\,\rangle + \Psi^{B_{10}}_{132}\,|\,f^{a_1}_{1+}f^{a_2}_{1+}f^{a_3}_{2-}\,\rangle\Big\}\ . \quad (3.1b)$$

For the Δ^+ (Σ^{*-}) f_1 represents an u (d) quark and f_2 a d (s) one. Again, the states of the Δ^0, Δ^- and $\Sigma^{\star+}$ are obtained from those of the Δ^+, Δ^{++} and $\Sigma^{\star-}$ by exchanging $u \leftrightarrow d$, respectively. The generalization to the other helicity states is trivial. There is obviously only one independent scalar wave function in the case of the Δ and we assume that the same shall apply to the Σ^*. The scalar wave functions $\Psi^{B_{10}}_{123}$ of the decuplet baryons B_{10} ($= \Delta, \Sigma^\star$) are parameterized in a fashion similar to the octet baryon case:

[1] We omit the discussion of the decuplet states $\Xi(1530)$ and Ω^- since the perturbative approach is not applicable to the decays $J/\psi \to \Xi(1530)\overline{\Xi}(1530)$ due to almost zero momentum transfer. The decay $J/\psi \to \Omega^-\overline{\Omega}^+$ is even kinematically forbidden. The decay into Σ^* is the borderline case for the application of the perturbative approach

$$\Psi^{B_{10}}_{123}(x,\mathbf{k}_\perp) = \frac{f_{(10)}(\mu_F)}{24\sqrt{2}}\,\phi^{B_{10}}_{123}(x,\mu_F)\,\Omega_{(10)}(x,\mathbf{k}_\perp)\,. \quad (3.2)$$

$f_{(10)}$ is assumed to be equal for all members of the baryon decuplet. We again adopt the form (2.6) for the transverse momentum dependent part of the wave function, $\Omega_{(10)}$, with the transverse size parameter $a_{(8)}$ replaced by $a_{(10)}$.

Decuplet baryons in helicity 1/2 states have the same eigenfunctions of the evolution kernel and the same anomalous dimensions as the octet baryons (see Table 1). Evolution of the helicity 3/2 baryons is, on the other hand, different [12]. We refrain from giving details on that case here since we do not consider such baryons.

In the particular case of the Δ it is tempting to use a completely permutation symmetric distribution amplitude since the Δ is composed of three light quarks in symmetric spin and flavour states. Noting that the permutation symmetric part of the nucleon distribution amplitude (2.10) is just equal to the asymptotic distribution amplitude, we take $\phi^{\Delta}_{123}(x) = \phi_{\rm AS}(x)$ and construct $\phi^{\Sigma^\star}(x)$ analogously to $\phi^{\Sigma}(x)$

$$\phi^{\Sigma^\star}_{123}(x) = N_{\Sigma^*}\,\phi_{\rm AS}(x)\,\exp\left(-\frac{a^2_{(10)}m_s^2}{x_3}\right)\,. \quad (3.3)$$

The evolution of the Σ^* distribution amplitude is treated as in the case of the octet baryons by expanding (3.3) upon the eigenfunctions $\tilde\phi^n_{123}(x)$ up to $n=5$. In distinction from the octet baryon case, the QCD sum rule based distribution amplitudes for the decuplet baryons [13,17] are not very different from the ones we are proposing.

4 Decays of the J/ψ into octet baryons

Now, with the model wave functions at hand, we can calculate the decay width of J/ψ into octet baryon-antibaryon pairs within the modified perturbative approach. The helicity amplitudes of these processes may be decomposed covariantly to read

$$\mathcal{M}^{B_8}_{\lambda_1\lambda_2\lambda} = \quad (4.1)$$
$$\overline{u}_{B_8}(p_1,\lambda_1)\left[\mathcal{B}^{B_8}\,\gamma_\mu + \mathcal{C}^{B_8}\,\frac{(p_1-p_2)_\mu}{2\,m_{B_8}}\right]v_{B_8}(p_2,\lambda_2)\,\epsilon^\mu(\lambda)$$

where m_{B_8} is the mass of an octet baryon. p_1 (p_2) and λ_1 (λ_2) denote the momentum and the helicity of an octet baryon (antibaryon), respectively. u_{B_8} and v_{B_8} are their spinors (normalized as $\overline{u}_{B_8}u_{B_8} = 2m_{B_8}$) and ϵ is the polarization vector of the J/ψ. In a leading twist perturbative approach the helicity amplitudes are determined by the invariant $\mathcal{B}^{B_8}$ ($\mathcal{C}^{B_8}=0$). Implicitly this ensures hadronic helicity conservation. The $J/\psi\to B_8\overline{B}_8$ decay width reads

$$\Gamma(J/\psi\to B_8\overline{B}_8) = \frac{\rho_{\rm p.s.}(m_{B_8}/M_\psi)}{48\,\pi\,M_\psi}\,\sum|\,\mathcal{M}^{B_8}_{\lambda_1\lambda_2\lambda}|^2 \quad (4.2)$$

in the J/ψ rest frame (M_ψ is the J/ψ mass). If the J/ψ is produced in e^+e^- annihilations it is transversely polarized with respect to the beam direction and the angular distribution of the baryons emitted in the J/ψ rest frame then exhibits a $1+\cos^2\theta$ dependence (up to corrections of $\mathcal{O}(m^2_{B_8}/M^2_\psi)$) characteristic of perturbative QCD [1]. The function $\rho_{\rm p.s.}$ in (4.2) is the usual phase space factor

$$\rho_{\rm p.s.}(z) = \sqrt{1-4z^2}\,. \quad (4.3)$$

As we said in the introduction we are going to calculate the invariant $\mathcal{B}^{B_8}$ within the modified perturbative approach proposed in [8], thus generalizing our analysis of the J/ψ decay into nucleon-antinucleon pairs [6]. As in previous perturbative calculations [1–3] the J/ψ meson is treated as a non-relativistic $c\bar c$ system and $\mathcal{O}(v^2/c^2)$ corrections are neglected. In contrast to the decays of P-wave charmonium [18] the $c\bar cg$ Fock state is suppressed in exclusive decays by inverse powers of the large scale which is provided by the c-quark mass m_c [19,20], relative to the $c\bar c$ Fock state and is, therefore, neglected in our analysis. We will comment on this approximation in Sect. 7. The use of m_c, strictly speaking $2m_c$, as the large scale rather than the charmonium mass is consistent with the neglect of relativistic corrections. It is also well in the spirit of a perturbative approach since in the internal c-quark propagators the c-quark mass appears. The J/ψ state is written in a covariant fashion

$$|\,J/\psi;\,q,\lambda\,\rangle \;=\; \frac{\delta_{ab}}{\sqrt{3}}\left(\frac{f_\psi}{2\sqrt{6}}\right)\frac{1}{\sqrt{2}}(\not q + M_\psi)\not\epsilon(\lambda) \quad (4.4)$$

where a and b are colour indices and f_ψ is the J/ψ decay constant being related to the J/ψ wave function at the origin of the configuration space. Merely that part of the wave function is, to a reasonable approximation, required in the calculation of $\mathcal{B}$ since, as we already mentioned in the introduction, the c and the $\bar c$ quark only annihilate if their mutual distance is less than about $1/m_c$ [4] which is smaller than the J/ψ radius [21]. M_ψ is replaced by $2m_c$ in the calculation of $\mathcal{B}^{B_8}_{3g}$ and the baryon masses are neglected. Only the phase space factor in (4.2) is evaluated with the physical masses, M_ψ and m_{B_8}.

The invariant $\mathcal{B}^{B_8}$ receives its dominant contribution from the graphs with three intermediate gluons, see Fig. 2. Within the modified perturbative approach the three-gluon contribution $\mathcal{B}^{B_8}_{3g}$ to the J/ψ decay into $B_8\overline{B}_8$ is of the form

$$\mathcal{B}^{B_8}_{3g} = \frac{f_\psi}{2\sqrt{6}}\int[{\rm d}x][{\rm d}x']\int\frac{{\rm d}^2\mathbf{b}_1}{(4\pi)^2}\,\frac{{\rm d}^2\mathbf{b}_3}{(4\pi)^2}\,\hat T_H(x,x',\mathbf{b})$$
$$\times\exp[-S(x,x',\mathbf{b},2m_c)]\big[\hat\Psi^{B_8}_{123}(x,\mathbf{b})\hat\Psi^{B_8}_{123}(x',\mathbf{b})$$
$$+\frac{1}{2}\big(\hat\Psi^{B_8}_{123}(x,\mathbf{b})+\hat\Psi^{B_8}_{321}(x,\mathbf{b})\big)\big(\hat\Psi^{B_8}_{123}(x',\mathbf{b})+\hat\Psi^{B_8}_{321}(x',\mathbf{b})\big)\big] \quad (4.5)$$

for $B_8 = N,\Sigma,\Xi$. This contribution is the same for all octet baryons belonging to the same isospin multiplet. Since the form of the Λ Fock state (2.1b) is slightly different, the three-gluon contribution reads

$$\mathcal{B}^{\Lambda}_{3g} = \sqrt{\frac{3}{2}}\,\frac{f_\psi}{2}\int[{\rm d}x][{\rm d}x']\int\frac{{\rm d}^2\mathbf{b}_1}{(4\pi)^2}\,\frac{{\rm d}^2\mathbf{b}_3}{(4\pi)^2}\,\hat T_H(x,x',\mathbf{b})$$

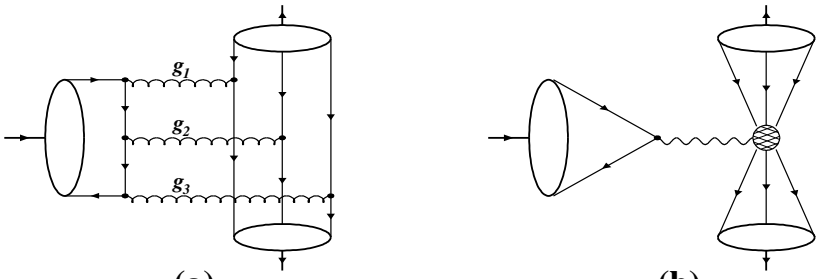

Fig. 2. Feynman graphs for the J/ψ decay into baryon-antibaryon. **a** Three-gluon contribution (graphs with permutated gluon lines are not shown), **b** electromagnetic contribution

$$\times \exp[-S(x,x',\mathbf{b},2m_c)]\big[\hat{\Psi}^{\Lambda}_{123}(x,\mathbf{b})\hat{\Psi}^{\Lambda}_{123}(x',\mathbf{b}) + \frac{1}{2}\big(\hat{\Psi}^{\Lambda}_{123}(x,\mathbf{b})-\hat{\Psi}^{\Lambda}_{321}(x,\mathbf{b})\big)\big(\hat{\Psi}^{\Lambda}_{123}(x',\mathbf{b})-\hat{\Psi}^{\Lambda}_{321}(x',\mathbf{b})\big)\big] \tag{4.6}$$

in the case of the Λ. The representation of $\mathcal{B}_{3g}$ as a convolution of wave functions and a hard scattering amplitude $\hat{T}_H$ can formally be derived by using the methods described in detail by Botts and Sterman [8]. The $\mathbf{b}_i$, canonically conjugated to the transverse momenta $\mathbf{k}_{\perp i}$, are the quark separations in the transverse configuration space. $\mathbf{b}_1$ and $\mathbf{b}_3$ correspond to the locations of quarks 1 and 3 in the transverse plane relative to quark 2 and $\mathbf{b}_2 = \mathbf{b}_1 - \mathbf{b}_3$. $\hat{\Psi}^{B_8}_{ijk}$ represents the Fourier transform of the wave function $\Psi^{B_8}_{ijk}$ (see Sect. 2).

$\hat{T}_H$ is the Fourier transform of the usual momentum space hard scattering amplitude to be calculated from the Feynman graphs shown in Fig. 2a. Up to corrections of order α_s^4, $m^2_{B_8}/(4m_c^2)$ and $b^2/(4m_c^2)$ the hard scattering amplitude in $\mathbf{b}$ space reads

$$\hat{T}_H(x,x',\mathbf{b}) = -\sqrt{\frac{2}{3}}\,\frac{5}{9}\,2^{12}\,m_c^5\,\frac{(x_1x_3'+x_3x_1')}{[\tilde{q}_1^2+\tilde{g}_1^2][\tilde{q}_3^2+\tilde{g}_3^2]} \times\Big(\prod_{i=1}^{3}\alpha_S(t_i)\Big)\int \mathrm{d}^2\mathbf{b}_0 \times\left[\frac{i\pi}{2}\mathrm{H}_0^{(1)}(\tilde{g}_1|\mathbf{b}_1+\mathbf{b}_0|)-\mathrm{K}_0(\tilde{q}_1\,|\mathbf{b}_1+\mathbf{b}_0|\,)\right]\frac{i\pi}{2}\mathrm{H}_0^{(1)}(\tilde{g}_2b_0) \times\left[\frac{i\pi}{2}\mathrm{H}_0^{(1)}(\tilde{g}_3|\mathbf{b}_3+\mathbf{b}_0|)-\mathrm{K}_0(\tilde{q}_3|\mathbf{b}_3+\mathbf{b}_0|)\right]. \tag{4.7}$$

The quantities

$$\tilde{q}_i^2 = 2[x_i\,(1-x_i')+(1-x_i)\,x_i']\,m_c^2\,, \qquad \tilde{g}_i^2 = 4x_ix_i'\,m_c^2 \tag{4.8}$$

represent the virtualities of the internal quarks and gluons at zero transverse momenta. Since, as shown in [6], the hard scattering amplitude only depends on the sum of transverse momenta, $\mathbf{k}_{\perp i}+\mathbf{k'}_{\perp i}$, the transverse separation of any two quarks inside the baryon is the same as that of the corresponding antiquarks inside the antibaryon ($\mathbf{b}_i = \mathbf{b'}_i$). Physically, this property of the hard scattering amplitude means that baryon and antibaryon are created with identical transverse configurations of the quarks and antiquarks, respectively. The auxiliary variable $\mathbf{b}_0$ in (4.7) serves as a Lagrange multiplier to the constraint $\sum \mathbf{k}_i + \mathbf{k'}_i = \mathbf{0}$. Since the virtualities of the gluons are timelike, $\hat{T}_H$ includes complex-valued Hankel functions $\mathrm{H}_0^{(1)}$ that are related to the usual modified Bessel functions K_0, appearing for space-like propagators, by analytic continuation.

The Sudakov factor $\exp[-S]$ entering (4.5) and (4.6) takes into account those gluonic radiative corrections not accounted for in the QCD evolution of the wave function as well as the renormalization group transformation from the factorization scale μ_F to the renormalization scales t_i at which the hard amplitude $\hat{T}_H$ is evaluated. The Sudakov factor, originally derived by Botts and Sterman [8] and later on slightly improved, can be found for instance in [22]. The renormalization scales t_i are defined in analogy to the case of electromagnetic form factors [8] as the maximum scale of either the longitudinal momentum or the inverse transverse separation associated with each of the gluons

$$t_1 = \max(\tilde{q}_1,\tilde{g}_1,1/b_3)\,, \quad t_2 = \max(\tilde{g}_2,1/b_2)\,, \quad t_3 = \max(\tilde{q}_3,\tilde{g}_3,1/b_1)\,. \tag{4.9}$$

Infrared cut-off parameters $\tilde{b}_i$ appear in the Sudakov factor which are naturally related to, but not uniquely determined by the mutual separations of the three quarks [23]. Following [5] we chose $\tilde{b}_i = \tilde{b} = \max\{b_1,b_2,b_3\}$. With this "MAX" prescription the three-gluon contribution $\mathcal{B}^{B_8}_{3g}$ is unencumbered by α_S singularities in the soft end-point regions. As a consequence of the regularizing power of the "MAX" prescription, the perturbative contribution saturates in the sense that the results become insensitive to the inclusion of the soft regions. A saturation as strong as possible is a prerequisite for the self-consistency of the perturbative approach. The infrared cut-off $\tilde{b}$ marks the interface betweeen the non-perturbative soft gluons, which are implicitly accounted for in the baryon wave function, and the contributions from soft gluons, incorporated in a perturbative way in the Sudakov factor. Obviously, the

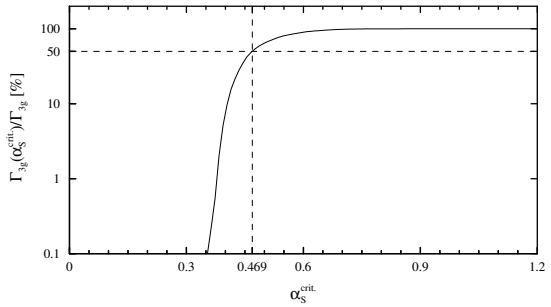

Fig. 3. Percental accumulation of the three-gluon contribution to the width of the $N\overline{N}$ channel from regions of internal momenta where $\prod_{i=1}^{3}\alpha_s(t_i) < (\alpha_s^{\rm crit})^3$

gliding factorization scale to be used in the evolution of the wave function, has to be chosen as $\mu_F = 1/\tilde{b}$.

As an inspection of (4.5), (4.6) and (4.7) reveals, a nine dimensional numerical integration has to be performed.[2] Although this is a rather involved technical task it can be carried through with sufficient accuracy if some care is put into it. The numerical results are obtained from the wave functions discussed in Sect. 2 and for the following values of the J/ψ decay constant and the c-quark mass: $f_\psi = 409$ MeV, $m_c = 1.5$ GeV. We evaluate α_s in the one-loop approximation with $n_f = 4$ and $\Lambda_{QCD} = 210$ MeV [24].

Before turning to the detailed discussion of the numerical results for the decay widths, we want to focus on an important feature of the modified perturbative approach. As mentioned in the introduction, one of the motivations for including the transverse hadronic structure and the Sudakov factor in the analysis is to achieve a theoretically self-consistent calculation in the sense that the bulk of the perturbative contribution is accumulated in regions where the strong coupling constant α_s is sufficiently small. A method to check whether or not this is the case is to set the integrand in (4.5) or (4.6) equal to zero in those regions where $\prod_{i=1}^{3}\alpha_s(t_i) > (\alpha_s^{\rm crit})^3$ and to evaluate the three-gluon contribution to the decay width as a function of $\alpha_s^{\rm crit}$. In Fig. 3 we show the accumulation profile for the nucleon case; it is typical of all baryons. Consulting Fig. 3, one sees that almost the entire result is accumulated in the comparatively narrow region of α_s between 0.4 and 0.6. The regions with $\prod_{i=1}^{3}\alpha_s(t_i) < 0.469^3$ provide 50% of the total result. Hence, our calculation of the J/ψ decay widths into octet baryon-antibaryon pairs is theoretically self-consistent.

In Table 3 we present our results for the $J/\psi \to B_8\overline{B}_8$ decay widths using the distribution amplitudes discussed in Sect. 2. For the sake of comparison we also expose the

[2] Taking into account relativistic corrections to the J/ψ wave function, i.e. its transverse momentum dependence, one would have to perform a 14 dimensional numerical integration which seems impossible with present day computers to a sufficient degree of accuracy

Table 3. Results for the decay widths of J/ψ into octet baryon-antibaryon pairs for the four sets of distribution amplitudes defined by the expansion coefficients B_n quoted in Table 2 ($f_\psi = 409$ MeV, $m_c = 1.5$ GeV). For comparison we also quote the experimental results and, in the column labelled $\phi_{\rm AS}$, predictions evaluated with distribution amplitudes constructed from the asymptotic distribution amplitude instead from (2.10) (with $m_s = 350$ MeV)

	Γ_{3g} [eV]					Data [24]
Channel	Set 1	Set 2	Set 3	Set 4	$\phi_{\rm AS}$	$\Gamma_{\rm exp}$ [eV]
$p\overline{p}$	174	174	174	174	140	186 ± 14
$\Sigma^0\overline{\Sigma}^0$	136	128	113	108	97.8	110 ± 15
$\Lambda\overline{\Lambda}$	140	133	117	107	99.7	117 ± 14
$\Xi^-\overline{\Xi}^+$	107	92.8	62.5	47.4	46.9	78 ± 18

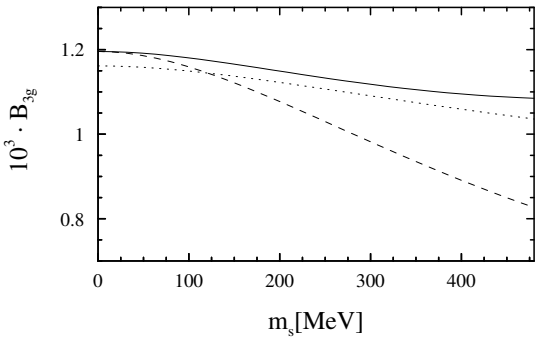

Fig. 4. The three-gluon contribution $\mathcal{B}_{3g}^{B_8}$ vs. the mass parameter m_s for the decay channels $\Sigma\overline{\Sigma}$ (*solid*), $\Xi\overline{\Xi}$ (dashed) and $\Lambda\overline{\Lambda}$ (*dotted*)

available experimental data [24][3]. As can be seen from the results obtained with the distribution amplitudes termed set 1 the phase space factor $\rho_{\rm p.s.}$ is an important but not sufficient element for the suppression of the hyperon channels. Since $m_s = 0$ for these distribution amplitudes the differences in the predictions for the widths, except for the $\Lambda\overline{\Lambda}$ case, are only due to $\rho_{\rm p.s.}$. For $m_s > 0$ the additional suppression of the end-point regions leads to smaller decay widths for the hyperon channels. In order to demonstrate the strength of that reduction we display the invariant $\mathcal{B}_{3g}^{B_8}$ versus m_s in Fig. 4. The number of strange quarks embodied in a given baryon is reflected in differently strong m_s dependences of $\mathcal{B}_{3g}^{B_8}$. As inspection of Table 3 brings to view, the phase space corrected three-gluon contributions nicely reproduce the experimental pattern of the decay widths provided they are computed with the distribution amplitudes of set 3. The value of 350 MeV for the parameter m_s used in the construction of these distribution amplitudes appears reasonable, considering the interpretation of the mass factor (2.11) as the BHL exponential [15]. Note that, as a consequence of the use of the c-quark mass in the calculation of $\mathcal{B}_{3g}^{B_8}$, the result for the decay width of the $p\overline{p}$ channel given in Table 3 differs from that one reported in [6].

[3] We use PDG averages throughout. The original data are from [25–28]

The distribution amplitudes defined by (2.10-2.12c) exhibit a little asymmetry with respect to permutations of the x_i. In order to study the importance of this asymmetry we also show in Table 3 results computed with the asymptotic distribution amplitude for the nucleon and with modifications of it for the hyperons that are constructed analogously to the distribution amplitudes described in Sect. 2. The asymmetry in the distribution amplitudes is seen to increase the magnitudes of the decay widths by about 30% for the $\Xi\overline{\Xi}$ channel and about 20% for the other channels while the pattern of the predictions remains unchanged.

Let us now assess the uncertainties of our predictions. The value of the J/ψ decay constant used by us is determined from the leptonic J/ψ decay width. Since the J/ψ decay constant, or more generally, the decay constant, f_n, of a n^3S_1 quarkonium state is defined by

$$\langle 0|j^\mu_{\rm em}|n^3S_1\rangle \;=\; f_n M_n \epsilon^\mu, \tag{4.10}$$

the leptonic decay width of a n^3S_1 state reads

$$\Gamma(nS\to e^+e^-) \;=\; \frac{4\pi}{3}\,\frac{e_Q^2\alpha^2 f_n^2}{M_n} \tag{4.11}$$

where e_Q is the charge of the heavy quark the charmonium state consists of. Re-expressing (4.11) in terms of the non-relativistic wave function at the origin of the configuration space, one arrives at the famous van Royen-Weisskopf width [29]. The use of the decay constant instead of the wave function at the origin implies that we relate the $J/\psi\to B\overline{B}$ (or the $n^3S_1\to B\overline{B}$) widths to the leptonic width. By that means the uncertainties in the determination of the wave function at the origin via the usual van Royen-Weisskopf width cancel to a large extend. These uncertainties arise from relativistic and QCD corrections which seem to be large [30] but are not well known [21,31].

The next uncertainty to be mentioned arises from the choice of the c-quark mass value. In accordance with calculations of the charmonium spectrum within non-relativistic potential approaches [21] and with a global fit of charmonium parameters [32] we take 1.5 GeV as the favoured value. That value has, for instance, also been used in a recent analysis of P-wave charmonium decays into two pions [18]. In spite of this, little changes of the m_c value cannot be excluded and lead to an approximate rescaling of the decay widths by the factor $(1.5\,{\rm GeV}/m_c)^8$.

The value of Λ_{QCD} is also subject to uncertainties. A change of that value by, say, ± 20 MeV which roughly represents the inaccuracy of our present knowledge of Λ_{QCD} [24], would alter the theoretical decay widths by about $\pm 25\%$. We stress that for any changes of the m_c and Λ_{QCD} values the ratios of any two decay widths calculated by us remain approximately unchanged. This assertion does not only refer to the $J/\psi\to B_8\overline{B}_8$ decay widths but it also applies to the still to be discussed $J/\psi\to B_{10}\overline{B}_{10}$ and $\psi'\to B\overline{B}$ widths.

In addition to the three-gluon contribution $\mathcal{B}_{3g}^{B_8}$ there is a subdominant, although in some cases perhaps sizeable, isospin-violating electromagnetic one, $\mathcal{B}_{\rm em}^{B_8}$ [33], arising from the graph shown in Fig. 2b. This contribution is proportional to the time-like magnetic form factor of the baryon. For the proton $\mathcal{B}_{\rm em}^{p}$ amounts to about 15% (in absolute value) of the total $\mathcal{B}\checkmark$ as is estimated from the recent measurement of the proton magnetic form factor in the time-like region [34]. Since the relative phase between $\mathcal{B}_{3g}^{p}$ and $\mathcal{B}_{\rm em}^{p}$ is unknown we cannot simply add the two contributions [4]. Therefore, we merely can state that, depending on the value of the relative phase, the electromagnetic contribution (including the interference term between it and $\mathcal{B}_{3g}^{p}$) to the $J/\psi\to p\overline{p}$ decay width can be as large as 30% or only 2% . The comparison of our result with data indicates a relative phase close to $\pm\pi/2$ in the $p\overline{p}$ case.

At this point a remark concerning the $n\overline{n}$ decay channel is in order. Since, as said repeatedly, the three-gluon contribution respects isospin symmetry any difference between the $p\overline{p}$ and $n\overline{n}$ decay widths must be due to the electromagnetic contribution. From experiment it is known that the widths for $J/\psi\to p\overline{p}$ and $J/\psi\to n\overline{n}$ decays agree within the experimental errors [28] and that the time-like form factors of the proton and the neutron are approximately equal in modulus at least at $s=5.4$ GeV^2 [35]. Thus, one may conclude that the relative phases between the three-gluon and the electromagnetic contributions are the same (up to a possible sign) for the proton and the neutron channel.

The size of the electromagnetic contribution to the hyperon channels may be estimated from a recent analysis of the octet baryon form factors within a diquark model [36]. With the help of a few rather well determined parameters that model is able to describe a large number of exclusive observables. In particular relevant for the present work is the prediction that, in the space-like region, the magnetic form factors of the Σ^+ and Σ^- have opposite signs and are comparable in magnitude to that of the proton. The form factors of the Λ and Σ^0, on the other hand, turn out to be very small. Predictions for the Ξ^- form factor are not reported in [36] but that form factor is presumably smaller in absolute value than the proton form factor. Assuming similar relative magnitudes of the form factors in the time-like region, we expect that, in the case of the hyperon channels listed in Table 3, the three-gluon contributions should match with the experimental data. This may not be the case for the $\Sigma^-\overline{\Sigma}^+$ channel; while the three-gluon contribution is the same as for the $\Sigma^0\overline{\Sigma}^0$ channel, the electromagnetic contribution may be large.

There is also a small contribution to the invariant $\mathcal{B}^{B_8}$ from $c\bar{c}$ annihilations mediated by two gluons and a photon. The $gg\gamma$ contribution to $\mathcal{B}^{B_8}$ being proportional to the ggg contribution [33], amounts to less than about 1% of the latter and is therefore neglected.

Finally, from the measurement of the angular distribution of $B_8\overline{B}_8$ pairs produced in $e^+e^-\to J/\psi\to B_8\overline{B}_8$ [26] one observes small violations of the helicity sum rule: The fraction of $p\overline{p}$ and $\Lambda\overline{\Lambda}$ pairs with equal helicities amounts

[4] The time-like nucleon form factor is likely not under the regime of perturbative QCD in the energy region of interest; it is about a factor of 3 larger in absolute value than the form factor in the space-like region at $Q^2=M_\psi^2$

to about 10% of the total number of pairs with, in particular in the Λ case, large errors. For the other hyperon channels the errors are so large that no conclusion can be drawn. The small amount of equal helicity pairs is what is to be expected if the process is dominated by perturbative QCD: Each of the virtual gluons creates a quark and an antiquark with opposite helicities. Since our baryon wave functions do not embody any non-zero orbital angular momentum component the quark helicities sum up to the baryon helicity. Hence, baryon and antibaryon are produced with opposite helicities. The small amount of $B_8\bar{B}_8$ pairs with the wrong helicity combination observed experimentally, while indicating the presence of some soft contributions, can be considered as a hint that perturbative QCD is the dominant dynamical mechanism in the $J/\psi \to B_8\overline{B}_8$ decays. One should, however, be aware of these contributions when theoretical results for the $J/\psi \to B_8\overline{B}_8$ decays are compared with experiment. The production of $B_8\overline{B}_8$ pairs with equal helicities can perhaps be explained as an constituent quark mass and/or baryon mass effect [33,37].

5 J/ψ decays into decuplet baryons

We are now going to apply the modified perturbative approach to the J/ψ decays into the decuplet baryon-antibaryon channels $\Delta^{++}\overline{\Delta}^{--}$ and $\Sigma^{\star-}\overline{\Sigma}^{\star+}$ in the same manner as for the octet baryon case. As we said above, within a perturbative approach and with wave functions of zero orbital angular momentum components in the direction of the baryon momentum, B_{10} and $\overline{B}_{10}$ are produced with opposite helicities. For the same reason only helicities $\pm 1/2$ are possible. Thus, as for the octet baryon case, the only non-zero helicity amplitudes are the $\mathcal{M}^{B_{10}}_{\pm\mp\lambda}$. They are fed by only one invariant $\mathcal{B}^{B_{10}}$ (or, depending on the definitions of the covariants, only one linear combination of the invariants) out of the five the general covariant decomposition of the $J/\psi \to B_{10}\overline{B}_{10}$ helicity amplitudes comprises. Within the modified perturbative approach, the three-gluon contribution to the invariant $\mathcal{B}^{B_{10}}$ reads

$$\mathcal{B}^{B_{10}}_{3g} = \sqrt{\frac{3}{2}}\frac{f_\psi}{4}\int[\mathrm{d}x][\mathrm{d}x']\int\frac{\mathrm{d}^2\mathbf{b}_1}{(4\pi)^2}\frac{\mathrm{d}^2\mathbf{b}_3}{(4\pi)^2}\,\hat{T}_H(x,x',\mathbf{b}) \times\exp[-S(x,x',2m_c)]\hat{\Psi}^{B_{10}}_{123}(x,\mathbf{b})\hat{\Psi}^{B_{10}}_{123}(x',\mathbf{b})\,, \tag{5.1}$$

for $B_{10} = \Delta, \Sigma^\star$. The three-gluon contribution is evaluated with the decuplet wave functions introduced in Sect. 3 and with the hard scattering amplitude (4.7). Insertion of $\mathcal{B}^{B_{10}}_{3g}$ into (4.2) provides the wanted decay widths. It still remains to choose plausible values of the parameters $a_{(10)}$ and $f_{(10)}$. The fact that we assume the same form for the $k_\perp$ dependence of the Δ and the nucleon wave functions, and that we use $\phi_{\mathrm{A}S}$ for the Δ distribution amplitude which does not differ from the actual nucleon distribution amplitude (2.10) greatly, suggests, as a first attempt, the ansatz $a_{(10)} = a_{(8)}$ $(= 0.75\ \mathrm{GeV}^{-1})$ and $P^\Delta_{3q} = P^N_{3q}$. Thereby, it is perhaps plausible to evaluate the valence quark probability of the nucleon from $\phi_{\mathrm{A}S}$ instead from (2.10). Doing so we find $P^N_{3q}(\mathrm{A}S) = 0.163$ and the requirement $P^\Delta_{3q} = P^N_{3q}(\mathrm{A}S)$ leads to $f_{(10)} = 0.0163\,\mathrm{GeV}^2$ which is larger than $f_{(8)}$ by a factor of $\sqrt{6}$, i.e., the SU(6) result. The results for the J/ψ decay widths into $B_{10}\overline{B}_{10}$ pairs are presented in Table 4. As can be seen our result for the $\Sigma^\star\overline{\Sigma}^\star$ channel is too large as compared to the data while agreement is achieved for the $\Delta\overline{\Delta}$ case.

Bearing in mind that the Δ is in completely symmetric flavour and spin states and that the Pauli principle effectively induces an additional repulsive interquark force, one may expect a larger radius, and hence a larger value of $a_{(10)}$, for the Δ than for the nucleon. Therefore, we also try values for $a_{(10)}$ slightly larger than that for a_{B_8} and fix in each case $f_{(10)}$ from the requirement $P^\Delta_{3q} = 0.163$ as before. In Table 4 we list results obtained with the values 0.80 and 0.85 GeV^{-1} for $a_{(10)}$. For both these values of $a_{(10)}$ satisfactory agreement with experiment is found. The uncertainties of the theoretical results are the same as for the octet baryons. In particular, one has to consider the possibility of large electromagnetic contributions (see Fig. 2b).

6 Decays of the ψ' and other quarkonia

The extension of our approach to baryonic decays of the ψ' $(= \psi(2\,^3S_1))$ is now a simple matter. It is however important to realize that, in contrast to other authors [1], we evaluate the three-gluon contribution with the c-quark mass and not with the charmonium mass. This is, as we said, legitimate in a non-relativistic treatment of the charmonia. Hence, in order to get the ψ' widths in our approach we have not to rescale the corresponding J/ψ widths by $(M_\psi/M_{\psi'})^8$ but rather by

$$\Gamma(\psi' \to B\overline{B}) = \frac{\rho_{\mathrm{p.s.}}(m_B/M_{\psi'})}{\rho_{\mathrm{p.s.}}(m_B/M_\psi)}\,\frac{\Gamma(\psi' \to e^+e^-)}{\Gamma(J/\psi \to e^+e^-)} \times\Gamma(J/\psi \to B\overline{B}) \tag{6.1}$$

which holds for both, octet and decuplet baryons. The charmonium decay constants are replaced by the leptonic decay widths by means of formula (4.11).

As an immediate examination of our approach one may apply the scaling relation (6.1) directly to the experimental data (see Tables 3, 4, 5). Considering the uncertainties due to the electromagnetic contributions, which in one or the other case may be large, (6.1) works quite well in particular for the octet baryons. For the decuplet baryons, on the other hand, it seems that the suppression of the ψ' decay widths is a slightly underestimated, although the large experimental errors prevent any definite conclusion at present. One may suspect the neglect of the decuplet baryon masses to be responsible for that possible imperfection. In any case, an additional strong suppression, as provided by $(M_\psi/M_{\psi'})^8$ [1], is in clear conflict with the data [9]. This observation supports our attempt of using

Table 4. $f_{(10)}$, the expansion coefficients B_n for the $\Sigma^\star$ distribution amplitude and results for the three-gluon contribution to the $J/\psi \to \Delta^{++}\overline{\Delta}^{--}$, $\Sigma^{\star-}\overline{\Sigma}^{\star+}$ decay widths for various values of the transverse size parameter $a_{(10)}$ ($m_s = 350$ MeV). $f_{(10)}$ is fixed by the requirement $P_{3q}^{\Delta} = 0.163$

$a_{(10)}$ [GeV^{-1}]	$f_{(10)}$ [GeV^2]	B_1	B_2	B_3	B_4	B_5	$\Gamma_{3g}^{\Delta\overline{\Delta}}$ [eV]	$\Gamma_{3g}^{\Sigma^\star\overline{\Sigma}^\star}$ [eV]
0.75	0.0163	-0.494	0.165	-0.203	-1.013	0.058	105	66.1
0.80	0.0143	-0.547	0.182	-0.216	-1.081	0.062	82.6	51.8
0.85	0.0127	-0.601	0.200	-0.229	-1.142	0.065	65.1	40.8
			$\Gamma_{\rm exp}$ [eV]	[24]			96 ± 26	45 ± 6

Table 5. The three-gluon contribution to the $\psi' \to B\overline{B}$ decay widths (in [eV]) computed through (6.1) from the set 3 J/ψ widths (see Table 3). $a_{(10)} = 0.85$ GeV^{-1}

channel	$p\overline{p}$	$\Sigma^0\overline{\Sigma}^0$	$\Lambda\overline{\Lambda}$	$\Xi^-\overline{\Xi}^+$	$\Delta^{++}\overline{\Delta}^{--}$	$\Sigma^{*-}\overline{\Sigma}^{*+}$
Γ_{3g}	76.8	55.0	54.6	33.9	32.1	24.4
$\Gamma_{\rm exp}$ [9]	76 ± 14	26 ± 14	58 ± 12	23 ± 9	25 ± 8	16 ± 8
[24]	53 ± 15					

the c-quark mass in the calculation of the decay amplitudes rather than the mass of the charmonium state in question.

Results for baryonic decay widths of the ψ', evaluated through (6.1) from the set 3 J/ψ widths, are listed in Table 5 where also recent experimental results of the BES collaboration [9] are quoted. The data are still preliminary. The agreement between theoretical results (with $a_{(10)} = 0.85$ GeV^{-1} in the decuplet baryon case) and experiment is generally good although our results seem to be a bit too large for the $\Sigma\overline{\Sigma}$ and $\Sigma^*\overline{\Sigma}^*$ channels. For a discussion of uncertainties we refer to Sect. 3.

Computation of the $\psi(3\,^3S_1)$ decay widths are difficult within our approach. The relativistic corrections are presumably larger since the $\psi(3\,^3S_1)$ mass is above the threshold for open charm production. The $B\overline{B}$ decay widths are likely to be very small and it is hardly conceivable that they will be measured. Therefore, we refrain from estimating these decay widths.

Results for bottonium decays, on the other hand, can safely be calculated within our approach. The hard scale, provided by the b-quark mass, is larger than in the charmonium case and relativistic corrections are smaller. But, as it turns out, the predicted decay widths for the baryonic channels are also very small. Approximately, i.e. ignoring the fact that the $k_\perp$-dependent suppression of the three-gluon contribution is a little bit different in the two cases, we find the following rescaling formula

$$\Gamma(\Upsilon \to B\overline{B}) = 4\,\frac{\rho_{\rm p.s.}(m_B/M_\Upsilon)}{\rho_{\rm p.s.}(m_B/M_\psi)}\,\frac{\Gamma(\Upsilon \to e^+e^-)}{\Gamma(J/\psi \to e^+e^-)} \times \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^6 \left(\frac{m_c}{m_b}\right)^8 \Gamma(J/\psi \to B\overline{B}) \quad (6.2)$$

Using $m_b = 4.5$ GeV we obtain, for instance, a value of 0.02 eV for the $\Upsilon \to p\overline{p}$ decay width which value corresponds to a branching ratio of 0.3×10^{-7} well below the experimental upper bound [24]. The decay widths for the other $B\overline{B}$ channels are even smaller.

7 Summary

In this investigation, we applied the modified perturbative approach to the decays of J/ψ and ψ' into baryon-antibaryon pairs. We demonstrated that, on the basis of plausible baryon wave functions for which SU(3)$_{\rm F}$ symmetry is only mildly broken by quark mass effects and for which even SU(6) symmetry, in sharp contrast to the QCD sum rule based wave functions, approximately holds, the experimental data for octet and decuplet baryon channels are quite well reproduced by the phase space corrected three-gluon contributions. The perturbative contributions to the decay widths are calculated self-consistently in the sense that the bulk of a perturbative contribution is accumulated in regions of reasonably small values of α_s. Besides the form of the wave functions used by us our analysis differs from previous ones in the following points:

i) The use of the modified perturbative approach allows to take into account the running α_s and the evolution of the wave functions properly, in contrast to the usual leading twist analysis. The virtualities of the internal c quarks and gluons can be chosen as the arguments of α_s. Hence, α_s reflects the characteristic scale of the process under question. The running coupling constant is therefore not a quasi-free parameter that can, within a certain range, be chosen arbitrarily. Since the decay widths are proportional to α_s^6, a large factor of uncertainty is therefore hidden in the standard perturbative analysis.

ii) The hard scattering amplitude is computed with the c-quark mass instead of the charmonium mass. The use of the c-quark mass is consistent with the non-relativistic treatment of the charmonium state and with the perturbative approach. Hence the ψ' - J/ψ scaling relation (6.1) holds in our approach. That scaling relation is nicely confirmed by the data in contrast to the usual $(M_\psi/M_{\psi'})^8$ scaling.

Although our results for the decay widths agree with the data, we are aware of a number of uncertainties in our calculation. Above all, we mention as a source of uncertainty the value of the c-quark mass. The value we have chosen (1.5 GeV) is consistent with other constraints on that mass. Another uncertainty arises from the electromagnetic contribution which, at least in the $p\overline{p}$ case, can be large. In virtue of the unknown relative phase between the electromagnetic and the three-gluon contribution, the first cannot be taken into account properly. We emphasize that most of the uncertainties cancel in ratios of widths to a large extent.

At this point a comment on higher Fock state contributions is in order. The fact that, with plausible values of the parameters, the valence Fock states alone suffice to describe the data on the baryonic J/ψ and ψ' decay widths, can be regarded as a hint at fairly small contributions from higher Fock states, in particular from the $c\bar{c}g$ states of the charmonia, the so-called colour-octet contributions. This is in marked contrast to P-wave charmonium decays into pairs of pseudoscalar mesons where the colour-octet contributions have been found to be large [18]. Both these findings are nevertheless in agreement with our theoretical understanding of charmonium decays. In the case of the P-wave charmonia the colour-octet contributions are not suppressed by powers of either v, the typical velocity of the charm quark in the heavy meson[5], or $1/m_c$ as compared to the contributions from the valence Fock states [18]. Hence, the colour-octet contributions have to be included for a consistent analysis of P-wave charmonium decays. The situation is different for the S-wave charmonium decays into baryon-antibaryon pairs: Higher Fock state contributions are suppressed by powers of $1/m_c$ and first start at $\mathcal{O}(v^4)$. Moreover, there is no obvious enhancement of the corresponding hard parts of the decay processes in the HSA, they appear with at least the same power of α_s as the valence Fock state contributions. Thus, despite of the fact that m_c is not very large and v not small ($v^2 \simeq 0.3$), it seems reasonable to expect small higher Fock state contributions to the decay processes considered in this work. Or, turning the argument around, the $B\overline{B}$ channels are not suitable for an investigation of higher Fock state contributions to exclusive J/ψ and ψ' decays. The process $J/\psi \to \pi\pi$, for instance, seems to be more appropriate for this aim. That decay is customarily assumed to be dominated by the $c\bar{c}$ annihilation into a photon and subsequent creation of a light $q\bar{q}$ pair; the strong process with three intermediate gluons cancels to zero if the light quark masses are assumed to be zero. Thus, in this case, higher Fock state contributions, although still suppressed by powers of velocity and inverse c-quark masses, are enhanced by a large factor α_s^p/α_{em}^2 and may, therefore, be large [18].

An interesting class of J/ψ decays are the $B_8\overline{B}_{10}$ channels. While the three-gluon contributions to the $p\overline{\Delta}^-$ and $\Sigma^{*0}\overline{\Lambda}$ channels are strictly zero [38] they are non-zero - although small - for other $B_8\overline{B}_{10}$ channels, since our wave functions exhibit only a mild breaking of $\mathrm{SU(3)_F}$ symmetry. Experimentally, only upper bounds are known for the first two channels [24] saying that these decays are indeed suppressed by at least an order of magnitude as compared to, say, the $p\overline{p}$ or the $\Lambda\overline{\Lambda}$ channels. The experimental widths for the other $B_8\overline{B}_{10}$ channels are surprisingly large [24]. Thus, in accordance with [38], we expect as the dominant $\mathrm{SU(3)_F}$ breaking mechanism for these reactions a sizeable electromagnetic contribution.

Finally, we have to mention that there are a few exclusive charmonium decays which cannot be described within the standard or the modified perturbative approach (in valence quark approximation). Thus, the relatively large branching ratio of the process $J/\psi \to \rho\pi$ observed experimentally, for instance, indicates a substantial violation of hadronic helicity conservation while only mild violations are observed in the baryonic J/ψ decays (see the discussion in Sect. 3). For a discussion of this puzzle and a possible solution of it by means of intrinsic charm of the ρ meson see [39].

Acknowledgements. We thank S.J. Brodsky, J.G. Körner, D. Lichtenberg and N.G. Stefanis for useful discussions and suggestions. We are also grateful to Y. Zhu for comments on the BES data.

[5] As recent developments in the theory of heavy quarkonia have shown [20], corrections to the quark-potential model description of the charmonia can be organized in a systematic expansion in powers of the velocity v

References

1. S.J. Brodsky and G.P. Lepage, Phys. Rev. D **24**, 2848 (1981)
2. V.L. Chernyak, A.A. Ogloblin and I.R. Zhitnitsky, Z. Phys. C **42**, 583 (1989)
3. N.G. Stefanis and M. Bergmann, Phys. Rev. D **47**, 3685 (1993); Phys. Lett. B **304**, 24 (1994)
4. Th. Appelquist and H.D. Politzer, Phys. Rev. Lett. **34**, 43 (1975)
5. J. Bolz, R. Jakob, P. Kroll, M. Bergmann, and N.G. Stefanis, Z. Phys. C **66**, 267 (1995)
6. J. Bolz and P. Kroll, Z. Phys. A **356**, 327 (1996)
7. V.A. Nesterenko and A.V. Radyushkin, Yad. Fiz. **39**, 1287 (1984)
8. J. Botts and G. Sterman, Nucl. Phys. B **325**, 62 (1989); H.-N. Li and G. Sterman, Nucl. Phys. B **381**, 129 (1992)
9. Y. Zhu for the BES coll., talk presented at the XXVIII Int. Conf. on High Energy Physics, 25-31 July 1996, Warsaw, Poland
10. M.G. Sotiropoulos and G. Sterman, Nucl. Phys. B **425**, 489 (1994)
11. Z. Dziembowski, Phys. Rev. D **37**, 768 (1988)

12. G.P. Lepage and S.J. Brodsky, Phys. Rev. D **22**, 2157 (1980)
13. N.G. Stefanis, Acta Phys. Pol. B **25**, 1777 (1994)
14. N.G. Stefanis, Phys. Rev. D **40**, 2305 (1989); D **44**, 1616(E) (1991)
15. S.J. Brodsky, T. Huang and G.P. Lepage, Banff Summer Institute, Particles and Fields 2, p. 143, A.Z. Capri, A.N. Kamal (eds.) (1983)
16. V.L. Chernyak, A.A. Ogloblin and I.R. Zhitnitsky, Z. Phys. C **42**, 569 (1989)
17. G.R. Farrar, H. Zhang, A.A. Ogloblin and I.R. Zhitnitsky, Nucl. Phys. B **311**, 585 (1988/89)
18. J. Bolz, P. Kroll and G.S. Schuler, Phys. Lett. B **392**, 198 (1997) and preprint hep-ph/9704378, to be published in Eur. Phys. J. C
19. R. Barbieri, R. Gatto and E. Remiddi, Phys. Lett. B **106**, 497 (1981)
20. G.T. Bodwin, E. Braaten and G.P. Lepage, Phys. Rev. D **51**, 1125 (1995)
21. W. Buchmüller and S.-H. Tye, Phys. Rev. D **24**, 132 (1981)
22. M. Dahm, R. Jakob and P. Kroll, Z. Phys. C **68**, 595 (1995)
23. J.C. Collins and D.E. Soper, Nucl. Phys. B **193**, 381 (1981)
24. R.M. Barnett et al., Phys. Rev. D **54**, 1 (1996) (Review of Particle Properties)
25. M.W. Eaton et al., MARK2 collaboration, Phys. Rev. D **29**, 804 (1984)
26. D. Pallin et al., DM2 collaboration, Nucl. Phys. B **292**, 653 (1987)
27. P. Henrard et al., DM2 collaboration, Nucl. Phys. B **292**, 670 (1987)
28. A. Antonelli et al., FENICE collaboration, Phys. Lett. B **301**, 317 (1993)
29. R. van Royen and V.F. Weisskopf, Nuovo Cimento **50**, 617 (1967)
30. R. Barbieri, R. Kögerler, Z. Kunszt and R. Gatto, Nucl. Phys. B **105**, 125 (1976)
31. E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane and T.-M. Yan, Phys. Rev. D **17**, 3090 (1978) and D **21**, 203 (1980)
32. M.L. Mangano and A. Petrelli, Phys. Lett. B **352**, 445 (1995)
33. M. Claudson, S.L. Glashow and M.B. Wise, Phys. Rev. D **25**, 1345 (1982)
34. T.A. Armstrong et al., E760 collaboration, Phys. Rev. Lett. **70**, 1212 (1993)
35. A. Antonelli et al., FENICE collaboration, Phys. Lett. B **313**, 283 (1993)
36. R. Jakob, P. Kroll, M. Schürmann and W. Schweiger, Z. Phys. A **347**, 109 (1993)
37. C. Carimalo, Int. J. Mod. Phys. A **2**, 249 (1987)
38. J.G. Körner, Z. Phys. C **33**, 529 (1987)
39. S.J. Brodsky and M. Karliner, Phys. Rev. Lett. **78**, 4682 (1997)