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QCD structure of multiparticle production: quark and gluon jets

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Abstract. The discrete non linear structure of QCD for multiparticle production, with multi-gluon branching, is applied to the study of quark and gluon jets and the results compared to very recent LEP data.

The theory of strong interactions, QCD, in order to make clear predictions on multiparticle production and particle correlations has to solve first two very hard problems. One, is the problem of the need of controlling the non-linear equations of the theory to all orders in the coupling parameter α_s, as one requires, on one hand, to work down to very low virtual parton masses, and, on the other, to keep track of energy-momentum balance. The second problem is the problem of hadronization which is usually avoided by invoking the principle of local parton hadron duality (LPHD), a principle that, strictly speaking, cannot in general be true. For an overall discussion of these matters see [1] and for more recent developments [2-5].

Our strategy here is to concentrate on the abstracted mathematical discrete structure of the QCD branching equations for producing n partons and to solve them exactly. This is to be contrasted with the usual strategy of starting with rigorous equations in energy-momentum space and to look for approximate solutions, to some order in α_s. Our believe is that the relevant features of multiparticle production and particle correlations are a direct consequence of the non-linearity of the theory. A good example is given by KNO scaling [6], first obtained with a non-linear bootstrap equation by Polyakov [7], and shown, in a simple model, to result of the non-Abelian, self-interacting character of the theory [8].

The purpose of the present paper is, on the theoretical side, to give the general equations of gluon jet and quark jet multiparton evolution in SU (N_c), N_c being the number of colours, in the framework of the discrete branching calculus. In previous work, only the simplified $N_c \to \infty$ limit was considered [9]. On the phenomenological side, we present the kind of phenomenology that can be extracted from our equations, in comparison with very recent experimental results on gluon and quark jets [10, 11, 12], with an emphasis on jet energy dependence of multiplicity distributions.

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At a formal level, what controls the ability of a parton to produce another parton (soft gluon) is the strength of the colour charge. When one compares a gluon to a quark the ratio R of the squares of their colour charges, in SU (N_c), is given by

$$R \equiv 2N_c^2/(N_c^2-1)\ . \tag{1}$$

In the large $N_c \to \infty$ limit this number is 2 and essentially this means that regarding parton production a gluon behaves as two quarks (a quark-antiquark pair). We want to give the general description for the situation when a gluon behaves as R quarks, even when R is not an integer.

If a gluon behaves as R quarks, meaning R independent quarks, then the parton distribution correlation functions $f_m, f_1 = \bar{n}, f_2 = \overline{n(n-1)} - \bar{n}^2$, etc, or the normalized cumulants $k_m \equiv f_m/\bar{n}^m$, for gluon and quark jets are very simply related:

$$f_m^g = R f_m^q\ , \tag{2}$$

or, equivalentely,

$$K_m^g = \frac{1}{R^{m-1}} K_m^q\ . \tag{3}$$

In the QCD discrete branching description and in the $N_c \to \infty$, or $R = 2$, limit it is straightforward to find a solution to (2) and (3). Let us start by introducing a single parameter α such that $0 \le \alpha \le 1$. This parameter α is related to α_s and in the small α limit it is proportional to α_s: α_s times integrals in energy and momentum. If $(1-\alpha)$ is the probability, at a given step i, of a quark remaining a quark, and α the probability of emitting a gluon, with $\Sigma P = (1-\alpha) + \alpha = 1$, then, for a 2-quark gluon, there is a probability $(1-\alpha)^2$ to remain a gluon, a probability $2\alpha(1-\alpha)$ to emit an additional gluon, and a probability α^2 to emit two gluons, with $\Sigma P = (1-\alpha)^2 + 2\alpha(1-\alpha) + \alpha^2 = ((1-\alpha)+\alpha)^2 = 1$. The natural generalization to a R-quark gluon is the following: $(1-\alpha)^R$ is the probability of the gluon to remain a gluon, $R\alpha(1-\alpha)$ is the probability to emit one additional gluon, $\frac{1}{2}R(R-1)\alpha^2(1-\alpha)^{R-2}$ the

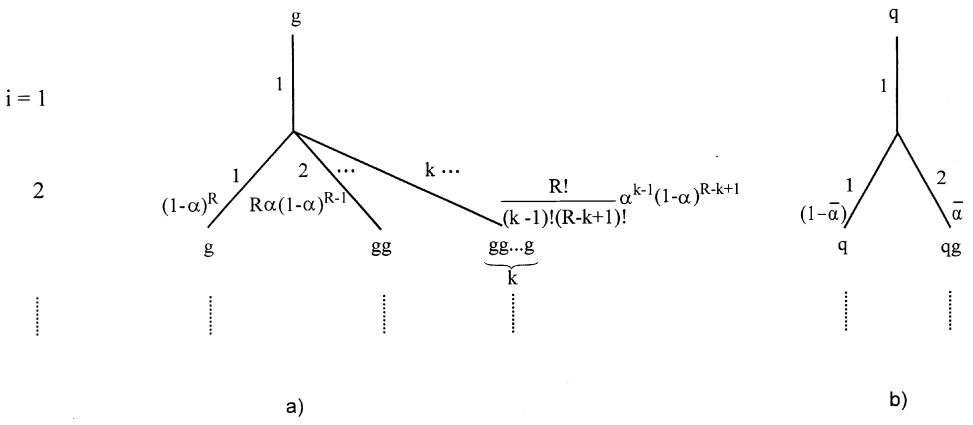

Fig. 1. The probability branching tree in **a** gluon jet and **b** quark jet

probability to emit two gluons, and so on, with $\Sigma P = ((1-\alpha)+\alpha)^R = 1$. We then have, in general, starting with one gluon, for the probability to produce n gluons (see Fig. 1a):

$$P(1g \to ng) = \frac{R!}{(n-1)!(R-n+1)!}\alpha^{n-1} \times (1-\alpha)^{R-n+1} \ , \tag{4}$$

with

$$\sum_{n=1}^{\infty} \frac{R!}{(n-1)!(R-n+1)!}\alpha^{n-1}(1-\alpha)^{R-n+1} \equiv ((1-\alpha)+\alpha)^R = 1 \ . \tag{5}$$

Before discussing the physical meaning of (4) let us check that with (4) and (5) one indeed satisfies relations (2) and (3).

We start by introducing the generating function

$$G(z) = \sum_{\nu=0}^{\infty}(1+z)^{\nu}P_{\nu} \ , \tag{6}$$

where ν is the number of emitted gluons ($n = \nu+1$), such that the normalized factorial moments, $F_m \equiv \overline{\nu(\nu-1)\ldots(\nu-m+1)}/\bar{\nu}^m$, are given by

$$F_m = \frac{1}{\bar{\nu}^m}\frac{d^m G(z)}{dz^m}|_{z=0} \ , \tag{7}$$

and the normalized cumulants K_m by

$$K_m = \frac{1}{\bar{\nu}^m}\frac{d^m \ln G(z)}{dz^m}|_{z=0} \ . \tag{8}$$

In the case of a quark, with a probability $(1-\alpha)$ to remain a quark and a probability α to emit a gluon, one obtains

$$G^q(z) = (1-\alpha)+(1+z)\alpha = 1+z\alpha \ , \tag{9}$$

and,

$$F_m^q = 1, m = 1 \quad ; \ F_m^q = 0, m > 1 \ , \tag{10}$$

$$K_m^q = (-1)^{m-1}\ (m-1)! \ . \tag{11}$$

In the case of a gluon, (4) and (5), by noting that the factorial moments are simply related to partial derivates of (5) with respect to α, one obtains

$$F_m^g = R(R-1)\ldots(R-m+1)/R^m \ , \tag{12}$$

and,

$$K_m^g = (-1)^{m-1}(m-1)!/R^{m-1} \ . \tag{13}$$

It is clear, by comparing (3) and (13), that relations (2) and (3) are exactly satisfied, R being an integer or not. In other words, $G^g(z) = (G^q(z))^R$.

However, there is a price to pay. While (5) is always true, (4) is not always positive (see also (12)). For n =INT (R)+3, R non-integer, (4) is negative and oscillates in sign for larger values of n. With integer R such problem does not occur: for $n \geq R+2$ (4) is identically zero. As we are interested not in the first step in the gluon cascade, from $i = 1$ to $i = 2$, but in the evolution after many steps, i.e., $i \gg 1$, this anomaly in the multiparton distribution is pushed to very large values of n (or very large values of m, in the moments), $n \simeq (\mathrm{INT}(R)+3)^{i-1}$, and will not affect the bulk of the multiparticle distribution. For a prescription to cure this problem in the generation of multiparton distributions see [16].

When comparing our results, in particular (2) and (3), with conventional perturbative QCD calculations it looks as if we are at the level of jet calculus [13] – see also [14] – and of the double logarithmic approximation (DLA), including coherence [15]. However, it is not exactly so. To be more precise, the agreement with jet calculus and DLA exists in the $\alpha \to 0$ limit, when, from step $(i-1)$ to step i one gluon produces at most one additional gluon and multiparton transitions are excluded. In this limit the KNO function is of exponential type. In the same limit, and similarly to DLA results, no changes in sign are observed in the K_m moments (or H_m moments) as functions of m. In general, when multigluon branching is in operation and contrary to DLA calculations, changes in sign occur in our approach, for large enough values of α and $m \geq 3$ [8, 16], and the KNO function is narrower than the exponential function.

If we want to be more realistic, in the next step, similarly to the modified leading logarithmic approximation (MLLA) of perturbative QCD and next to leading-log ap-

proximation [17, 3], we have to take into account phase-space and interference effects. Quark and gluon jets will no longer be simply related by the ratio R of their colour charges. As the proliferation of gluons is strongly restricted by phase-space (energy-momentum conservation) and interference (shadowing) effects we simulate this situation by introducing a parameter $\bar{\alpha}$, $0 \leq \bar{\alpha} \leq 1$, for the quark branching tree (see Fig. 1b), different from the parameter α of gluon branching with, naturally,

$$\alpha \lesssim \bar{\alpha} \tag{14}$$

As energy increases phase space constraints become weaker and α approaches $\bar{\alpha}$ [9].

We are now ready to write the probability $P_i(n)$ of obtaining n partons at level i from k partons at level $(i-1)$, with $k = 1, \ldots, n$.

Gluon jet:

$$P_i^g(n) = \sum_k \sum \frac{k!}{k_1!k_2!\ldots} \times \prod_{\ell=1} \left[\frac{R!}{(R-\ell+1)!(\ell-1)!} (1-\alpha)^{R-\ell+1} \alpha^{\ell-1} \right]^{k_\ell} \times P_{i-1}^g(k) \ , \tag{15}$$

with $k_1 + k_2 + \ldots = k,\ k_1 + 2k_2 + \ldots = n,\ P_1^g(1) = 1$.

Quark jet:

$$\begin{aligned} P_i^q(n) = (1-\bar{\alpha}) & \sum_k \sum \frac{k!}{k_1!k_2!\ldots} \\ & \times \prod_{\ell=1} \left[\frac{R!}{(R-\ell+1)!(\ell-1)!} (1-\alpha)^{R-\ell+1} \alpha^{\ell-1} \right]^{k_\ell} \\ & \times P_{i-1}^q(k) \\ & \{k_1 + k_2 + \ldots = k-1,\ k_1 + 2k_2 + \ldots = n-1\} \\ & + \bar{\alpha} \sum_k \sum \frac{k!}{k_1!k_2!\ldots} \\ & \times \prod_{\ell=1} \left[\frac{R!}{(R-\ell+1)!(\ell-1)!} (1-\alpha)^{R-\ell+1} \alpha^{\ell-1} \right]^{k_\ell} \\ & \times P_{i-1}^q(k), \\ & \{k_1 + k_2 + \ldots = k-1,\ k_1 + 2k_2 + \ldots = n-2\} \ , \end{aligned} \tag{16}$$

where n is the number of partons, $(n-1)$ gluons and 1 quark, and $P_1^q(1) = 1$.

From (15) and (16) we obtain the ratio r of the average multiplicities for gluon and quark jets,

$$\bar{n}^g/\bar{n}^q \equiv r = R\alpha/\bar{\alpha}, \tag{17}$$

and the squares of the widths of the respective KNO distributions,

$$\Delta_g^2 \equiv D^2/\bar{n}^2|_g = \frac{1-\alpha}{1+R\alpha}, \tag{18}$$

$$\Delta_q^2 \equiv D^2/\bar{n}^2|_q = \frac{(r-R\alpha)(1+R\alpha)+r(1-\alpha)}{(2+R\alpha)(1+R\alpha)}, \tag{19}$$

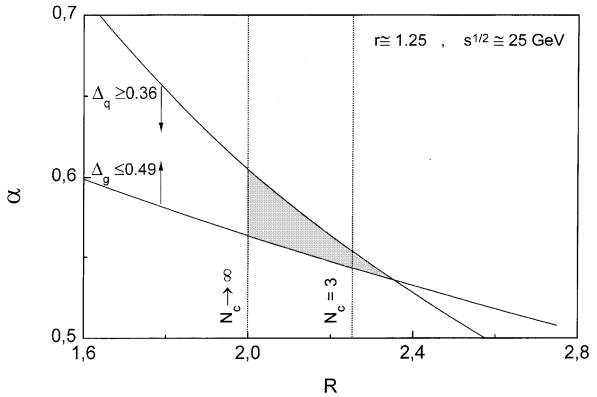

Fig. 2. Test of the model, (9), (10) and (11), in comparison with experimental data and bounds on $r, D/\bar{n}|_g \equiv \Delta_g$ and $D/\bar{n}|_q \equiv \Delta_q$ [10]. The region of validity is the dark one

where $D^2 \equiv \bar{n^2} - \bar{n}^2 = f_2 + \bar{n}$.

In the DLA-like limit, $\bar{\alpha} = \alpha$ and

$$\bar{n}^g = R\bar{n}^q \ , \tag{20}$$

$$\Delta_g^2 = \frac{1}{R}\Delta_q^2 \ . \tag{21}$$

in agreement with (2) and (3). In general, $\bar{\alpha} > \alpha$ and, from (17), (18) and (19),

$$\bar{n}^g < R\bar{n}^q \ , \tag{22}$$

$$\Delta_g^2 > \frac{1}{R}\Delta_q^2 \ . \tag{23}$$

As $R = 9/4 > 1$, what relations (22) and (23) tell us, concerning average multiplicity and KNO distribution, is that gluon jets and quark jets are more similar to each other than what one would have expected from the asymptotic relations (20) and (21). Result (21) has been known for sometime [1], result (23) was very recently experimentally confirmed [10]. In a preliminary presentation of our work [18] we gave estimates of Δ_g^2 and Δ_q^2 which are in agreement with [10].

We turn next to data, and to the recent DELPHI collaboration results on gluon and quark jet multiparticle distributions, from the analysis of $q\bar{q}g$ Y and Mercedes events [10]. From that data, reasonable, conservative estimates for r, Δ_q and Δ_g are, at $\sqrt{s} \simeq 25$ GeV, $r \simeq 1.25$, Δ_g and Δ_q in the range $0.36 \leq \Delta_q,\ \Delta_g \leq 0.49$. In Fig. 2, using (17), (18) and (19), and the values for r and the bounds just quoted we obtained a constrained region of validity of the model in the α, R plane. $R = 2$ was taken as the lowest acceptable value, corresponding to $N_c \to \infty$. The value $R = 9/4$ falls into the region of validity. If one accepts LPHD, what is remarkable is to see how models can already be so much constrained.

Concerning the energy dependence of r and of the KNO distribution, from experiment [10, 11, 12], it is becoming clear that:

i) The quantity r is rapidely increasing with energy ($r \simeq 1$ at $\sqrt{s} \simeq 10$ GeV, $r = 1.22 \pm 0.01$ at 23.5 GeV,

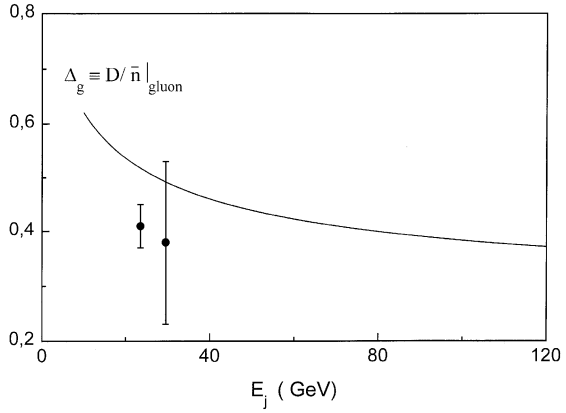

Fig. 3. Predicted behaviour for the energy dependence of the width of the gluon jet KNO distribution, $\Delta_g \equiv D/\bar{n}|_g$. It decreases with energy and is not approaching the asymptotic DLA limit of $\Delta_g = 0.58$. Data points are from [10]

$r = 1.30 \pm 0.03$ at 29.5 GeV, $r = 1.552 \pm 0.041 \pm 0.06$ at 39.2 GeV);

ii) The ratio $D^2/\bar{n}^2 \equiv \Delta^2_{e^+e^-}$ in e^+e^-, in aggreement with KNO scaling, is quite independent of energy, in the $\sqrt{s} \simeq 10 - 160$ GeV region. Note that $\Delta^2_{e^+e^-} \simeq 1/2\Delta^2_q$.

In order to understand simultaneously i) and ii) in our approach it is clear, see (17) and (19), that α is rising with energy. In fact, for r to increase with energy, (15), either α increases or $\bar{\alpha}$ decreases with energy, or both. On the other hand, from (19) $\Delta^2_q \equiv D^2/\bar{n}^2|_q$ is an increasing function of r and a decreasing function of α. Then, in order to have $\Delta^2_q \simeq const.$ with energy one necessarily requires α to increase.

This implies that, at present energies, it is not the running of α_s that directly controls multiparticle production (it should make α to decrease), but rather the vanishing of the explicit energy dependent corrections, higher powers of α_s, that make $\alpha < \bar{\alpha}$ at low energy. The quantity α is approaching $\bar{\alpha}$, while $\bar{\alpha}$ remains almost constant, the final decrease required by asymptotic freedom being far away.

As, by making use of the experimental information i) and ii), the parameter α is increasing with energy, from (18) one sees that the gluon jet KNO distribution does not satisfy scaling: it shrinks with energy. This is the contrary of what expected in the approach to DLA asymptopia.

In order to make a rough estimate of the energy dependence of $\Delta_g \equiv D/\bar{n}|_{gluon}$ we have parameterized the energy dependence of r in the form

$$r = R/(1 + 50/E_j)^{0.5}, \tag{24}$$

E_j being the jet energy in GeV, and used the experimental constraint $2\Delta^2_{e^+e^-} \simeq \Delta^2_q \simeq 0.193 = const.$ [11] in the $10 < E_j < 100$ GeV region. The predicted energy dependence of the width of the KNO distribution Δ_g is shown in Fig. 3.

It is clear that the experimental points tend to fall bellow the curve. We can find several arguments to justify it. On one hand rescale factors may be present, even within LPHD, as we are dealing with charged hadrons not charged + neutral hadrons, on the other hand, we have not yet included in our approach quark-antiquark pair production which may affect results, in particular at low energy.

In any case our main point, and that is independent of previous remarks, is that consistency of data, i) and ii), with our equations, (17), (18) and (19), requires Δ_g to decrease with energy, at least in the range $10 \lesssim E_j \lesssim 100$ GeV, and not to increase and to approach the DLA asymptotic limit $\Delta_g = 0.58$. It would be important to measure $\Delta_g \equiv D/\bar{n}|_{gluon}$ at higher energy, at LEP and in hadron colliders, in order to understand better the low energy constraints imposed on multigluon branching.

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