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Mesonic parton densities derived from constituent quark model constraints

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Abstract. Using constituent quark model constraints we calculate the gluon and sea content of pions solely in terms of their valence density and the known sea and gluon densities of the nucleon. The resulting small-x predictions for $g^\pi(x,Q^2)$ and $\bar{q}^\pi(x,Q^2)$ are unique and parameter free, being entirely due to QCD dynamics. Similar ideas are applied for calculating the gluon and sea content of kaons which, for our suggested choice of the kaon's valence densities, turn out to be identical to the ones of the pion.

1 Introduction

The parton content of the mesons, π, K, ρ, ..., is not well known due to the scarce experimental information solely from Drell-Yan dilepton production processes as compared to the rich and accurate data which exist for the nucleon from various different reactions. One can try to improve the situation by relating the (rather) well known nucleonic parton distributions to the poorly known mesonic ones utilizing a plausible constituent quark description of the hadrons in which the partons are considered as *universal* parts of the constituent quarks [1-3]. This model was applied recently [4] to predict the pion structure from the known nucleon structure functions utilizing the constituent wave functions in [1].

As noted in [4], the choice of the constituent wave functions introduces some ambiguity in the prediction of the pion structure functions from those of the proton. We shall therefore employ a slightly different approach which eliminates the dependence on the constituent wave functions without sacrificing completely the predictive power of the model. In Sect. 2 we apply our approach to the pion in leading order (LO) and next-to-leading order (NLO) of QCD, while Sect. 3 is devoted to the K meson.

2 The pion structure

Following [1,4], the constituent building blocks of the nucleon and the pion will be denoted by U and D, i.e. $p = UUD$, $\pi^+ = U\bar{D}$, etc., and their distributions within the proton [pion] will be denoted by $U^p(x)$ [$U^\pi(x)$], etc., which are scale (Q^2) independent. Their *universal* (i.e., hadron independent) partonic content will be denoted by $v_c(x,Q^2)$, $g_c(x,Q^2)$, and $\bar{q}_c(x,Q^2)$ representing the valence, gluon, and sea components of the constituent U^h, D^h distributions, respectively. The usual parton content of the proton is then given by

$$u_v^p = U^p \otimes v_c \tag{1a}$$
$$d_v^p = D^p \otimes v_c \tag{1b}$$
$$\bar{q}^p = (U^p + D^p) \otimes \bar{q}_c \tag{1c}$$
$$g^p = (U^p + D^p) \otimes g_c \tag{1d}$$

where $u_v^p \equiv u^p - \bar{u}^p$ and $d_v^p \equiv d^p - \bar{d}^p$ are the valence quark densities, $\bar{q}^p = \left(\bar{u}^p + \bar{d}^p\right)/2$, and $\otimes$ denotes the usual convolution which becomes a simple product for the corresponding Mellin n-moments, henceforth utilized in our discussion and for our explicit calculations. We rewrite these equations in Mellin n-moment space as follows:

$$v^p \equiv u_v^p + d_v^p = (U^p + D^p)\, v_c \tag{2a}$$
$$\bar{q}^p = (U^p + D^p)\, \bar{q}_c \tag{2b}$$
$$g^p = (U^p + D^p)\, g_c \tag{2c}$$

with $v^p = v^p(n,Q^2) \equiv \int_0^1 x^{n-1} \left[u_v^p(x,Q^2) + d_v^p(x,Q^2)\right] dx$, $v_c = v_c(n,Q^2) \equiv \int_0^1 x^{n-1} v_c(x,Q^2) dx$, etc., and where we omit the obvious n and Q^2 dependence in $v^p(n,Q^2)$, $v_c(n,Q^2)$, etc., whenever possible. Similarly we obtain for the pion [1]

$$v^\pi \equiv u_v^{\pi^+} + \bar{d}_v^{\pi^+} = \left(U^{\pi^+} + \bar{D}^{\pi^+}\right) v_c \tag{3a}$$
$$\bar{q}^\pi = \left(U^{\pi^+} + \bar{D}^{\pi^+}\right) \bar{q}_c \tag{3b}$$
$$g^\pi = \left(U^{\pi^+} + \bar{D}^{\pi^+}\right) g_c \tag{3c}$$

[1] It should be recalled that $u_v^{\pi^+} = \bar{d}_v^{\pi^+} = \bar{u}_v^{\pi^-} = d_v^{\pi^-}$, $\bar{u}^{\pi^+} = d^{\pi^+} = u^{\pi^-} = \bar{d}^{\pi^-}$ and $q^{\pi^0} = (q^{\pi^+} + q^{\pi^-})/2$. Similarly, $u_v^{K^+} = \bar{u}_v^{K^-}$, $\bar{s}_v^{K^+} = s_v^{K^-}$ and $\bar{u}^{K^+} = u^{K^-} = d^{K^\pm} = \bar{d}^{K^\pm}$

with $\bar{q}^\pi = \left(\bar{u}^{\pi^+} + d^{\pi^+}\right)/2$ and $\bar{u}^{\pi^+} = d^{\pi^+}$ due to the common neglect of $\mathrm{SU}(2)_{flavor}$ breaking effects in the π. The above equations are *conceived to apply at the low resolution scale* $Q^2 = \mu^2$ ($\mu^2_{LO} = 0.23\,\mathrm{GeV}^2$, $\mu^2_{NLO} = 0.34\,\mathrm{GeV}^2$) of [5] where the strange quark distribution is considered to be negligible, i.e.,

$$s^p(x,\mu^2) = \bar{s}^p(x,\mu^2) = 0 \quad . \tag{4}$$

We shall adopt the same approximation also for the pion, i.e.,

$$s^\pi(x,\mu^2) = \bar{s}^\pi(x,\mu^2) = 0 \quad . \tag{5}$$

Note that in contrast to our previous analysis [6] of the pion structure, we now start with a *non*-vanishing sea $\bar{q}^\pi$ at $Q^2 = \mu^2$ as follows from (2b) and (3b).

It is easily seen that (2) and (3) yield the following *wave function independent* relations

$$\frac{v^\pi}{v^p} = \frac{\bar{q}^\pi}{\bar{q}^p} = \frac{g^\pi}{g^p} \tag{6}$$

which, together with (5), fix the pion structure in terms of the proton structure as soon as v^π is reasonably well determined:

$$g^\pi = \frac{v^\pi}{v^p}\, g^p \quad , \quad \bar{q}^\pi = \frac{v^\pi}{v^p}\, \bar{q}^p \quad . \tag{7}$$

These n-moment relations are our basic predictions for the gluon and sea densities of the pion at the input scale $Q^2 = \mu^2$. The required LO and NLO input densities of the proton are taken from [5], with $\bar{q}^p$ referring to the average of the $\bar{u}$ and $\bar{d}$ sea densities of [5], i.e., $\bar{q}^p = \left(\bar{u}^p + \bar{d}^p\right)/2$. Furthermore the sum rules

$$\int_0^1 v^\pi(x,Q^2)\,dx = 2 \tag{8}$$

$$\int_0^1 x v^\pi(x,Q^2)\,dx = \int_0^1 x v^p(x,Q^2)\,dx \tag{9}$$

impose strong constraints on $v^\pi(x,\mu^2)$ which are very useful for its almost unambiguous determination from experimental Drell-Yan data in πN collisions. Independent analyses of the valence structure of protons and pions within the framework of the radiative parton model [5,6] suggest that the valence quarks in the proton and the pion carry similar total fractional momentum, as implied by (9). In practice we can therefore utilize the $v^\pi(x,\mu^2)$ of [6] slightly modified so as to comply with the new constraint in (9). This yields

$$v^\pi_{LO}(x,\mu^2_{LO}) = 0.942\, x^{-0.501}\left(1 + 0.632\sqrt{x}\right) \times (1-x)^{0.367} \tag{10}$$

$$v^\pi_{NLO}(x,\mu^2_{NLO}) = 1.052\, x^{-0.495}\left(1 + 0.357\sqrt{x}\right) \times (1-x)^{0.365} \quad . \tag{11}$$

The total momentum fractions carried by these LO and NLO input valence densities are given by

$$\int_0^1 x v^\pi_{LO}(x,\mu^2_{LO})\,dx = 0.603 \quad ,$$

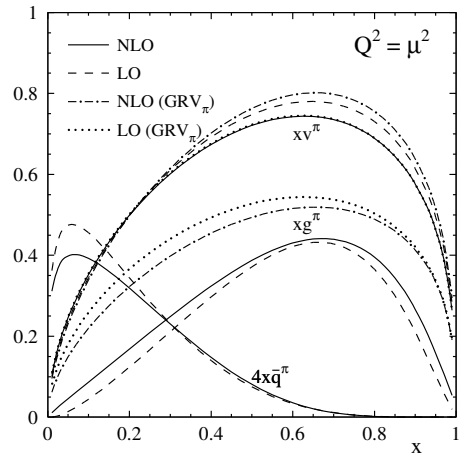

Fig. 1. The valence-like input distributions $xf^\pi(x,Q^2=\mu^2)$ with $f = v, \bar{q}, g$ as compared to those of [6] denoted by GRV_π. Note that GRV_π employs a vanishing $\mathrm{SU}(3)_{flavor}$ symmetric $\bar{q}^\pi$ input. Our present $\mathrm{SU}(3)_{flavor}$ broken sea densities refer to a vanishing s^π input in (5), similarly to GRV_π [6]

$$\int_0^1 x v^\pi_{NLO}(x,\mu^2_{NLO})\,dx = 0.582 \tag{12}$$

which coincide, as they should, with the ones of the proton [5], cf. (9).

Having completely fixed the input for g^π and $\bar{q}^\pi$ in (7), we perform the LO and NLO evolutions of $g^\pi(n,Q^2)$ and $\bar{q}^\pi(n,Q^2)$ to $Q^2 > \mu^2$ in Mellin n-moment space, followed by a straightforward numerical Mellin-inversion [7] to Bjorken-x space for obtaining $g^\pi(x,Q^2)$ and $\bar{q}^\pi(x,Q^2)$. The same is done for s^π, starting from the vanishing input in (5). It should be noted that the evolutions are always performed in the fixed (light) $f = 3$ flavor factorization scheme [5,8], i.e., we refrain from generating radiatively massless 'heavy' quark densities $h(x,Q^2)$ where $h = c, b$, etc. Hence heavy quark contributions have to be calculated in fixed-order perturbation theory via, e.g., $g^\pi g^p \to h\bar{h}$, $\bar{u}^\pi u^p \to h\bar{h}$, etc.

In Fig. 1 we compare our present LO and NLO input parton distributions at $Q^2 = \mu^2$ with those of [6], while Fig. 2 shows our resulting predictions for various larger fixed values of Q^2 as compared again to our [6] former results denoted by GRV_π. In contrast to our former [6] $\mathrm{SU}(3)_{flavor}$ symmetric sea $\bar{q}^\pi$, the present one is merely $\mathrm{SU}(2)_{flavor}$ symmetric and $\bar{q}^\pi$ refers now to the quantity in (3b) while $s^\pi = \bar{s}^\pi$ is not shown in the figure since it practically coincides with our previous GRV_π [6] $\bar{q}^\pi$ for the following reasons: Our unique parameter free small-x ($x \lesssim 10^{-2}$) predictions for xg^π and $x\bar{q}^\pi$ at $Q^2 > \mu^2$ in Fig. 2 are entirely due to QCD dynamics since they are radiatively generated from the valence-like input densities at $Q^2 = \mu^2$ which vanish as $x \to 0$. Thus the results for g^π and $\bar{q}^\pi$ almost coincide with the ones of [6] except for $x\bar{q}^\pi$ at $x \gtrsim 10^{-2}$ which has been generated from a vanishing input [6], in contrast to the present analysis. Furthermore our predictions for s^π, resulting also from the vanishing input in (5), almost coincide with our previous GRV_π $\bar{q}^\pi$ shown in Fig. 2.

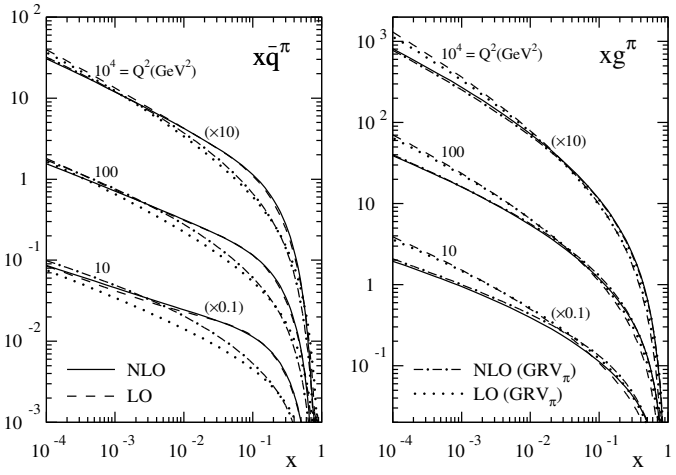

Fig. 2. The radiatively generated pionic gluon and sea-quark distributions at various fixed values of Q^2 as compared to those of [6] denoted by GRV_π. The predictions for the strange sea density $s^\pi = \bar{s}^\pi$ are similar to the LO and NLO GRV_π results for $\bar{q}^\pi$

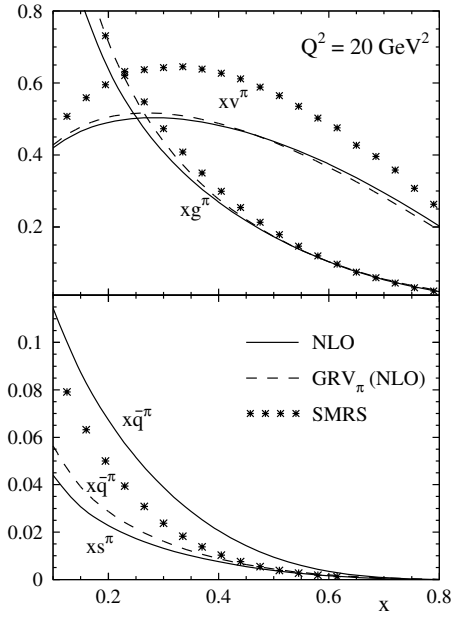

Fig. 3. A detailed comparison of NLO pionic parton distributions at $Q^2 = 20\,\mathrm{GeV}^2$. The stars (SMRS) refer to the distributions of [9]. It should be noted that our present $\bar{q}^\pi \equiv \bar{u}^{\pi^+} = d^{\pi^+}$ (*solid line*), while the GRV_π and SMRS $\bar{q}^\pi \equiv \bar{u}^{\pi^+} = d^{\pi^+} = s^\pi = \bar{s}^\pi$

In Fig. 3 we present a more detailed comparison of our present (solid lines) and previous NLO [6] results at a specific value of $Q^2 = 20\,\mathrm{GeV}^2$ where we also show the corresponding distributions of [9]. Finally our predictions are confronted in Fig. 4 with a representative sample of the experimental data [10,9] from the Drell-Yan process ($\pi^- W$ reactions) as also done in [9,4]. It should be noted that our NLO K-factors, i.e. K', are similar to the ones obtained in [4,9]. The relevant NLO differential Drell-Yan

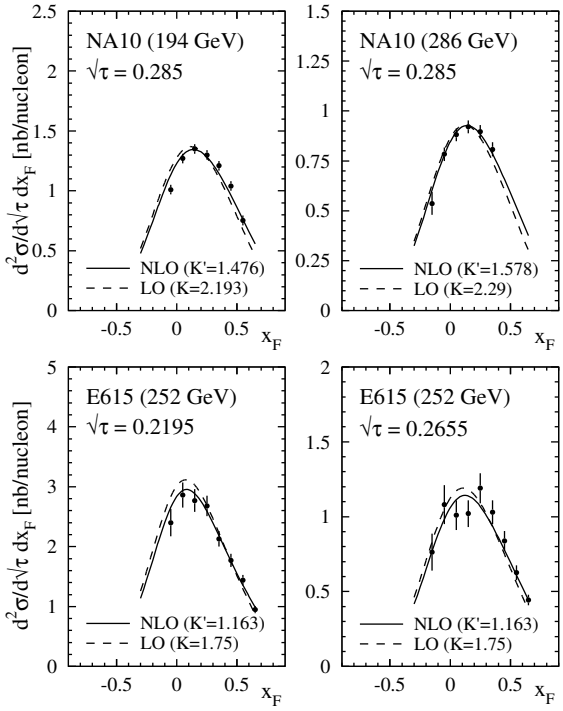

Fig. 4. A comparison of a sample of experimental Drell-Yan data ($\pi^- W$ reactions) [10,9] with LO and NLO predictions based on the present pion distributions together with the nucleon distributions of [5]

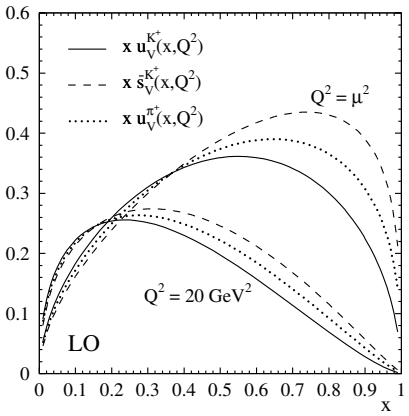

Fig. 5. The LO valence distributions of the K^+ meson at the input scale $Q^2 = \mu^2$ and at $Q^2 = 20\,\mathrm{GeV}^2$ as compared to the corresponding valence distribution of the pion

cross section $d^2\sigma/d\sqrt{\tau}dx_F$ has been presented in the Appendix of [9] except for (A8) which has to be modified [11, 12] in order to conform with the usual $\overline{\mathrm{MS}}$ convention for the number of gluon polarization states $2(1-\epsilon)$ in $4-2\epsilon$ dimensions.

3 The kaon structure

Here (3a)–(3c) are obviously replaced by [1]

$$v^K \equiv u_v^{K^+} + \bar{s}_v^{K^+} = \left(U^{K^+} + \bar{S}^{K^+}\right) v_c \qquad (13a)$$

$$\bar{q}^K = \left(U^{K^+} + \bar{S}^{K^+}\right) \bar{q}_c \tag{13b}$$

$$g^K = \left(U^{K^+} + \bar{S}^{K^+}\right) g_c \tag{13c}$$

where as in the case of the pion, $\bar{q}^K \equiv \bar{u}^{K^+} = d^{K^+} = \bar{d}^{K^+}$, and for the strange sea input we take again

$$s^{K^+} = \bar{s}^{K^-} = 0 \quad . \tag{14}$$

They yield together with (2a)–(2c)

$$\frac{v^K}{v^p} = \frac{\bar{u}^{K^+}}{\bar{u}^p} = \frac{g^K}{g^p} \tag{15}$$

just as for the corresponding pion-proton relations in (6). Thus our basic predictions for the gluon and sea content of kaons at the input scale $Q^2 = \mu^2$ in Mellin n-moment space are

$$g^K = \frac{v^K}{v^p}\, g^p \;\;,\;\; \bar{q}^K = \frac{v^K}{v^p}\, \bar{q}^p \tag{16}$$

which is analogous to (7). Taking again the input parton densities of the proton from [5], only the total valence density of the kaon, $v^K \equiv u_v^{K^+} + \bar{s}_v^{K^+}$, remains to be fixed. In contrast to the pion, the constituent quarks have now different masses, i.e., $M_s > M_u$ so that the valence distribution $\bar{s}_v^{K^+}$ is expected to *differ* from $u_v^{K^+}$ in being somewhat harder: $\bar{s}_v^{K^+} > u_v^{K^+}$ as $x \to 1$, i.e., the heavier $\bar{s}$ in K^+ should carry more momentum than the lighter u (d). Unfortunately, the details of this difference are not yet explored experimentally nor reliably predictable theoretically [13-15]. The only experimental information available concerns $u_v^{K^+}$ which derives from the Drell-Yan process $K^- p \to \mu^+\mu^- X$ at $4.1\,\mathrm{GeV} \le M_{\mu^+\mu^-} \le 8.5\,\mathrm{GeV}$ [16]. It indicates that $u_v^{K^+} < u_v^{\pi^+}$ for $x > 0.6$ or $u_v^{K^+}/u_v^{\pi^+} \to 1/2$ as $x \to 1$ at $\langle Q^2 \rangle = 20 - 40\,\mathrm{GeV}^2$. This requirement can be easily accounted for by the ansatz $u_v^{K^+}(x,\mu^2) = N_u(1-x)^\kappa v^\pi(x,\mu^2)$ with κ being fitted to the (scarce) NA3 data [16] and N_u follows from

$$\int_0^1 u_v^{K^+}(x,Q^2)\,dx = \int_0^1 \bar{s}_v^{K^+}(x,Q^2)\,dx = 1 \quad . \tag{17}$$

In analogy to (9), we have furthermore

$$\int_0^1 x v^K(x,Q^2)\,dx = \int_0^1 x v^p(x,Q^2)\,dx = \int_0^1 x v^\pi(x,Q^2)\,dx \quad . \tag{18}$$

In view of the absence of any experimental information about $\bar{s}^{K^+}(x,Q^2)$, we take as our input

$$\bar{s}_v^{K^+}(x,\mu^2) = v^\pi(x,\mu^2) - u_v^{K^+}(x,\mu^2) \tag{19}$$

which satisfies trivially the expectation discussed above as well as the sum rules (17) and (18). Fitting now our ansatz for $u_v^{K^+}$ to the NA3 data [16] yields

$$u_{v,LO}^{K^+}(x,\mu_{LO}^2) = 0.541\,(1-x)^{0.17} v_{LO}^\pi(x,\mu_{LO}^2) \tag{20}$$

$$u_{v,NLO}^{K^+}(x,\mu_{NLO}^2) = 0.540\,(1-x)^{0.17} v_{NLO}^\pi(x,\mu_{NLO}^2) \quad . \tag{21}$$

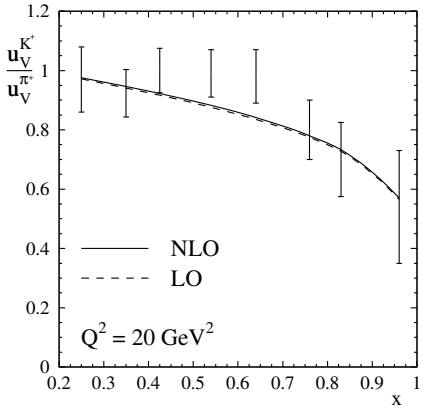

Fig. 6. The ratio of the up-quark valence distributions of the K^+ and π^+ mesons as compared with the corresponding experimental NA3 Drell-Yan data [16]

The total momentum fractions carried by these LO and NLO light valence input densities are

$$\int_0^1 x u_{v,LO}^{K^+}(x,\mu_{LO}^2)\,dx = 0.276 \;\;,$$

$$\int_0^1 x u_{v,NLO}^{K^+}(x,\mu_{NLO}^2)\,dx = 0.267 \quad . \tag{22}$$

Therefore the total momentum fractions carried by the heavy strange input densities are, according to (19) and (12),

$$\int_0^1 x \bar{s}_{v,LO}^{K^+}(x,\mu_{LO}^2)\,dx = 0.327 \;\;,$$

$$\int_0^1 x \bar{s}_{v,NLO}^{K^+}(x,\mu_{NLO}^2)\,dx = 0.315 \quad . \tag{23}$$

These LO input valence densities as well as the ones evolved to $Q^2 = 20\,\mathrm{GeV}^2$ are shown in Fig. 5, and $u_v^{K^+}/u_v^{\pi^+}$ at $Q^2 = 20\,\mathrm{GeV}^2$ is compared with the NA3 data [16] in Fig. 6. The NLO valence densities are very similar to the LO ones shown in Fig. 5. Our expectations are similar to the ones derived from non-relativistic bound state (potential) and Nambu-Jona-Lasino-type models [13-15]. Due to our ansatz (19), i.e., $v^{K^+} \equiv u_v^{K^+} + \bar{s}_v^{K^+} = v^{\pi^+}$, our predictions (16) for $g^K(x,Q^2)$ and $\bar{q}^K(x,Q^2)$ become identical to the ones for the pion in (7), i.e., $g^K = g^\pi$ and $\bar{q}^K = \bar{q}^\pi$, which are shown in Figs. 1–3. It is thus obvious that also $s^{K^+} = s^\pi$.

4 Discussion

The present approach to the constituent quark model replaces the previous [4] reliance on theoretically inferred

constituent wave functions [1,4] by experimentally extracted mesonic valence quark distributions. It is shown that these distributions together with the rather well known nucleon parton distributions provide all the required information for predicting the gluon and sea distributions within a meson. A confrontation with available experimental data supports the basic correctness of the underlying constituent quark model and of our approach to its practical, wave-function independent, implementation. Future improvements in our knowledge of the nucleon and meson parton distributions are required to test how reliable and correct the constituent quark model actually is.

A FORTRAN package containing our LO and NLO($\overline{\rm MS}$) pion densities can be obtained by electronic mail on request.

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References

1. G. Altarelli, N. Cabibbo, L. Maiani, R. Petronzio, Nucl. Phys. **B69** (1974) 531
2. A. Le Yaouanc, L. Oliver, O. Pene, J.C. Raynal, Phys. Rev. **D12** (1975) 2137; Erratum **D13** (1976) 1519
3. R.C. Hwa, Phys. Rev. **D22** (1980) 1593
4. G. Altarelli, S. Petrarca, F. Rapuano, Phys. Lett. **B373** (1996) 200
5. M. Glück, E. Reya, A. Vogt, Z. Phys. **C67** (1995) 433
6. M. Glück, E. Reya, A. Vogt, Z. Phys. **C53** (1992) 651
7. M. Glück, E. Reya, A. Vogt, Z. Phys. **C48** (1990) 471
8. M. Glück, E. Reya, M. Stratmann, Nucl. Phys. **B422** (1994) 37
9. P.J. Sutton, A.D. Martin, W.J. Stirling, R.G. Roberts, Phys. Rev. **D45** (1992) 2349
10. NA10 collab., B. Betev et al., Z. Phys. **C28** (1985) 9; NA10 collab., P. Bordalo et al., Phys. Lett. **B193** (1987) 368; K. Freudenreich, Int. J. Mod. Phys. **A5** (1990) 3643; E615 collab., J.S. Conway et al., Phys. Rev. **D39** (1989) 92
11. W. Furmanski, R. Petronzio, Z. Phys. **C11** (1982) 293, and references therein
12. M. Glück, E. Reya, A. Vogt, Phys. Lett. **B285** (1992) 285
13. F. Martin et al., XVth Rencontre de Moriond, Les Arcs, 1980, ed. Tran Thanh Van, Edition Frontieres, p. 487
14. T. Shigetani, K. Suzuki, H. Toki, Phys. Lett. **B308** (1993) 383
15. J.T. Londergan, G.Q. Liu, A.W. Thomas, Phys. Lett. **B380** (1996) 393
16. NA3 collab., J. Badier et al., Phys. Lett. **93B** (1980) 354; K. Freudenreich, Int. J. Mod. Phys. **A5** (1990) 3643.