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Measuring the interface tension when the electroweak phase transition becomes weak

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Abstract. We measure the interface tension near the phase transition endpoint of the $3d$ $SU(2)$–Higgs model. The tunnel correlation length method is used and compared to other approaches. A modified scaling behaviour for the mass gap as function of the transverse area is proposed.

1 Introduction

It is now almost established that the symmetry restoring electroweak phase transition changes into a smooth crossover if the mass of the so far elusive Higgs particle would be equal to the W-mass (or less within 10 per cent) [1–4]. This emerges from lattice studies in the effective 3-dimensional model as well as from 4-dimensional simulations, the latter so far only with a rough lattice spacing of order $O(1/(2T))$ with temperature T.

In order to quantify the strength of the first order transition near to the critical Higgs mass several quantities have been considered. Measuring the discontinuity of the Higgs condensate is practically tantamount (in three dimensions) to calculating the latent heat. Both quantities are easy to obtain but are very sensitive to finite volume effects. It is even more difficult to measure the interface tension for weak transitions. This we have observed in our recent work [5] where we have estimated the interface tension for a Higgs mass near to its critical value.

In the literature mainly three methods are discussed to extract the interface tension from Monte Carlo studies. In the present work we shall employ and critically discuss the tunneling correlation length method [6,7], which has been used so far only in the analysis of 4–dimensional simulations [8,9] in the context of the electroweak phase transition.

In Sect. 2 we define the model and discuss the methods used to extract the interface tension. The results obtained by the tunneling correlation method are presented and critically discussed in Sect. 3. They are put into perspective with all available lattice measurements for arbitrary Higgs masses in Sect. 4, which contains also our conclusion on the capability of the method.

2 The model and how to measure the interface tension

The model under study is a 3–dimensional $SU(2)$–Higgs system with one complex Higgs doublet of variable modulus. The gauge fields are represented by unitary 2×2 link matrices $U_{x,\alpha}$ and the Higgs fields are written as $\Phi_x = \rho_x V_x$. $\rho_x^2 = \frac{1}{2}tr(\Phi_x^+\Phi_x)$ is the Higgs modulus squared, V_x an element of the group $SU(2)$, U_p denotes the $SU(2)$ plaquette matrix. The lattice action is

$$S = \beta_G \sum_p \left(1 - \frac{1}{2}\mathrm{tr}U_p\right) - \beta_H \sum_{l=(x,\alpha)} \frac{1}{2}tr(\Phi_x^+ U_{x,\alpha}\Phi_{x+\alpha}) + \sum_x \left(\rho_x^2 + \beta_R(\rho_x^2-1)^2\right) \quad (1)$$

(summed over plaquettes p, links l and sites x), with the gauge coupling β_G, the lattice Higgs self–coupling β_R and the hopping parameter β_H. To relate the lattice couplings to continuum quantities the notations of [5] are used.

Let us recall the procedures used for extracting the interface tension of the electroweak phase transition. The first one, the two–coupling method, requires a careful extrapolation to the limits $\beta_{Hc} \pm \varepsilon$ of simulations in two coupled subvolumes [9–12] supposed to be kept in the two phases. The variants differ in the way this limit is taken. In our version of the method [11] we have used a simultaneous multihistogram technique for a system consisting of two parts in order to estimate the free energy difference between the homogeneous and mixed states at β_{Hc}.

The majority of results for the interface tension has been obtained with the second method based on the two–state signal in the histogram of an order parameter like quantity o [10,13,5,3]. Here the whole system is simulated at the pseudocritical point. The bulk variable under consideration is, say, the volume average of the modulus

squared $o = \overline{\frac{1}{2}tr(\Phi^+\Phi)}$ or the average link $o = \overline{\frac{1}{2}tr(\Phi^+U\Phi)}$. Usually, the interface tension α is estimated comparing the minimum and the two maxima of the doubly peaked histogram $p(o)$ by

$$\alpha a^2/T_c = 1/\left(2L^2\right)\log\left(p_{max}/p_{min}\right) \quad , \tag{2}$$

where L is a typical linear extension of a surface eventually dividing the system into different phases (the smaller extensions of a cylindrical system), a denotes the lattice spacing and T_c the corresponding transition temperature. Different entropy factors in the thermodynamic weight of the mixed state have been ignored for simplicity in (2), but are necessary to extract a reasonable estimate for the interface tension, in particular if histogram data of lattices with various geometries are used simultaneously [14]. Applying a formula like (2) one tacitly assumes that (*i*) the order parameter o is well–chosen in order to yield a histogram with clearly separated maxima and a broad minimum in between, (*ii*) the case of equal height of the maxima is near to the phase equilibrium, (*iii*) a minimal surface spanning through the lattice separates the pure phases from each other.

Lattice studies of the electroweak transition in the case of the Standard Model for realistic values of the Higgs mass are known to possess a transition which is very asymmetric and weak. Therefore the conditions (*i*) to (*iii*) are hardly fulfilled. By "asymmetric" we mean that the fluctuations of suitable order parameters o are stronger in one (the Higgs) phase than in the other (symmetric) phase. This fact makes it more and more difficult to use histogram methods based on (2) when approaching the critical Higgs mass.

In order to cope with overlapping histograms we have recently proposed a method to split histograms $p(o)$ into pure phase and mixed phase contributions [5] even under realistic circumstances of a very weak transition. This has enabled us to employ the histogram reweighting technique to find the pseudocritical coupling by the equal weight criterion. As a by–product, we have also obtained the latent heat from the variation of both pure phase thermodynamic weights near to the transition point and the thermodynamic weight of mixed states. Extracting, however, the interface tension from the relative weights of pure and mixed phases still depends on the simplifying assumption (*iii*) above. Collecting data from lattices with different aspect ratios and extrapolating in the smallest linear extension to infinity we [5] obtained at the physical Higgs mass $m_H = 64.77$ GeV (which corresponds to $T_c = 150.9$ GeV) for the $SU(2)$–Higgs theory without fermions an estimate of $\alpha/T_c^3 = 2.1 \times 10^{-4}$.

All methods discussed so far are focusing on changes of (volume or subvolume) averaged variables o in mixed phase systems with minimal interfaces. There is a third method [6,7] to deduce the interface tension α from a tunneling correlation length ξ_{tunnel} at the phase transition. To be more precise, it is the dependence of this correlation length on the geometry of the system which allows to extract α. A first test of the credibility of the method in the case of the electroweak phase transition has been undertaken in [8] far from the critical Higgs mass and in [9] at even smaller Higgs mass, both within the 4–dimensional framework.

In contrast to measurements of the temperature dependent Higgs mass on both sides of the transition [5] (which requires to separate pure phase samples) the emphasis is here on correlations due to different phases in coexistence. The tunneling correlation length is measured in a very elongated volume $L^2 \times L_z$, stretched along the z–direction. In 4–dimensional simulations the additional Euclidean temporal extent is understood to represent the (inverse) temperature and cannot be modified in practice. The connected correlator $C_{conn}(z_1 - z_2)$ of $o(z) = \sum_x \delta_{x_3,z}\ tr\ (\Phi_x^+\Phi_x)$ between two equal–z slices defines the correlation length ξ_{tunnel} which is expected to vary with the transversal extent of the lattice as

$$\xi_{tunnel} \propto \exp\left(\alpha_3 A\right), \qquad A = (aL)^2 \quad . \tag{3}$$

The interface tension of the original 4–dimensional theory α is related to α_3 through $\alpha = \alpha_3 T_c$.

Using a semiclassical expansion for a scalar φ^4 theory in $3d$ including quadratic fluctuations around a kink solution [7], the mass (energy) gap $m = \xi_{tunnel}^{-1}$ has been calculated (in lattice units) as

$$m_{lat} = m\ a = C\ \sqrt{\alpha a^2/T_c}\ \exp\left(-\alpha L^2 a^2/T_c\right) \tag{4}$$

with no additional L dependence in front of the exponent. Therefore, expressed in terms of the dimensionless parameter

$$x = \alpha L^2 a^2/T_c\ , \tag{5}$$

the following scaling behaviour is expected to hold for the tunneling correlation lengths (if they are measured in units given by the transverse lattice size)

$$m_{lat}\ L = C\sqrt{x}\ \exp(-x) \quad . \tag{6}$$

Strictly speaking, this result is valid for systems within the universality class of the Ising model, but it has been confronted with the 4–dimensional $SU(2)$–Higgs model in [8]. In this first application in the electroweak context (at smaller Higgs mass and, consequently, higher α) it has been shown that the simple perturbative one–loop result (4) is reached from above with increasing transverse lattice size and is valid only beyond $x \simeq 1$ (cf. Fig. 4 below). It has been argued already in [15] that higher order corrections may lead to a pre–exponential power in L with an exponent different from zero in (4)[1]. Concentrating on the *roughening* of the interface in a capillary wave model beyond the Gaussian approximation [17] it has been found that (4) gets a correction factor $\left(1 + O\left(T_c/(\alpha L^2 a^2)\right)\right)$.

Recall also that in all derivations it has been assumed that the tunneling correlation length is much larger than the typical correlation lengths in the pure phases. In our recent studies at a physical Higgs mass of roughly 65 GeV [5] we have measured the Higgs correlation lengths near

[1] This has been confirmed in a recent two–loop calculation [16]

to the phase transition in the pure phases as $\xi_{broken}/a = 13.40(41)$ and $\xi_{symm}/a = 9.71(29)$ which are not so small compared to the tunneling correlation length $1/(m_{lat}a)$ as one will see later. Furthermore we have visualised there a typical mixed–phase configuration which had very rough interfaces separating different phases.

Therefore, in order to extract an interface tension at this very weak first order transition (with very rough interfaces) we assume that the tunneling mass gap can be parametrised in a more general form as function of the transverse extent L

$$m_{lat}\ L = c\ L^{\gamma}\ \exp\left(-\ \alpha_{lat}\ L^2\right)\quad , \tag{7}$$

with the fit parameters c, γ and α_{lat}. Using the $3d$ continuum gauge coupling $g_3^2 = 4/(\beta_G a)$, we can put α_{lat} into relation to the $3d$ dimensionless interface tension α_3/g_3^4 by

$$\alpha_3/g_3^4 = (\beta_G/4)^2\,\alpha_{lat}\ . \tag{8}$$

As a check of this assumption we have to compare the interface tension with the result of other methods when these are available.

3 Results and discussion

The lattice model (2) is used as in [5], in particular with the same update algorithm as described there. We are dealing with the two cases corresponding to the Higgs masses $M_H^* = 70$ and $M_H^* = 57.4423$ GeV (denoted in the following by 57 GeV). These cases correspond to $\lambda_3/g_3^2 \approx 0.095703$ and $\lambda_3/g_3^2 \approx 0.0644457$, respectively ($\lambda_3$ is the $3d$ continuum Higgs self coupling) via

$$\lambda_3/g_3^2 = 1/8\,(M_H^*/80\mathrm{GeV})^2\ . \tag{9}$$

The Higgs mass M_H^* (in GeV) differs numerically only slightly from the physical Higgs mass m_H in the $4d$ theory without top.

The smaller Higgs mass is chosen in accordance with the work [13] (in their notation referred to as $m_H^* = 60$ GeV). Correlation function measurements have been taken after each 10th iteration. The maximum of the integrated autocorrelation time for this quantity was about 26 at the smaller Higgs mass and 16 at the larger Higgs mass (each in case of the largest measured transverse size).

To obtain the tunneling correlation length ξ_{tunnel} requires some tuning. At first, the appropriate hopping parameter value β_H has to be tuned, separately for each transverse size of the system while keeping near to the bulk critical value β_{Hc}, to the maximum of the tunneling correlation length. The actual longitudinal size of the lattice has been chosen three to four times larger than the correlation length one is going to measure.

In the tables we quote the statistics for all lattice geometries $L^2 \times L_z$ at the respective β_H (having the maximal tunneling correlation length) and its corresponding value m_{lat}. We indicate also the inverse transverse correlation length $m_{\perp\ lat}$ and the total statistics which went into the

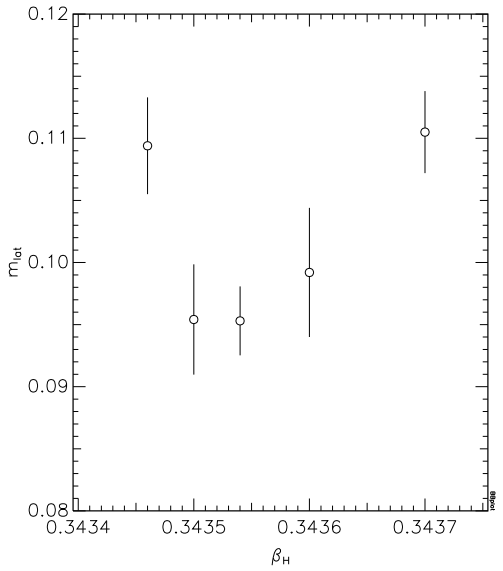

Fig. 1. Inverse correlation length at $M_H^* = 70$ GeV and $12^2 \times 64$ as function of β_H

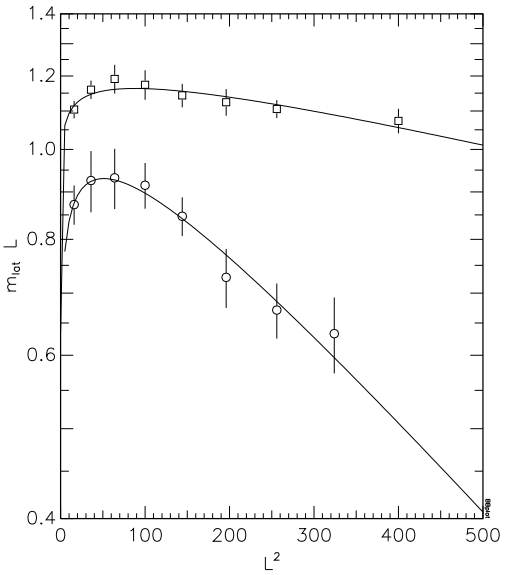

Fig. 2. Fits for the inverse tunneling correlation lengths at $M_H^* = 57$ GeV (*circles*) and $M_H^* = 70$ GeV (*squares*)

search for the minimum. Additionally, we have checked at the larger Higgs mass for geometries $12^2 \times 96$ and $20^2 \times 128$ that m_{lat} does not change within the errors for larger L_z.

In order to extract the correlation length ξ_{tunnel} we first check by inspection whether the local mass has a plateau. We define a local lattice mass $m_{lat}(z)$ at correlation distance z through a fit of three subsequent values $C_{conn}(z-1)$, $C_{conn}(z)$ and $C_{conn}(z+1)$ of the correlation functions of $o(z) = \sum_x \delta_{x_3,z}\ tr\ (\Phi_x^+ \Phi_x)$ to a hyperbolic cosine shape

$$A\ \{\exp\left[-m_{lat}(z)\ z\right] + \exp\left[-m_{lat}(z)\ (L_z - z)\right]\}. \tag{10}$$

This ansatz is essential in order to observe a plateau in the local mass versus z. In Fig. 3 $m_{lat}(z)$ is shown for one par-

Table 1. Statistics at $M_H^* = 57$ GeV

$L^2 \times L_z$	m_{lat}	# msmts	β_H	$m_{\perp lat}$	total # msmts
$4^2 \times 32$	.2180(106)	8000	.343600	—	36000
$6^2 \times 64$	.1543(117)	10000	.343000	—	24000
$8^2 \times 64$	.1165(081)	6000	.342700	—	8000
$10^2 \times 64$	.09150(531)	6000	.342700	.1008(29)	10000
$12^2 \times 64$	.07060(339)	6000	.342694	.0782(21)	12000
$14^2 \times 96$	.05200(380)	10000	.342700	.0632(21)	19000
$16^2 \times 128$	.04195(286)	10000	.342688	.0509(15)	21000
$18^2 \times 128$	.03517(331)	12000	.342686	.0425(14)	20000

Table 2. Statistics at $M_H^* = 70$ GeV

$L^2 \times L_z$	m_{lat}	# msmts	β_H	$m_{\perp lat}$	total # msmts
$4^2 \times 128$	.276(6)	3000	.345000	—	18400
$6^2 \times 64$	.1933(044)	10000	.344200	—	60000
$8^2 \times 64$	.1489(053)	20000	.343800	—	40000
$10^2 \times 64$	.1174(043)	25000	.343540	.1345(17)	80000
$12^2 \times 64$	.09530(278)	50000	.343540	.1080(13)	110000
$14^2 \times 64$	.08031(267)	25000	.343540	.0889(16)	25000
$16^2 \times 64$	.06910(154)	25000	.343540	.0761(09)	40000
$20^2 \times 64$	.05367(163)	15000	.343560	.0599(12)	51000

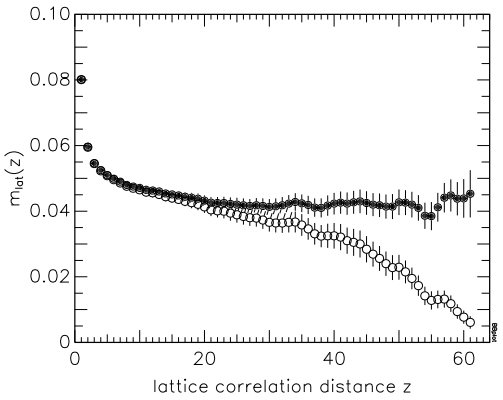

Fig. 3. Example of the local lattice mass at $M_H^* = 57$ GeV and $16^2 \times 128$ as function of z

ticular example. We contrast this with a local mass defined through a single exponential fit to the three neighbouring values of $C_{conn}(z)$ above. No plateau at all can be identified using the latter definition of a local mass. Finally, the (inverse of the) correlation length ξ_{tunnel} is obtained by a global hyperbolic cosine fit over the plateau range that we have identified. The behaviour of the inverse correlation length near to its lowest value is demonstrated in Fig. 1. In Fig. 2 we show the inverse tunneling correlation length multiplied by the transverse extension, $m_{lat}\ L$ as function of the transverse lattice area L^2 for both values of the Higgs mass under discussion. The strongly different exponential slopes reflect the weakening of the phase transition. The curves correspond to a least square fit with the ansatz (7).

From this fit we extract the dimensionless $3d$ interface tensions $\alpha_3/g_3^4 = 0.0224(56)$ for the lower Higgs mass $M_H^* = 57$ GeV, and $\alpha_3/g_3^4 = 0.0049(18)$ for $M_H^* = 70$ GeV. The first number is in very good agreement with the value presented in Fig. 15 of [13] (and – with the use of Eqs. (11.6) and (2.8) there – translated into) $\alpha_3/g_3^4 = 0.0217(22)$.

Using these numbers we find the interface tensions α

$$\alpha/T_c^3 = 3.24(80) \times 10^{-3}, \quad M_H^* = 57\text{GeV} \tag{11}$$

and

$$\alpha/T_c^3 = 70(26) \times 10^{-5}, \quad M_H^* = 70\text{GeV} \tag{12}$$

corresponding to the 4–dimensional model without top quark. Taking into account the top quark the numbers change to $3.57(89) \times 10^{-3}$ and $77(29) \times 10^{-5}$. The relations between 3–dimensional and 4–dimensional quantities have been obtained as in [5] using (in the same notations as there) Table 3 which is recalculated here for $M_H^* = 57$ GeV. Though the top contribution apparently changes the interface tension only insignificantly (Δ_g is small) some of the fermionic one-loop corrections to the $4d$ couplings are already too large and the other physical numbers should be taken with great care. The reported pseudocritical β_H values for the largest transverse sizes from Tables 1 and 2 are related to critical temperatures $T_c = 127.2$ and 150.1 GeV and physical Higgs masses $m_H = 52.35$ and 64.55 GeV of the $4d$ theory without top for the lower and larger M_H^*. The four dimensional $\overline{\text{MS}}$ gauge coupling $g^2(m_W)$ has the value 0.423 and 0.422, respectively, which is close to that of the standard model.

Our present result for the case of $M_H^* = 70$ GeV is larger by a factor 3.3 than the previous rough estimate [5]. The latter was obtained as a result of our equal weight histogram method, finally based on a global infinite volume extrapolation of the mixed phase thermodynamical weight for lattices of various aspect ratios.

Table 3. Corrections in (6) of [5]

	$M_H^* = 57$ GeV without fermions	$M_H^* = 57$ GeV with top
Δ_g	-0.01322	6.712×10^{-5}
Δ_λ	-0.02383	1.494
Δ_ν	-0.02938	0.9443
Δ_Y		-0.01301

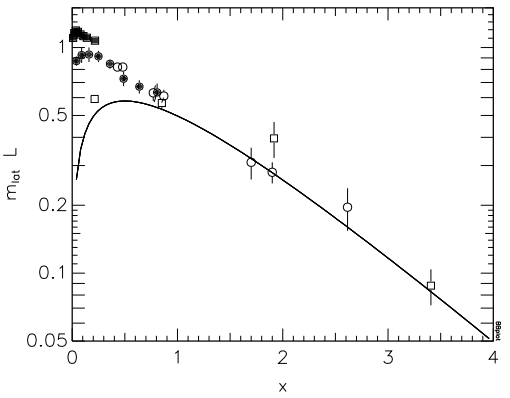

Fig. 4. Scaling law for the mass gap followed by data at $m_H = 35$ GeV (*open squares*) [9], $m_H = 49$ GeV (*open circles*) [8], $M_H^* = 57$ GeV (*full circles*) and $M_H^* = 70$ GeV (*full squares*) [this work]

In the present fits we obtain effective exponents $\gamma = 0.25(11)$ and $0.095(45)$ of L, respectively, which become smaller with decreasing strength of the transition. Fixing the exponent to $\gamma = 1$ as suggested by (4) we would be able to present only a local fit to the few highest transverse areas L^2. The interface tensions would be estimated by this fit as follows: $\alpha_3/g_3^4 = 0.023$ for $M_H^* = 57$ GeV and 0.016 for $M_H^* = 70$ GeV. In the first case this would be still acceptable comparing with the result of the Helsinki group [13] for that Higgs mass. But the interface tension evaluated at $M_H^* = 70$ GeV in this way does not follow the general trend of the interface tensions from $d = 3$ simulations which have been collected in [18].

In view of the arguments above, we consider the fit with the free ansatz (7) more serious than the results of the fit confined to $\gamma = 1$. We recall that also in [8] the latter fit has been successful only at large values of the scaling variable $x = \alpha L^2 a^2/T_c$. The one–loop scaling law (6) was meant to hold independent of the particular system, irrespective of the actual value of the interface tension. With the values of α obtained now and in [8,9] the underlying mass gaps for various Higgs masses and transverse lattice sizes seem to follow another universal law if expressed through the scaling variable x

$$m_{lat}\ L = C'\ \exp(-x) \tag{13}$$

instead of (13), except for the smallest transverse extensions L in each case. This can be seen in Fig. 4. In this figure we also show the asymptotic behaviour (6) with $C = 1.352$ as predicted by [7,19].

There is one warning in the data concerning the applicability of the analytical formulae used throughout the literature in order to extract the interface tension from lattice data of the tunneling correlation length. As mentioned above it is implicitly assumed that the correlation lengths of the two phases in equilibrium are small compared to the tunneling correlation length. From this point of view the present data on m_{lat} for $M_H^* = 70$ GeV (where we have measured Higgs masses separately for both phases at the transition temperature) indicate that transverse sizes $L \leq 10$ would have to be excluded from the analysis.

The other concern is caused by the fact that the *transverse* correlation length $\xi_\perp$ does not decouple from the transverse lattice extension at small L and keeps growing for all transverse sizes considered. For instance, at $M_H^* = 70$ GeV, the transverse correlation length amounts from 75 to 83 per cent of L on our lattices with transverse sizes $L \geq 10$ which are, on the other hand, the only acceptable ones in view of the criticism discussed before. In the case of $M_H^* = 57$ GeV the transverse correlation length is even larger compared with the transverse size L (100 to 130 per cent) on lattices with $L \geq 10$. It should be mentioned that the ratio of the transverse correlation length to the transverse size is consistent with the ratio of the bulk correlation length to the system size for cubic symmetries on top of the phase transition (measured without separating the Monte Carlo sample into pure phase configurations).

4 Overview and conclusions

To compare our $3d$ results for the interface tension with those of $4d$ measurements we follow the procedure outlined in [20]. The measurements in the $4d$ approach have been performed at a different gauge coupling. The measured renormalised gauge couplings do not seem to change significantly with the Higgs mass in the so far reported region from 18 to 49 GeV [10,21] and vary from 0.56 to 0.59. For simplicity (and due to missing calculation) it is assumed as in [20] that the measured renormalised gauge coupling roughly corresponds to the $\overline{\rm MS}$ running coupling. It is then different from the value $g^2(m_W) = 0.42$ corresponding to our calculations described in the last section.

Using the prescription to relate $3d$ and $4d$ parameters [5] we calculate Higgs masses, critical temperatures and interface tensions for increasing $4d$ gauge couplings keeping the respective λ_3/g_3^2 fixed. The numbers are collected in Table 4. Note that the Higgs mass is slightly moving to lower values while the ratio α/T_c^3 becomes much bigger, largely due to the critical temperature getting smaller.

Having the caveats of the last section in mind we compare now in Fig. 5 our interface tensions α with those measured by different methods in the $4d$ theory at various Higgs masses as function of the physical Higgs mass m_H. In order to do this we read for our $3d$ data (full points) the corresponding Higgs mass (without fermion contributions) from Table 4, for example at $g^2(m_W) = 0.58$. This value has been deduced as mentioned above from the measured renormalised gauge coupling.

Table 4. Higgs masses, critical temperatures and interface tensions for various $4d$ running gauge couplings

M_H^*/GeV	$g^2(m_W)$	m_H/GeV	T_c/GeV	α/T_c^3
57	0.423	52.35	127.2	0.00324(081)
	0.560	50.86	108.2	0.00539(134)
	0.570	50.75	107.1	0.00556(138)
	0.580	50.64	106.0	0.00574(143)
	0.590	50.54	104.9	0.00592(147)
70	0.422	64.55	150.1	0.00070(26)
	0.560	62.99	128.1	0.00115(43)
	0.570	62.88	126.8	0.00119(45)
	0.580	62.76	125.5	0.00122(46)
	0.590	62.65	124.3	0.00126(47)

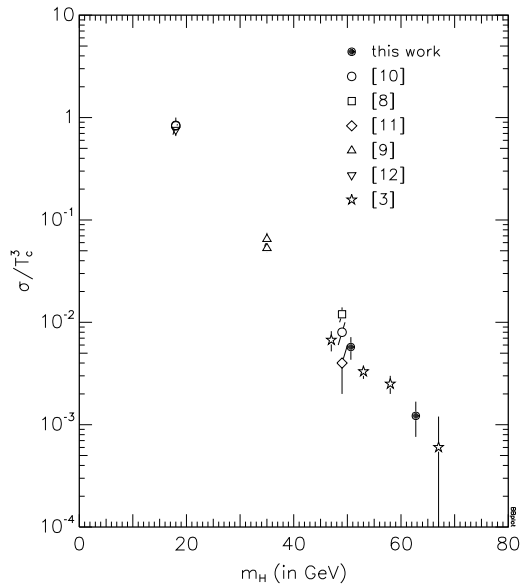

Fig. 5. Collected results on the interface tension as function of the Higgs mass

In the $4d$ data the Higgs masses are either measured [10,8,11,9,12] (no errors are taken into account in the horizontal scale) or estimated [3]. The agreement between the various approaches in $3d$ and $4d$ is remarkable if the $3d$ and $4d$ parameters are correctly mapped onto each other. This nicely demonstrates the validity of dimensional reduction in the considered Higgs mass range.

Our new data points are the following ones, expressed in terms of the 3–dimensional and the 4–dimensional interface tensions. We find at $\lambda_3/g_3^2 = 0.0644457$, $g^2(m_W) = 0.423$ and $\beta_G = 12$ ($M_H^* = 57$ GeV)

$$\alpha_3/g_3^4 = 0.0224(56), \quad \alpha/T_c^3 = 3.24(80) \times 10^{-3} \qquad (14)$$

and at $\lambda_3/g_3^2 = 0.095703$, $g^2(m_W) = 0.422$ and $\beta_G = 12$ ($M_H^* = 70$ GeV)

$$\alpha_3/g_3^4 = 0.0049(18), \quad \alpha/T_c^3 = 70(26) \times 10^{-5}. \qquad (15)$$

We observe an approximate scaling law expressing the energy gap for all Higgs masses in terms of the dimensionless variable x (see (5)) without the prefactor $\sqrt{x}$ which had been suggested by one–loop perturbation theory for interfaces in the case of binary systems.

We emphasise that the tunneling correlation length method works well even near to the critical Higgs mass where other methods relying on discrimination of histogram peaks and minima are already difficult to apply.

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