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Combined $B \to X_s\psi$ and $B \to X_s\eta_c$ decays as a test of factorization

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Abstract. We calculate the inclusive decays $B \to X_s\psi$ and $B \to X_s\eta_c$ using factorization assumption. To investigate the bound state effect of the decaying B meson in these inclusive decays we take into account the motion of the b quark using a Gaussian momentum distribution model. The resulting correction to free quark decay approximation is around 6% at most. Utilizing a potential model evaluation of the ratio of the decay constants $f_{\eta_c}^2/f_\psi^2$, it is shown that the ratio $R = \Gamma(B \to X_s\eta_c)/\Gamma(B \to X_s\psi)$ can be used as a possible test of factorization assumption.

Exclusive and inclusive nonleptonic B decays to charmonium states are of special interest theoretically and experimentally. These decay channels, among other things, provide a powerful testing ground for color suppression and factorization in hadronic B decays [1]. At the same time, exclusive modes of two body B decay into K meson resonances and charmonium states can provide an alternative examination of models for the treatment of hadronic form factors [2].

In this work, we focus on the inclusive two body decays $B \to X_s\psi$ and $B \to X_s\eta_c$, where X_s is a final state hadron containing a strange quark. There is no experimental data on the latter decay at this time. However, as we point out, the eventual measurement of this inclusive decay channel can be used to test the validity of the factorization assumption in nonleptonic B decays. In fact, one can show that the ratio $R = \Gamma(B \to X_s\eta_c)/\Gamma(B \to X_s\psi)$, calculated by using factorization, is independent of the QCD corrections and the scale ambiguity of the Wilson coefficients. In this context, R depends only on the ratio of the decay constants f_{η_c}/f_ψ for which we use an improved estimate obtained in a previous work [3].

The inclusive decays $B \to X_s\psi(\eta_c)$ are usually approximated with the free quark decays $b \to s\psi(\eta_c)$. To improve upon, in the present paper, we estimate the correction to this approximation by taking into account the motion of the b quark inside the B meson. For this purpose, we use a one parameter Gaussian momentum distribution for the b quark which has previously been applied to inclusive semileptonic [4], rare dileptonic [5] and nonleptonic B decays [6] (commonly known as ACCMM model in the literature). We present results for a range of the model parameter obtained from fits to experimental data.

Neglecting penguin operators, the relevant effective Hamiltonian for $B \to X_s\psi(\eta_c)$ can be written as:

$$\begin{aligned} H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} \big[& C_1(\mu)\bar{c}^i\gamma^\mu(1-\gamma_5)b^i\bar{s}^j\gamma_\mu(1-\gamma_5)c^j \\ & + C_2(\mu)\bar{s}^i\gamma^\mu(1-\gamma_5)b^i\bar{c}^j\gamma_\mu(1-\gamma_5)c^j\big] \\ & + H.C.\,, \end{aligned} \tag{1}$$

where i and j are color indices and $C_1(\mu)$ and $C_2(\mu)$ are QCD improved Wilson coefficients. One then can use a Fierz transformation to write (1) in the following form:

$$\begin{aligned} H_{eff} = & \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} \\ & \times \left[\left(C_2(\mu) + \frac{1}{3} C_1(\mu) \right) \bar{s}^i\gamma^\mu(1-\gamma_5)b^i \right. \\ & \bar{c}^j\gamma_\mu(1-\gamma_5)c^j + C_1(\mu)\bar{s}^i\gamma^\mu(1-\gamma_5)T^a_{in}b^n\bar{c}^j\gamma_\mu \\ & \left. (1-\gamma_5)T^a_{jm}c^m \right] + H.C.\,, \end{aligned} \tag{2}$$

where T^a $(a = 1..8)$ are generators of $SU(3)_{\rm color}$. We note that the first term in (2) is the product of two color singlet currents but the second term consists of two color octet currents. In the factorization assumption, only the color singlet quark current contributes to the production of colorless $c\bar{c}$ final state ψ and η_c. Using the definition for the decay constant (f_ψ) for the vector meson ψ:

$$f_\psi \epsilon^\mu = < 0|\bar{c}\gamma^\mu c|\psi >\,, \tag{3}$$

(ϵ^μ is the polarization vector of ψ) the effective Hamiltonian for $B \to X_s\psi$ is obtained as follows:

$$H_{eff}^{B\to X_s\psi} = C f_\psi \bar{s}\gamma^\mu(1-\gamma_5)b\epsilon_\mu\,, \tag{4}$$

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where

$$C = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} \left(C_2(\mu) + \frac{1}{3} C_1(\mu) \right) . \tag{5}$$

Similarly, utilizing the definition for the decay constant (f_{η_c}) for the psuedoscalar meson η_c:

$$f_{\eta_c} q^\mu = < 0 | \bar{c} \gamma^\mu \gamma_5 c | \eta_c(q) > , \tag{6}$$

results in the following effective Hamiltonian for $B \to X_s\eta_c$ decay:

$$H_{eff}^{B \to X_s \eta_c} = C f_{\eta_c} \bar{s} \gamma^\mu (1 - \gamma_5) b q_\mu . \tag{7}$$

The Wilson coefficients C_1 and C_2 in (5) are calculated to the next-to-leading order in reference [7] resulting in $a_2 = C_2(\mu) + 1/3 C_1(\mu) = 0.155$ for $\mu = m_b \approx 5$ GeV. However, the branching ratio for the exclusive decay $B \to K\psi$ obtained from (4) requires a_2 to be roughly by a factor of two larger than the above value in order to agree with the experimental data [8]

$$BR(B^+ \to K^+ \psi) = (0.101 \pm 0.014)\% .$$

This discrepancy between theoretical prediction and measurement could be due to two factors. On one hand, the μ-dependence of the Wilson coefficients which arises from short distance QCD results in a significant uncertainty in the calculated decay rate in the context of factorization. In fact, phenomenologically, a_2 is treated as a free parameter to be determined from experiment [9]. On the other hand, one may question the validity of the factorization assumption which allows to infer (4) from the effective Hamiltonian (2). In other words, the second term in (2) which is nonfactorizable could have a significant contribution to the matrix element [10]. To disentangle these two factors and examine the factorization assumption, the ratio of the inclusive decays R can serve as a crucial testing ground. Aside from the cancellation of the Wilson coefficients in R, this ratio is also free from the nonperturbative hadronic uncertainties which is usually associated with the theoretical calculations of exclusive decays [11].

Using (4) and (7), one can calculate the decay rates $\Gamma(b \to s\psi(\eta_c))$:

$$\Gamma(b \to s\psi) = \frac{C^2 f_\psi^2}{8\pi m_b m_\psi^2} g(m_b, m_s, m_\psi) \left[m_b^2 (m_b^2 + m_\psi^2) - m_s^2 (2m_b^2 - m_\psi^2) + m_s^4 - 2m_\psi^4 \right] , \tag{8}$$

$$\Gamma(b \to s\eta_c) = \frac{C^2 f_{\eta_c}^2}{8\pi m_b} g(m_b, m_s, m_{\eta_c}) \times \left[(m_b^2 - m_s^2)^2 - m_{\eta_c}^2 (m_b^2 + m_s^2) \right] , \tag{9}$$

where

$$g(x,y,z) = \left[(1 - \frac{y^2}{x^2} - \frac{z^2}{x^2})^2 - 4\frac{y^2 z^2}{x^4} \right]^{1/2} , \tag{10}$$

and m_b and m_s are b and s quark masses, respectively. The inclusive decay rates $\Gamma(B \to X_s\psi(\eta_c))$ are usually approximated by (8) and (9). However, in this work we estimate the bound state corrections to this approximation by taking into accout the motion of the heavy b quark inside the B meson. We follow the ACCMM method [4] which incorporates the bound state effect in semileptonic B decays by assuming a virtual b quark inside B meson accomponied by an on-shell light quark. In the meson rest frame, the energy-momentum conservation leads to the following relation for b quark mass W:

$$W^2(\mathbf{p}) = m_B^2 + m_q^2 - 2m_B \sqrt{\mathbf{p}^2 + m_q^2} , \tag{11}$$

where m_q is the light quark mass and $\mathbf{p}$ is the 3-momentum of the b quark. Following [4], we also consider a Gaussian momentum distribution for the Fermi motion of b quark:

$$\phi(\mathbf{p}) = \frac{4}{\sqrt{\pi} p_F^3} \mathrm{e}^{-\mathbf{p}^2/p_F^2} . \tag{12}$$

The model parameter p_F determines the distribution width, and is related to the average momentum $< p >$.

At this point we would like to remark on the consistency of the above model with heavy quark expansion. Let us consider the average b quark mass $\overline{m}_b$ defined as:

$$\overline{m}_b = \int_0^{p_{\max}} W(\mathbf{p}) \phi(\mathbf{p}) p^2 dp , \tag{13}$$

where $p_{\max}$ is the maximum kinematically allowed momentum. Using (13), one can derive an expansion of the B meson mass m_B in powers of $\overline{m}_b$ as follows:

$$m_B = \overline{m}_b + \frac{2p_F}{\sqrt{\pi}} + \frac{3p_F^2}{4\overline{m}_b} + O\left(\frac{1}{\overline{m}_b^2}\right) , \tag{14}$$

in which $m_q = 0$ is assumed. A comparison of (14) with the usual heavy quark expansion formula for heavy-light mesons, i.e.

$$m_M = m_Q + \bar{\Lambda} - \frac{\lambda_1 + d_M \lambda_2}{2m_Q} + O\left(\frac{1}{m_Q^2}\right) , \tag{15}$$

reveals that once $\overline{m}_b$ is identified with the mass of the heavy quark m_Q in (15), the nonperturbative parameters $\bar{\Lambda}$ and λ_1 of the heavy quark expansion and the model parameter p_F are connected as follows:

$$\bar{\Lambda} = \frac{2p_F}{\sqrt{\pi}} \ , \ \lambda_1 = -\frac{3p_F^2}{2} . \tag{16}$$

The ACCMM model does not provide a corresponding term for the nonperturbative parameter λ_2 ($d_M = 3, -1$ for psuedoscalar and vector mesons, respectively) which is due to the spin interactions. This could be considered as a shortcoming of the model. However, the constraint imposed by (16), i.e. $\lambda_1 = -3\pi/8\bar{\Lambda}^2$, is in reasonable agreement with quoted values for these parameters [12]. It is in this sense that we consider the above model to be consistent with heavy quark symmetries.

To incorporate the effects of the motion of the b quark in the inclusive decays $B \to X_s\psi(\eta_c)$, we replace the b

quark mass m_b in (8) and (9) with $W(\mathbf{p})$ defined in (11) and integrate over the kinematically allowed range of the b quark momentum $\mathbf{p}$, i.e.

$$\Gamma(B \to X_s\psi(\eta_c)) = \int_0^{p_{max}} \Gamma(b \to s\psi(\eta_c))_{m_b=W(\mathbf{p})}\phi(\mathbf{p})p^2dp. \tag{17}$$

As a result, the sensitivity to the heavy quark mass m_b is replaced with the model parameter p_F and the light quark mass m_q dependence. There are various determinations of p_F from fits to semileptonic B decays and also from comparison with the heavy quark effective theory approach. In [12], $\bar{\Lambda} = 0.55 \pm 0.05$ GeV and $\lambda_1 = -0.35 \pm 0.05$ GeV^2 have been extracted from CLEO data on inclusive semileptonic $B \to X\ell\bar{\nu}_\ell$ decay. A comparison with (16) leads to $p_F \approx 0.5$ GeV. However, in reference [13], a smaller central value $p_F = 0.27^{+0.22}_{-0.27}$ is obtained by an ACCMM model analysis of the ARGUS results for the lepton energy spectrum of $B \to X_c\ell\bar{\nu}_\ell$ decay. In order to investigate the sensitivity of our estimates to the model parameter p_F, we present results for $p_F = 0.3$ and $p_F = 0.5$ GeV. Using (14), these values of p_F correspond to $\overline{m}_b = 4.92$ GeV and $\overline{m}_b = 4.65$ GeV, respectively.

Following the prescription of (17) and introducing the notation

$$\Gamma\left(B \to X_s\psi(\eta_c)\right) = \frac{C^2 f^2_{\psi(\eta_c)}}{8\pi} N_{\psi(\eta_c)}\,, \tag{18}$$

we obtain $N_\psi = 7.64(8.14)$ GeV and $N_{\eta_c} = 44.29(48.18)$ GeV^3 for $p_F = 0.3$ GeV (the first number is calculated by taking the strange quark mass $m_s = 0.55$ GeV and the second number in parentheses is resulted from using $m_s = 0.15$ GeV). Comparing these results with the case where $m_b = \overline{m}_b = 4.92$ GeV is inserted in (8) and (9), i.e. $N_\psi = 7.55(8.04)$ GeV and $N_{\eta_c} = 43.55(47.42)$ GeV^3, indicates that only a small correction of order 1–2% arises from considering this bound state effect. On the other hand, a larger value of the model parameter $p_F = 0.5$ GeV (which is compatible with heavy quark effective theory approach) results in $N_\psi = 5.85(6.35)$ GeV and $N_{\eta_c} = 33.14$ (36.77) GeV^3. A comparison with the decay rates obtained from (8) and (9) by using $m_b = \overline{m}_b = 4.65$ GeV, i.e. $N_\psi = 5.57$ (6.06) GeV and $N_{\eta_c} = 31.01(34.62)$ GeV^3, reveals a larger bound state corrections of order 5–6%. We also notice that the corrections are almost independent of s quark mass m_s.

As we observe from (18), the ratio $R = \Gamma(B \to X_s\eta_c)/\Gamma(B \to X_s\psi)$ is independent of C in the context of factorization assumption, i.e.

$$R = \frac{\Gamma(B \to X_s\eta_c)}{\Gamma(B \to X_s\psi)} = \frac{N_{\eta_c} f^2_{\eta_c}}{N_\psi f^2_\psi} = (5.8 \pm 0.1\,\text{GeV}^2)\frac{f^2_{\eta_c}}{f^2_\psi}\,, \tag{19}$$

where our error estimate in the numerical factor represents the variation of the model parameter p_F in the range 0.3 to 0.5 GeV and also the s quark mass m_s from 0.15 to 0.55 GeV.

To evaluate the ratio of the decay constants in (19), we use the potential model relations which relate these form factors to the value of the meson wavefunction at the origin:

$$\begin{aligned} f_{\eta_c} &= \sqrt{\tfrac{12}{m_{\eta_c}}}\Psi_{\eta_c}(0)\,, \\ f_\psi &= \sqrt{12 m_\psi}\Psi_\psi(0)\,. \end{aligned} \tag{20}$$

The common assumption in the literature is that the wavefunction of the psuedoscalar and vector mesons at the origin are more or less identical. However, in [3], based on a simple perturbation theory argument, this ratio was estimated to be

$$\frac{|\Psi_{\eta_c}(0)|^2}{|\Psi_\psi(0)|^2} = 1.4 \pm 0.1\,. \tag{21}$$

Consequently, the ratio of the decay constants in (19) can be written as:

$$\frac{f^2_{\eta_c}}{f^2_\psi} = \frac{1}{m_{\eta_c} m_\psi}\frac{|\Psi_{\eta_c}(0)|^2}{|\Psi_\psi(0)|^2} \approx 0.15 \pm 0.01\ \text{GeV}^{-2}\,. \tag{22}$$

Inserting (22) into (19), we obtain:

$$R = \frac{\Gamma(B \to X_s\eta_c)}{\Gamma(B \to X_s\psi)} = 0.87 \pm 0.06\,. \tag{23}$$

We would like to point out again that the ratio R is free of hadronic model uncertainties which are normally encountered in calculating exclusive decays. The inclusive decay branching ratio $BR(B \to X_s\psi) = (1 \pm 0.25) \times 10^{-2}$ has been extracted from experimental data for direct production of ψ in nonleptonic B decays [14]. Therefore, a measurement of the inclusive $B \to X_s\eta_c$ decay along with reducing the error bar in the experimental result for $B \to X_s\psi$ can serve as an alternative test of the factorization assumption which is used in deriving (23). A significant deviation of the experimental value for R from the theoretical prediction in (23) could be an indication of the failure of the factorization assumption. We would like to emphasize that the cancellation of the Wilson coefficients in the ratio R results in a significant reduction of the uncertainty due to QCD corrections and scale dependence.

In conclusion, using factorization, we calculated the ratio of the inclusive nonleptonic B decays to a hadron containing a strange quark plus psuedoscalar and vector $c\bar{c}$ mesons. The bound state effect due to the motion of the b quark inside the B meson was also considered. We used an improved estimate of the ratio of the decay constants for η_c and ψ was obtained based on potential model and pertubation theory argument. Once the experimental results on $B \to X_s\eta_c$ decay are available, a comparison to the theoretical prediction presented in this paper could serve as a test of factorization assumption. Finally, we would like to emphasize that the pattern of deviations from factorization can give us important clues on the non-factorizable contributions, and therefore, it is important to examine the departures from this approximation using various experimental data.

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