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Inclusive semileptonic decays of polarized Λ_b Baryons into polarized τ-Leptons

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Abstract. We employ OPE techniques within HQET to calculate the inclusive semileptonic decays of polarized Λ_b baryons. Lepton mass effects are included which enables us to also discuss rates into polarized τ-leptons. We present explicit results for the longitudinal polarization of the τ in the Λ_b rest frame as well as in the $(\tau^-, \bar{\nu}_\tau)$ c.m. frame. In both the Λ_b rest frame and in the $(\tau^-, \bar{\nu}_\tau)$ c.m. frame we make use of novel calculational techniques which considerably simplify the calculations. The transverse polarization components of the τ are calculated in the $(\tau^-, \bar{\nu}_\tau)$ c.m. frame. We delineate how to measure the full set of 14 polarized and unpolarized structure functions of the decay process by angular correlation measurements. A set of observables are identified that allow one to isolate the contributions of the two $O(1/m_b^2)$ nonperturbative matrix elements K_b and ϵ_b.

1 Introduction

Large samples of the bottom baryon Λ_b have been produced on the Z_0 at LEP and are expected to be produced in future colliders. Advances in microvertexing techniques have allowed for efficient means of Λ_b identification. For example, when LEP was running on the Z one had $\approx 2.2 \times 10^5$ $b\bar{b}$ pairs per 10^6 Z-decays. Of these approximately 10% go into Λ_b baryons of which again $\approx 20\%$ decay semileptonically (e, μ and τ). Thus one can expect a sample of 4000 inclusive semileptonic Λ_b (or $\bar{\Lambda}_b$) decays for every 10^6 Z-decays. Plans at the SLC call for altogether 3×10^6 produced Z's. The quality of the Λ_b data from the SLC will even be better because its small beam size provides for an excellent definition of the Λ_b production vertex.

The Λ_b's produced on the Z-peak are expected to be quite strongly polarized [1]. This calls for a consideration of polarization effects in the Λ_b decays which could be used to determine the polarization of the Λ_b. Approximately 10% of the total $(e+\mu+\tau)$ semileptonic decay sample have a τ-lepton in the final state. The τ-lepton is sufficiently heavy which necessitates the inclusion of lepton mass effects in the dynamical rate calculations, apart from pure phase space effects. We will study also the polarization of the τ-lepton which, because of mass effects, differs from its naive $m_l = 0$ limiting value. The τ-polarization can be experimentally determined from its subsequent decay distributions.

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In this paper we present all the necessary tools to calculate inclusive semileptonic decays of polarized Λ_b's including τ-lepton polarization effects and spin-spin, spin-momentum and momentum-momentum correlation effects. Our calculation makes use of the heavy quark effective theory (HQET) and the operator product expansion (OPE) method as applied to heavy hadron decays. There is some overlap of our work with earlier results on inclusive Λ_b decays in [2–8] and the more recent paper by M. Gremm, G. Köpp and L.M. Sehgal on polarization effects in inclusive semileptonic Λ_b-decays [9]. At the technical level we differ from the analysis of the above papers in that we employ helicity techniques to derive compact forms for the differential decay distributions including polarization effects. Also, by using suitable phase space variables, the integration of the higher order terms in the operator product expansion become much simplified, i.e. there are no surface term contributions.

In Sect. II we list our results on the hadronic matrix elements needed for the subsequent semileptonic rate calculations. As the techniques of deriving the hadronic matrix elements are quite standard by now we do not dwell much on theoretical background but immediately proceed to the final results which we list in terms of a set of spin-dependent invariant structure functions. In Sect. III the invariant structure functions are related to helicity structure functions which determine the complete angular structure of the polarized decay distributions involving the three helicity angles of the decay process. We write down the full differential decay distribution of polarized Λ_b-decays into longitudinally polarized τ-leptons. We give analytical and numerical results on decay distributions in-

tegrated over phase space. In Sect. IV we discuss the case of transversely polarized τ-leptons.

Section V is dedicated to a calculation of the longitudinal polarization of the τ in the Λ_b rest frame as well as of its azimuthally averaged transverse polarization component in the plane spanned by the τ and the polarization vector of the Λ_b. The calculation is considerably simplified by extracting the rate as the absorptive part of the appropriate one-loop contribution to the $\bar{\tau}\Lambda_b \to \bar{\tau}\Lambda_b$ scattering amplitude. Section VI contains our conclusions. More detailed results have been collected in two Appendices. In Appendix A we list results on q_0-integrated helicity structure functions. In Appendix B we give fully integrated analytic results for the seven structure functions that describe the angular decay structure of unpolarized Λ_b decays.

2 Hadronic matrix elements

The dynamics of the hadron-side transitions is embodied in the hadronic tensor $W^{\mu\nu}$ which is defined as

$$W_j^{\mu\nu}(q_0,q^2,s) = (2\pi)^3 \sum_X \delta^4(p_1-q-p_x) \times \langle \Lambda_b(p_1,s)|J_j^{\mu\dagger}|X_j(p_x)\rangle\langle X_j(p_x)|J_j^{\nu}|\Lambda_b(p_1,s)\rangle\,, \quad (1)$$

where $J_j^{\mu}(j=c,u)$ is the hadronic current inducing $b\to c$ and $b\to u$ transitions. The hadron tensor is a function of two kinematic variables which we choose as q_0 and q^2. Note that we are not summing over the Λ_b-spin such that the hadron tensor depends also on its spin four-vector s. The structure of the hadron tensor can then be represented by an expansion along a standard set of covariants [10] ($v = p_1/m_{\Lambda_b}$)

$$\begin{aligned} W^{\mu\nu} = &-g^{\mu\nu}W_1 + v^\mu v^\nu W_2 - i\epsilon^{\mu\nu\rho\sigma}v_\rho q_\sigma W_3 \\ &+q^\mu q^\nu W_4 + (q^\mu v^\nu + v^\mu q^\nu)W_5 \\ &+\big[-q.s\,\big[-g^{\mu\nu}G_1 + v^\mu v^\nu G_2 - i\epsilon^{\mu\nu\alpha\beta}v_\alpha q_\beta G_3 \\ &+q^\mu q^\nu G_4 + (q^\mu v^\nu + q^\nu v^\mu)G_5\big] \\ &+(s^\mu v^\nu + s^\nu v^\mu)G_6 + (s^\mu q^\nu + s^\nu q^\mu)G_7 \\ &+i\epsilon^{\mu\nu\alpha\beta}v_\alpha s_\beta G_8 + i\epsilon^{\mu\nu\alpha\beta}q_\alpha s_\beta G_9\big] \\ &+(s^\mu v^\nu - s^\nu v^\mu)G_{10} + (s^\mu q^\nu - s^\nu q^\mu)G_{11} \\ &+(v^\mu\epsilon^{\nu\alpha\beta\gamma}q_\alpha v_\beta s_\gamma + v^\nu\epsilon^{\mu\alpha\beta\gamma}q_\alpha v_\beta s_\gamma)G_{12} \\ &+(q^\mu\epsilon^{\nu\alpha\beta\gamma}q_\alpha v_\beta s_\gamma + q^\nu\epsilon^{\mu\alpha\beta\gamma}q_\alpha v_\beta s_\gamma)G_{13}\,. \end{aligned} \quad (2)$$

The last four invariants G_{10}, G_{11}, G_{12} and G_{13} are so-called T-odd invariants which are fed by CP-odd and/or imaginary part contributions. They are zero for Standard Model couplings when loop effects are neglected. They will therefore be disregarded in the following. Note that the invariants $W_4, W_5, G_4, G_5, G_7, G_{11}$ and G_{13} do not contribute to inclusive semileptonic decays in the zero lepton mass case. Since we are also considering decays involving the massive τ-lepton we must keep the full set of invariants implied by (2).

In order to compute the spin-independent structure functions $W_1, ..., W_5$ and the spin-dependent structure functions $G_1, ..., G_9$ we resort to the well-known OPE techniques in HQET. The requisite steps in this calculation are so well documented in the literature [9–17] that we can forgo a description of the intermediate steps and immediately list the final result of the OPE analysis. The hadron tensor is obtained as the absorptive part of the forward matrix element ($W^{\mu\nu} = -\frac{1}{\pi}\mathrm{Im}\,T^{\mu\nu}$)

$$T^{\mu\nu}(q_0,q^2) = -i\langle\Lambda_b(p_1,s)| \int \mathrm{d}^4x\, e^{-iq\cdot x}\mathrm{T}J^{\mu\dagger}(x)J^{\nu}(0)|\Lambda_b(p_1,s)\rangle. \quad (3)$$

The amplitude $T^{\mu\nu}$ is decomposed into 14 invariant form-factors T_i, S_i analogous to the W_i, G_i in (2) and can be computed by using HQET methods [9,11–13,10,14]. Keeping terms up to $1/m_b^2$ one obtains

$$\begin{aligned} T_1 &= \frac{1}{2\Delta_0}(m_b - v.q)(1+X_b) \\ &\quad +\frac{2m_b}{3}(K_b+G_b)\left(\frac{-1}{2\Delta_0}+\frac{q^2-(v.q)^2}{\Delta_0^2}\right) \\ &\quad +\frac{m_b(K_b+G_b)}{2\Delta_0} - \frac{m_b^2 G_b}{3\Delta_0^2}(m_b - v.q), \\ T_2 &= \frac{m_b}{\Delta_0}(1+X_b) + \frac{2m_b}{3}(K_b+G_b)\left(\frac{1}{\Delta_0}+\frac{2m_b v.q}{\Delta_0^2}\right) \\ &\quad +\frac{m_b(K_b+G_b)}{\Delta_0} + \frac{4m_b^2 K_b v.q}{3\Delta_0^2} + \frac{2m_b^3 G_b}{3\Delta_0^2}, \\ T_3 &= \frac{1}{2\Delta_0}(1+X_b) - \frac{2m_b}{3}(K_b+G_b)\frac{m_b - v.q}{\Delta_0^2} \\ &\quad +\frac{2m_b^2 K_b}{3\Delta_0^2} - \frac{m_b^2 G_b}{3\Delta_0^2}, \\ T_4 &= \frac{4m_b}{3\Delta_0^2}(K_b+G_b), \\ T_5 &= \frac{-1}{2\Delta_0}(1+X_b) - \frac{2m_b}{3}(K_b+G_b)\frac{2m_b+v.q}{\Delta_0^2} + \frac{m_b^2 G_b}{3\Delta_0^2}, \\ S_1 &= -\frac{1+\epsilon_b}{2\Delta_0} - \frac{5m_b}{3\Delta_0^2}v.qK_b + \frac{4m_b^2 K_b}{3\Delta_0^3}(q^2-(v.q)^2)\,, \\ S_2 &= \frac{4m_b^2 K_b}{3\Delta_0^2}\,, \\ S_3 &= \frac{2m_b K_b}{3\Delta_0^2}\,, \\ S_4 &= 0\,, \\ S_5 &= \frac{-2m_b K_b}{3\Delta_0^2}\,, \\ S_6 &= -\frac{m_b(1+\epsilon_b)}{2\Delta_0} - \frac{5m_b K_b}{6\Delta_0} - \frac{5m_b^2}{3\Delta_0^2}v.qK_b \\ &\quad +\frac{4m_b^3 K_b}{3\Delta_0^3}(q^2-(v.q)^2)\,, \\ S_7 &= \frac{1+\epsilon_b}{2\Delta_0} + \frac{(2m_b+3v.q)m_b K_b}{3\Delta_0^2} \\ &\quad -\frac{4m_b^2 K_b}{3\Delta_0^3}(q^2-(v.q)^2)\,, \\ S_8 &= \frac{m_b(1+\epsilon_b)}{2\Delta_0} + \frac{m_b K_b}{6\Delta_0} + \frac{5m_b^2}{3\Delta_0^2}v.qK_b \end{aligned}$$

$$-\frac{4m_b^3K_b}{3\Delta_0^3}(q^2-(v.q)^2)\,,$$

$$S_9 = -\frac{1+\epsilon_b}{2\Delta_0} - \frac{(2m_b+3v.q)m_bK_b}{3\Delta_0^2} + \frac{4m_b^2K_b}{3\Delta_0^3}(q^2-(v.q)^2)\,, \quad (4)$$

where

$$X_b = \frac{-2(m_b-v.q)m_b(K_b+G_b)}{\Delta_0} - \frac{8m_b^2K_b}{3\Delta_0^2}(q^2-(v.q)^2) + \frac{2m_b^2K_b}{\Delta_0}\,, \quad (5)$$

We use the notation of [10] throughout the paper. K_b is related to the mean kinetic energy of the heavy quark in the Λ_b baryon

$$K_b = -\sum_s \langle \Lambda_b(p,s)|\bar{b}_v(x)\frac{(iD)^2}{2m_b^2}b_v(x)|\Lambda_b(p,s)\rangle = \frac{\mu_\pi^2}{2m_b^2} \approx 0.013, \quad (6)$$

where we used $\mu_\pi^2 \approx 0.6$ GeV^2 [18] and $m_b = 4.8$ GeV. The spin-dependent contribution ϵ_b is defined by

$$\langle \Lambda_b(p,s)|\bar{b}\gamma^\lambda\gamma_5 b|\Lambda_b(p,s)\rangle = (1+\epsilon_b)s^\lambda\,. \quad (7)$$

with $\epsilon_b \approx -\frac{2}{3}K_b$. An estimate of the spin-dependent parameter ϵ_b has been given in [19] with the result $\epsilon_b = -\frac{2}{3}K_b$, based on an assumption that the contribution of terms arising from double insertions of the chromomagnetic operator can be neglected. A zero recoil sum rule analysis gives the constraint $\epsilon_b \le -\frac{2}{3}K_b$ [20] which puts the estimate of [19] at the upper boundary of the constraint. We use the value of [19] keeping in mind that the numerical value of ϵ_b could be reduced in more realistic calculations.

In order to make our presentation as complete as possible we have retained the chromomagnetic contribution proportional to G_b in (4) although it is zero for the Λ_b system, i.e.

$$G_b := \sum_s \langle \Lambda_b(p,s)|\bar{b}_v(x)\left(\frac{-gF_{\alpha\beta}\sigma^{\alpha\beta}}{4m_b^2}\right)b_v(x)|\Lambda_b(p,s)\rangle = 0\,. \quad (8)$$

The reason is that the general helicity formalism introduced later on can also be applied to B meson and Ω_b baryon decays where $G_b \neq 0$.

The denominator factor Δ_0 is given by

$$\Delta_0 = (m_b v - q)^2 - m_j^2 + i\epsilon \quad (9)$$

The imaginary parts of inverse powers of Δ_0, which are needed for obtaining the structure functions W_i, G_i, can be obtained with the help of

$$\text{Im}\,\left(\frac{1}{\Delta_0}\right) = \frac{-\pi}{2m_b}\delta\left(q_0 - \left(\frac{-m_j^2+q^2+m_b^2}{2m_b}\right)\right)\,,$$

$$\text{Im}\,\left(\frac{1}{\Delta_0^2}\right) = \frac{-\pi}{4m_b^2}\delta'\left(q_0 - \left(\frac{-m_j^2+q^2+m_b^2}{2m_b}\right)\right)\,,$$

$$\text{Im}\,\left(\frac{1}{\Delta_0^3}\right) = \frac{-\pi}{16m_b^3}\delta''\left(q_0 - \left(\frac{-m_j^2+q^2+m_b^2}{2m_b}\right)\right) \quad (10)$$

Various subsets of (4) have appeared in the literature before [9,10,16]. We have recalculated them and collected them together for ease of reference.

3 Helicity structure functions and angular decay distributions

The hadronic structure for the transitions $\Lambda_b(s) \to X_j$ is fully specified by the absorptive parts of the 14 structure functions listed in (4). In order to obtain the full decay distribution for the inclusive decay $\Lambda_b(s) \to X_j + l^-(s_l) + \bar{\nu}_l$ one needs to contract the hadronic tensor $W^{\mu\nu}$ with the known leptonic tensor $L_{\mu\nu}$. Traditionally the contraction $L_{\mu\nu}W^{\mu\nu}$ is done in covariant fashion. Here we advocate a different approach and use helicity techniques to write down the relevant decay distributions. The advantage is that the angular decay distributions involving helicity angles are given by simple linear combinations of the helicity structure functions. The use of helicity techniques to describe angular decay distributions in exclusive semileptonic decays is widespread by now [2–4] and is easily generalized to inclusive semileptonic decays.

Let us begin by writing down the relation between the full spin-dependent set of 14 helicity structure functions and the set of 14 invariant structure functions in (2). One has

$$\begin{aligned}
W_{++}^{++} &= W_1 - pW_3 - pG_1 + p^2G_3 - G_8 - q_0G_9\,,\\
W_{++}^{--} &= W_1 - pW_3 + pG_1 - p^2G_3 + G_8 + q_0G_9\,,\\
W_{--}^{++} &= W_1 + pW_3 - pG_1 - p^2G_3 + G_8 + q_0G_9\,,\\
W_{--}^{--} &= W_1 + pW_3 + pG_1 + p^2G_3 - G_8 - q_0G_9\,,\\
q^2W_{tt}^{++} &= -q^2W_1 + q_0^2W_2 + q^4W_4 + 2q_0q^2W_5 - p(-q^2G_1 + q_0^2G_2 + q^4G_4 + 2q_0q^2G_5 - 2q_0G_6 - 2q^2G_7)\,,\\
q^2W_{tt}^{--} &= -q^2W_1 + q_0^2W_2 + q^4W_4 + 2q_0q^2W_5 + p(-q^2G_1 + q_0^2G_2 + q^4G_4 + 2q_0q^2G_5 - 2q_0G_6 - 2q^2G_7)\,,\\
q^2W_{00}^{++} &= q^2W_1 + p^2W_2 - pq^2G_1 - p^3G_2 + 2q_0pG_6\,,\\
q^2W_{00}^{--} &= q^2W_1 + p^2W_2 + pq^2G_1 + p^3G_2 - 2q_0pG_6\,,\\
q^2W_{0t}^{++} &= pq_0W_2 + pq^2W_5 - q_0p^2G_2 - q^2p^2G_5 + (p^2+q_0^2)G_6 + q_0q^2G_7\,,\\
q^2W_{0t}^{--} &= pq_0W_2 + pq^2W_5 + q_0p^2G_2 + q^2p^2G_5 - (p^2+q_0^2)G_6 - q_0q^2G_7\,,\\
W_{0+}^{+-} &= \sqrt{2/q^2}(-pG_6 - q_0G_8 - q^2G_9)\,,\\
W_{-0}^{+-} &= \sqrt{2/q^2}(pG_6 - q_0G_8 - q^2G_9)\,,\\
W_{t+}^{+-} &= \sqrt{2/q^2}(-q_0G_6 - q^2G_7 - pG_8)\,,
\end{aligned}$$

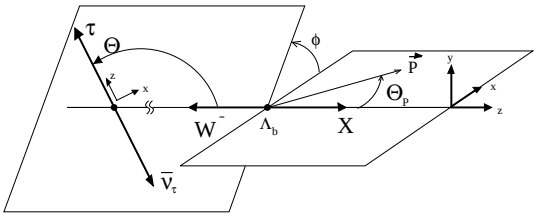

Fig. 1. Definition of the polar angles Θ and Θ_P and of the azimuthal angle ϕ in the decay $\bar{\Lambda}_b \to X + W^-(\to l^-\bar{\nu}_l)$ in the Λ_b rest system. We specify a z-axis which we take to be along $\mathbf{p}_X$. $\mathbf{P}$ denotes the polarization three-vector of the Λ_b

$$W_{-t}^{+-} = \sqrt{2/q^2}(q_0 G_6 + q^2 G_7 - pG_8)\,. \tag{11}$$

We denoted here $p = \sqrt{q_0^2 - q^2}$. The helicity structure functions $W_{\lambda_W \lambda'_W}^{\lambda_{\Lambda_b}\lambda'_{\Lambda_b}}$ are defined by

$$W_{\lambda_W \lambda'_W}^{\lambda_{\Lambda_b}\lambda'_{\Lambda_b}} = (2\pi)^3 \sum_X \delta^4(p_1 - q - p_x)\langle X_j|J_j^\mu|\Lambda_b, \lambda_{\Lambda_b}\rangle \times \epsilon_\mu^*(\lambda_W)\langle \Lambda_b, \lambda'_{\Lambda_b}|J_j^{\nu\dagger}|X_j\rangle \epsilon_\nu(\lambda'_W)\,. \tag{12}$$

Here $\lambda_{\Lambda_b} = \pm 1/2$ is the helicity of the Λ_b and $\lambda_W = 0, \pm 1, t$ are the helicities of the virtual W-boson (spatial: $\lambda_W = 0, \pm 1$; temporal: $\lambda_W = t$). Note that there are no zeroth order parton model and kinetic energy contributions to the structure functions W_{++}^{++} and W_{--}^{--} because of angular momentum conservation. We shall return to this point later in this section when we discuss the contribution of the spin-dependent matrix element ϵ_b. Note that from (12) one has the hermiticity relation

$$W_{\lambda_W \lambda'_W}^{\lambda_{\Lambda_b}\lambda'_{\Lambda_b}\,*} = W_{\lambda'_W \lambda_W}^{\lambda'_{\Lambda_b}\lambda_{\Lambda_b}}\,. \tag{13}$$

Since the helicity structure functions are real in our case one can drop the complex conjugation sign in (13). The helicity structure functions are defined in the Λ_b rest system. We therefore need to specify a z-axis which we take to be along $\mathbf{p}_X$ (see Fig. 1).

As noted before the full angular decay distribution of $\Lambda_b(s) \to X_j + l^-(s_l) + \bar{\nu}_l$ including all polarization effects is completely determined by the set of 14 helicity structure functions. The necessary manipulations involving Wigner's $D^J_{mm'}$ functions are standard and well documented in the literature [21,22] (see also Sect. IV). Here we closely follow the presentation of [5–7]. For example, for the five-fold decay distribution in $q_0, q^2, \cos\Theta, \cos\Theta_P$ and ϕ into negative ($\mathrm{d}\Gamma^-$) and positive ($\mathrm{d}\Gamma^+$) helicity leptons we obtain⋆

$$\frac{\mathrm{d}\Gamma^-}{\mathrm{d}q_0 \mathrm{d}q^2 \mathrm{d}\cos\Theta \mathrm{d}\cos\Theta_P \mathrm{d}\phi} = \frac{2G^2|V_{bj}|^2(q^2 - m_l^2)^2\sqrt{q_0^2 - q^2}}{3(2\pi)^4 q^2}$$

$$\Big[\left(\rho_{++}\left(W_{--}^{++} + W_{++}^{++}\right) + \rho_{--}\left(W_{--}^{--} + W_{++}^{--}\right)\right)\frac{3}{8}(1 + \cos^2\Theta) + \left(\rho_{++}W_{00}^{++} + \rho_{--}W_{00}^{--}\right)\frac{3}{4}\sin^2\Theta + \frac{3}{4}\left(\rho_{++}\left(W_{++}^{++} - W_{--}^{++}\right) + \rho_{--}\left(W_{++}^{--} - W_{--}^{--}\right)\right)\cos\Theta - \frac{3}{2\sqrt{2}}\rho_{+-}\left(W_{-0}^{+-} + W_{0+}^{+-}\right)\sin\Theta\cos\phi + \frac{3}{4\sqrt{2}}\rho_{+-}\left(W_{-0}^{+-} - W_{0+}^{+-}\right)\sin 2\Theta\cos\phi \Big] \tag{14}$$

$$\frac{\mathrm{d}\Gamma^+}{\mathrm{d}q_0 \mathrm{d}q^2 \mathrm{d}\cos\Theta \mathrm{d}\cos\Theta_P \mathrm{d}\phi} = \frac{2G^2|V_{bj}|^2(q^2 - m_l^2)^2\sqrt{q_0^2 - q^2}}{3(2\pi)^4 q^2} \frac{m_l^2}{2q^2}\Big[\left(\rho_{++}\left(W_{--}^{++} + W_{++}^{++}\right) + \rho_{--}\left(W_{--}^{--} + W_{++}^{--}\right)\right)\frac{3}{4}\sin^2\Theta + \left(\rho_{++}W_{00}^{++} + \rho_{--}W_{00}^{--}\right)\frac{3}{2}\cos^2\Theta + \frac{3}{2}\left(\rho_{++}W_{tt}^{++} + \rho_{--}W_{tt}^{--}\right) + 3\left(\rho_{++}W_{0t}^{++} + \rho_{--}W_{0t}^{--}\right)\cos\Theta - \frac{3}{2}\sqrt{2}\rho_{+-}\left(W_{-t}^{+-} - W_{t+}^{+-}\right)\sin\Theta\cos\phi - \frac{3}{4}\sqrt{2}\rho_{+-}\left(W_{-0}^{+-} - W_{0+}^{+-}\right)\sin 2\Theta\cos\phi \Big] \tag{15}$$

For $\Lambda_c \to X_s + l^+ + \nu_l$ decays one has to effect the replacement $\mathrm{d}\Gamma^\mp \leftrightarrow \mathrm{d}\Gamma^\pm$ and one has to change the signs of the contributions proportional to $\cos\Theta$ and $\sin\Theta\cos\phi$ in (14). The differential rate into unpolarized leptons is simply $\mathrm{d}\Gamma^+ + \mathrm{d}\Gamma^-$. The polar angles Θ and Θ_P and the azimuthal angle ϕ are defined in Fig. 1. We have rotated the density matrix of the Λ_b to the z-axis such that one has

$$\rho^{\Lambda_b}(\cos\Theta_P) = \frac{1}{2}\begin{pmatrix} 1 + P\cos\Theta_P & P\sin\Theta_P \\ P\sin\Theta_P & 1 - P\cos\Theta_P \end{pmatrix}\,. \tag{16}$$

Note that for Λ_b's from Z-decays one expects that the Λ_b's are longitudinally polarized with backward polarization. Thus, for Λ_b's from Z-decays, the direction of P in Fig. 1 coincides with the boost direction that brings Λ_b to rest ($P \geq 0$).

The longitudinal polarization of the τ-lepton is given by

$$P_\tau^l = \frac{d\Gamma^+ - d\Gamma^-}{d\Gamma^+ + d\Gamma^-} \tag{17}$$

We emphasize that the longitudinal polarization of the τ calculated from (14, 15) and (17) refers to the $(\tau, \bar{\nu}_\tau)$ c.m. frame which differs from the longitudinal polarization of the τ in the Λ_b rest system as calculated e.g. in

⋆ Similar decay distributions have been written down in [23], where the $O(\alpha_s)$ corrections to unpolarized $t \to b$ decays were evaluated.

[9,17]. A slight adaptation of the lepton-side density matrix elements in (14, 15) as described in [8] will yield the longitudinal polarization in the Λ_b rest system. Put in a different language the two respective polarizations are related to one another by a Wigner rotation (see e.g. [24]). A direct computation of the longitudinal polarization of the τ in the Λ_b rest frame including correlation effects will be presented in Sect. V.

The quasi three-body decay $\Lambda_b(s) \to X_j + l^- + \bar{\nu}_l$ is described by three kinematic invariants. A particularly convenient choice is the one used in (14, 15) in terms of q_0, q^2 and $\cos\Theta$. Working in $(q_0, q^2, \cos\Theta)$ phase space has big technical advantages as can be seen in the following. The integration over $\cos\Theta$ is trivial. The integration over q_0 is almost trivial as can be seen by the following reasoning. The leading order parton model or free quark decay contributions are proportional to the δ-function and the q_0-integration amounts to the substitution $q_0 = \frac{m_b^2 - m_j^2 + q^2}{2m_b}$ in these terms. The corrections to the parton model contributions involve derivatives of the δ-function. Using partial integration the derivatives can easily be shifted to the integrand functions without encountering surface term contributions because, to the requisite order in $1/m_b$, the two-dimensional parton phase space never touches the boundary of the three-dimensional particle phase space, except at maximal q^2 and q_0 where the integration measure is zero. The only nontrivial phase-space integration that remains to be done is with regard to q^2.

If desired, the transformation to the usual (q_0, q^2, E_l) set of variables can be done with the help of the relation

$$\cos\Theta = \frac{q_0(q^2 + m_l^2) - 2q^2 E_l}{\sqrt{q_0^2 - q^2}(q^2 - m_l^2)} \,. \tag{18}$$

However, as can be easily appreciated by substituting (18) in the differential rate (14, 15), the integration of the corresponding differential rate function is more cumbersome for the following two reasons. First, when doing the q_0-integration, one has to carefully consider the contributions from the surface terms induced by the derivatives of the δ-function from the OPE expansion (see (10)). These may lead to the appearance of spurious singularities when calculating polarization type observables [25,9]. Second, one remains with two nontrivial (E_l and q^2) integrations, apart from having to carefully consider surface term contributions when doing the q_0-integration.

Returning to the differential rate (14, 15) one first does the q_0-integration which, after integration by parts, amounts to a mere substitution as noted before. The results of the q_0-integration are listed in Appendix A. Next one integrates over q^2 in the limits $m_l^2 \le q^2 \le (m_1 - m_2)^2$. Numerical and analytical results of the q^2-integration can be found in Tables 1 and 2 and in Appendix B.

We use in the Tables 1, 2 the following numerical values: $m_\tau = 1.777$ GeV, $m_c = 1.451$ GeV, $m_b = 4.808$ GeV. K_b is given by $K_b = \mu_\pi^2/(2m_b^2)$ with $\mu_\pi^2 = 0.6$ GeV^2 and for ϵ_b we adopt the value $\epsilon_b = -2K_b/3$ as explained above. The values in brackets show the free quark decay (FQD) results.

Table 1. Numerical values for polarized and unpolarized helicity structure functions into negative helicity leptons. Column 2: $b \to c, l = \tau$; column 3: $b \to c, l = e$ ($m_e = 0$); column 4: $b \to u$ ($m_u = 0$), $l = \tau$; column 5: $b \to u, l = e$ ($m_e = 0$)

	$b \to c$		$b \to u$	
	$\eta = m_\tau^2/m_b^2$	$\eta = 0$	$\eta = m_\tau^2/m_b^2$	$\eta = 0$
$\hat{\Gamma}_U^-$	0.0389 (0.0376)	0.177 (0.172)	0.133 (0.124)	0.352 (0.333)
$\hat{\Gamma}_L^-$	0.0330 (0.0352)	0.332 (0.344)	0.124 (0.136)	0.635 (0.666)
$\hat{\Gamma}_F^-$	–0.0185 (–0.0208)	–0.105 (–0.112)	–0.093 (–0.124)	–0.282 (–0.333)
$\hat{\Gamma}_U^{P-}$	0.0183 (0.0208)	0.104 (0.112)	0.092 (0.124)	0.279 (0.333)
$\hat{\Gamma}_L^{P-}$	–0.0282 (–0.0306)	–0.322 (–0.331)	–0.116 (–0.136)	–0.629 (–0.666)
$\hat{\Gamma}_F^{P-}$	–0.0385 (–0.0376)	–0.176 (–0.172)	–0.132 (–0.124)	–0.349 (–0.333)
$\hat{\Gamma}_I^{P-}$	–0.0397 (–0.0349)	–0.208 (–0.213)	–0.123 (–0.128)	–0.418 (–0.431)
$\hat{\Gamma}_A^{P-}$	–0.0239 (–0.0263)	–0.179 (–0.186)	–0.103 (–0.128)	–0.390 (–0.431)

Table 2. Numerical values for polarized and unpolarized helicity structure functions into positive helicity τ-leptons for $b \to c$ and $b \to u$ transitions. Parameter values as in Table 1

	$b \to c$	$b \to u$
$\hat{\Gamma}_U^+$	0.0081 (0.0079)	0.0198 (0.0186)
$\hat{\Gamma}_L^+$	0.0076 (0.0080)	0.0218 (0.0234)
$\hat{\Gamma}_S^+$	0.0079 (0.0080)	0.0231 (0.0234)
$\hat{\Gamma}_{SL}^+$	0.0066 (0.0072)	0.0210 (0.0234)
$\hat{\Gamma}_U^{P+}$	0.0041 (0.0046)	0.0154 (0.0186)
$\hat{\Gamma}_L^{P+}$	-0.0067 (-0.0072)	-0.0213 (-0.0234)
$\hat{\Gamma}_S^{P+}$	-0.0066 (-0.0072)	-0.0208 (-0.0234)
$\hat{\Gamma}_{SL}^{P+}$	-0.0077 (-0.0080)	-0.0222 (-0.0234)
$\hat{\Gamma}_{ST}^{P+}$	-0.0077 (-0.0076)	-0.0209 (-0.0205)
$\hat{\Gamma}_A^{P+}$	-0.0055 (-0.0060)	-0.0181 (-0.0205)

In order to simplify our notation we introduce a set of unpolarized and polarized reduced differential rate functions $d\hat{\Gamma}_i^-$ and $d\hat{\Gamma}_i^{P-}$, respectively, for the decays into negative helicity leptons. We define scaled variables $\hat{q}^2 = q^2/m_b^2$, $\hat{q}_0 = q_0/m_b$, $\rho = m_j^2/m_b^2$ and $\eta = m_\tau^2/m_b^2$ and

write

$$\begin{aligned}
\frac{\mathrm{d}\hat{\Gamma}_U^{(P)-}}{\mathrm{d}\hat{q}^2} &= 16\frac{(\hat{q}^2-\eta)^2}{\hat{q}^2} \\
&\quad \times I(W_{++}^{++}+W_{--}^{++}+(-)(W_{++}^{--}+W_{--}^{--})) \\
\frac{\mathrm{d}\hat{\Gamma}_L^{(P)-}}{\mathrm{d}\hat{q}^2} &= 16\frac{(\hat{q}^2-\eta)^2}{\hat{q}^2} I(W_{00}^{++}+(-)W_{00}^{--}) \\
\frac{\mathrm{d}\hat{\Gamma}_F^{(P)-}}{\mathrm{d}\hat{q}^2} &= 16\frac{(\hat{q}^2-\eta)^2}{\hat{q}^2} \\
&\quad \times I(W_{++}^{++}-W_{--}^{++}+(-)(W_{++}^{--}-W_{--}^{--})) \\
\frac{\mathrm{d}\hat{\Gamma}_I^{P-}}{\mathrm{d}\hat{q}^2} &= 16\frac{(\hat{q}^2-\eta)^2}{\hat{q}^2} I(W_{-0}^{+-}+W_{0+}^{+-}) \\
\frac{\mathrm{d}\hat{\Gamma}_A^{P-}}{\mathrm{d}\hat{q}^2} &= 16\frac{(\hat{q}^2-\eta)^2}{\hat{q}^2} I(W_{-0}^{+-}-W_{0+}^{+-}) \qquad (19)
\end{aligned}$$

where $\mathrm{d}\hat{\Gamma}_U^{(P)-}$ stands for either $\mathrm{d}\hat{\Gamma}_U^{-}$ or $\mathrm{d}\hat{\Gamma}_U^{P-}$ etc. with the corresponding signs specified on the r.h.s. of (19). Accordingly we define reduced differential rate functions for decays into positive helicity leptons.

$$\begin{aligned}
\frac{\mathrm{d}\hat{\Gamma}_U^{(P)+}}{\mathrm{d}\hat{q}^2} &= \frac{\eta}{2\hat{q}^2}\frac{\mathrm{d}\hat{\Gamma}_U^{(P)-}}{\mathrm{d}\hat{q}^2} \\
\frac{\mathrm{d}\hat{\Gamma}_L^{(P)+}}{\mathrm{d}\hat{q}^2} &= \frac{\eta}{2\hat{q}^2}\frac{\mathrm{d}\hat{\Gamma}_L^{(P)-}}{\mathrm{d}\hat{q}^2} \\
\frac{\mathrm{d}\hat{\Gamma}_S^{(P)+}}{\mathrm{d}\hat{q}^2} &= 16\frac{\eta}{2\hat{q}^2}\frac{(\hat{q}^2-\eta)^2}{\hat{q}^2} I(W_{tt}^{++}+(-)W_{tt}^{--}) \\
\frac{\mathrm{d}\hat{\Gamma}_{SL}^{(P)+}}{\mathrm{d}\hat{q}^2} &= 16\frac{\eta}{2\hat{q}^2}\frac{(\hat{q}^2-\eta)^2}{\hat{q}^2} I(W_{0t}^{++}+(-)W_{0t}^{--}) \\
\frac{\mathrm{d}\hat{\Gamma}_{ST}^{P+}}{\mathrm{d}\hat{q}^2} &= 16\frac{\eta}{2\hat{q}^2}\frac{(\hat{q}^2-\eta)^2}{\hat{q}^2} I(W_{-t}^{+-}-W_{t+}^{+-}) \\
\frac{\mathrm{d}\hat{\Gamma}_A^{P+}}{\mathrm{d}\hat{q}^2} &= \frac{\eta}{2\hat{q}^2}\frac{\mathrm{d}\hat{\Gamma}_A^{P-}}{\mathrm{d}\hat{q}^2} \qquad (20)
\end{aligned}$$

Note that the structure function combination $I(W_{-t}^{+-}+W_{t+}^{+-})$ does not appear in (19) and (20). This combination can only be measured through the transverse spin components of the τ-lepton as discussed in the next section. The differential rate into negative helicity leptons can then be written as

$$\begin{aligned}
&\frac{\mathrm{d}\Gamma^-}{\mathrm{d}\hat{q}^2\mathrm{d}\cos\Theta\mathrm{d}\cos\Theta_P\mathrm{d}\phi} = \frac{\Gamma_b}{4\pi}\times \\
&\left[\frac{3}{8}\left(\frac{d\hat{\Gamma}_U^-}{d\hat{q}^2}+P\cos\Theta_P\frac{d\hat{\Gamma}_U^{P-}}{d\hat{q}^2}\right)(1+\cos^2\Theta)\right. \\
&+\frac{3}{4}\left(\frac{d\hat{\Gamma}_L^-}{d\hat{q}^2}+P\cos\Theta_P\frac{d\hat{\Gamma}_L^{P-}}{d\hat{q}^2}\right)\sin^2\Theta \\
&+\frac{3}{4}\left(\frac{d\hat{\Gamma}_F^-}{d\hat{q}^2}+P\cos\Theta_P\frac{d\hat{\Gamma}_F^{P-}}{d\hat{q}^2}\right)\cos\Theta \\
&+\frac{3}{2\sqrt{2}}\frac{d\hat{\Gamma}_I^{P-}}{d\hat{q}^2}P\sin\Theta_P\sin\Theta\cos\phi \\
&\left.-\frac{3}{4\sqrt{2}}\frac{d\hat{\Gamma}_A^{P-}}{d\hat{q}^2}P\sin\Theta_P\sin 2\Theta\cos\phi\right] \qquad (21)
\end{aligned}$$

where

$$\Gamma_b = \frac{|V_{bj}|^2 G_F^2 m_b^5}{192\pi^3}\ . \qquad (22)$$

For the rate into positive helicity leptons we have

$$\begin{aligned}
&\frac{\mathrm{d}\Gamma^+}{\mathrm{d}\hat{q}^2\mathrm{d}\cos\Theta\mathrm{d}\cos\Theta_P\mathrm{d}\phi} = \frac{\Gamma_b}{4\pi}\times \\
&\left[\frac{3}{4}\left(\frac{d\hat{\Gamma}_U^+}{d\hat{q}^2}+P\cos\Theta_P\frac{d\hat{\Gamma}_U^{P+}}{d\hat{q}^2}\right)\sin^2\Theta\right. \\
&+\frac{3}{2}\left(\frac{d\hat{\Gamma}_L^+}{d\hat{q}^2}+P\cos\Theta_P\frac{d\hat{\Gamma}_L^{P+}}{d\hat{q}^2}\right)\cos^2\Theta \\
&+\frac{3}{2}\left(\frac{d\hat{\Gamma}_S^+}{d\hat{q}^2}+P\cos\Theta_P\frac{d\hat{\Gamma}_S^{P+}}{d\hat{q}^2}\right) \\
&+3\left(\frac{d\hat{\Gamma}_{SL}^+}{d\hat{q}^2}+P\cos\Theta_P\frac{d\hat{\Gamma}_{SL}^{P+}}{d\hat{q}^2}\right)\cos\Theta \\
&-\frac{3}{2}\sqrt{2}\frac{d\hat{\Gamma}_{ST}^{P+}}{d\hat{q}^2}P\sin\Theta_P\sin\Theta\cos\phi \\
&\left.-\frac{3}{4}\sqrt{2}\frac{d\hat{\Gamma}_A^{P+}}{d\hat{q}^2}P\sin\Theta_P\sin 2\Theta\cos\phi\right] \qquad (23)
\end{aligned}$$

For quick reference we shall adopt a generic labelling for the various helicity structure functions, namely we write $U^{(P)-}$ for $\mathrm{d}\Gamma_U^{(P)-}/\mathrm{d}\hat{q}^2$, etc. An inspection of the q^2 dependence of the helicity structure functions shows that the structure functions $U^{(P)-}$, $L^{(P)-}$, $F^{(P)-}$, $U^{(P)+}$, $L^{(P)+}$, $S^{(P)+}$ and $SL^{(P)+}$ can be integrated analytically. In contrast to this, the integrations of the structure functions $I^{(P)-}$, $A^{(P)-}$, $ST^{(P)+}$ and $A^{(P)+}$ lead to incomplete elliptic functions and will therefore be performed numerically.

In Appendix B we list analytic results for the totally integrated unpolarized structure functions U^-, L^-, F^-, U^+, L^+ and S^+ as well as for the total rate function proportional to $(U^- + U^+ + L^- + L^+ + 3S^+)$. Numerical results are listed in Tables 1 and 2, where we have used the following set of input values: $m_b = 4.8$ GeV, $m_c = 1.45$ GeV, $m_u = 0$ GeV, $m_\tau = 1.777$ GeV, $K_b = \mu_\pi^2/(2m_b^2)$, $\mu_\pi^2 = 0.6$ GeV^2 and $\epsilon_b = -2K_b/3$. In order to assess the importance of the nonperturbative contributions we also list the zeroth order parton model values ($K_b = \epsilon_b = 0$) in brackets. Judging from the numerical entries in Tables 1 and 2 the effect of the nonperturbative contributions can go both ways, i.e. depending on the particular structure function, they can enhance or decrease the parton model results. The nonperturbative contributions are generally small, at the few percent level. Notable are the structure functions U^{P-}, A^{P-}, SL^+, U^{P+} and L^{P+} where the nonperturbative contributions exceed 10%.

Fully integrated values of the longitudinal polarization of the τ can easily be constructed in analogy to (17). It is also evident that there are no transitions into positive

helicities for $m_l = 0$ which explains why Table 2 has only two columns.

The structure function combinations $(U^- + F^{P-} \pm (U^{P-} + F^-))$ are of particular interest since they are directly proportional to the nonperturbative spin parameter ϵ_b. They receive contributions from the helicity $\pm 3/2$ configurations, which are neither populated by the parton model contribution nor by the spin neutral kinetic energy term K_b. In fact, for the rate combinations $(U^- + F^{P-})$ and $(U^{P-} + F^-)$ one finds

$$\begin{aligned}\hat{\Gamma}_U^- + \hat{\Gamma}_F^{P-} &= -\epsilon_b \hat{\Gamma}_U^-(K_b = 0) \\ &= -\epsilon_b \left[\frac{\sqrt{R}}{3}(1 - 7\eta - 7\eta^2 + \eta^3 - 7\rho + 12\eta\rho \right. \\ &\quad -7\eta^2\rho - 7\rho^2 - 7\eta\rho^2 + \rho^3) \\ &\quad +8\left[\rho^2(\eta^2 - 1)\ln\left(\frac{1 - \eta + \rho - \sqrt{R}}{2\sqrt{\rho}}\right)\right. \\ &\quad \left.\left. +\eta^2(\rho^2 - 1)\ln\left(\frac{1 + \eta - \rho - \sqrt{R}}{2\sqrt{\eta}}\right)\right]\right] \end{aligned} \tag{24}$$

and

$$\begin{aligned}\hat{\Gamma}_U^{P-} + \hat{\Gamma}_F^- &= -\epsilon_b \hat{\Gamma}_F^-(K_b = 0) \\ &= -\epsilon_b \Big[(-1 + \eta + 2\sqrt{\rho} - \rho) \\ &\quad \times(1 - 7\eta - 7\eta^2 + \eta^3 + 2\sqrt{\rho} - 12\eta\sqrt{\rho} \\ &\quad -2\eta^2\sqrt{\rho} - 17\rho + 38\eta\rho - 7\eta^2\rho + 28\sqrt{\rho}^3 \\ &\quad -12\eta\sqrt{\rho}^3 - 17\rho^2 - 7\eta\rho^2 + 2\sqrt{\rho}^5 + \rho^3)/3 \\ &\quad +4\eta^2(1 - \rho)^2 \ln\left(\frac{\eta}{(1 - \sqrt{\rho})^2}\right)\Big] \end{aligned} \tag{25}$$

where the analytical results on the r.h.s. of (24) and (25) can be obtained from the closed form expressions for $\hat{\Gamma}_U^-$ and $\hat{\Gamma}_F^-$ in Appendix B by setting $K_b = 0$.

Numerically one has

$$\begin{aligned}\hat{\Gamma}_U^- + \hat{\Gamma}_F^{P-} &= -0.0376 \cdot \epsilon_b \\ \hat{\Gamma}_U^{P-} + \hat{\Gamma}_F^- &= 0.0208 \cdot \epsilon_b \end{aligned} \tag{26}$$

It is clear that the contribution of K_b could be extracted from taking appropriate linear combinations of the fully integrated structure functions listed in Appendix B. Since the coefficients needed in this extraction involve high powers of the masses in the process, which are uncertain, it would be desirable to find an observable directly proportional to K_b. Such an observable appears in the measurement of the transverse polarization of the τ as discussed in the next section.

4 Transverse polarization of the τ-Lepton

In this section we present results on the transverse polarization components of the τ-lepton in the $(\tau, \bar{\nu}_\tau)$ c.m. frame. The two transverse components are conventionally divided into the transverse perpendicular component in the lepton-hadron plane and the transverse normal component out of the lepton-hadron plane. The latter component is not affected by the boost from the Λ_b rest frame to the $(\tau, \bar{\nu}_\tau)$ c.m. frame and, after the appropriate integrations, can therefore be compared with the corresponding calculation done in the Λ_b rest frame in [9].

The 2×2 density matrix of the τ-lepton can be obtained from the master formula

$$\begin{aligned}&\frac{\mathrm{d}\Gamma_{\lambda_l \lambda_l'}}{\mathrm{d}q_0 \mathrm{d}q^2 \mathrm{d}\cos\Theta \mathrm{d}\cos\Theta_P \mathrm{d}\phi} \\ &= \frac{2G^2|V_{bj}|^2(q^2 - m_\tau^2)\sqrt{q_0^2 - q^2}}{3(2\pi)^4 q^2}\{W(\Theta, \phi, \Theta_P)\}_{\lambda_l \lambda_l'}, \end{aligned} \tag{27}$$

where [2]

$$\begin{aligned}&\{W(\Theta, \phi, \Theta_P)\}_{\lambda_l \lambda_l'} \\ &= \frac{3}{16} \sum_{m,m',J,J',\lambda_b,\lambda_b'} \Big[(-1)^{J+J'} h_{\lambda_l \frac{1}{2}} h^*_{\lambda_l' \frac{1}{2}} e^{-i(m-m')\phi} \\ &\cdot d^J_{m,\lambda_l - \frac{1}{2}}(\pi - \Theta) d^{J'}_{m',\lambda_l' - \frac{1}{2}}(\pi - \Theta) W^{\lambda_b \lambda_b'}_{mm'} \rho_{\lambda_b \lambda_b'}(\Theta_P)\Big] . \end{aligned} \tag{28}$$

The helicity amplitudes $h_{\lambda_l \frac{1}{2}}$ $(\lambda_l = \pm\frac{1}{2})$ for the decay $W^-_{off-shell} \to \tau^- + \bar{\nu}_\tau$ appearing in the master formula are given by [4]

$$\begin{aligned} h_{-\frac{1}{2}\frac{1}{2}} &= \sqrt{8(q^2 - m_\tau^2)} = 2\sqrt{2} m_b \sqrt{\hat{q}^2 - \eta} \\ h_{\frac{1}{2}\frac{1}{2}} &= \sqrt{\frac{m_\tau^2}{2q^2}} h_{-\frac{1}{2}\frac{1}{2}} = \sqrt{\frac{\eta}{2\hat{q}^2}} h_{-\frac{1}{2}\frac{1}{2}} . \end{aligned} \tag{29}$$

The diagonal elements of the density matrix $\{W\}_{\lambda_l \lambda_l'}$ have already been written down in the main text. Here we list the nondiagonal density matrix element relevant for the transverse polarization components of the τ.

For the unnormalized transverse perpendicular component of the polarization vector one needs to calculate $\{W\}^x = \{W\}_{+-} + \{W\}_{-+}$. One has

$$\begin{aligned}&\{W\}^x = \{W(\Theta, \phi, \Theta_P)\}_{+-} + \{W(\Theta, \phi, \Theta_P)\}_{-+} \\ &= \frac{m_\tau}{\sqrt{2q^2}}(q^2 - m_l^2) \\ &\Big[\frac{3}{2\sqrt{2}}(\rho_{++}(W^{++}_{++} - W^{++}_{--}) \\ &+\rho_{--}(W^{--}_{++} - W^{--}_{--}))\sin\Theta \\ &-\frac{3}{4\sqrt{2}}(\rho_{++}(W^{++}_{++} + W^{++}_{--} - 2W^{++}_{00}) \\ &+\rho_{--}(W^{--}_{++} + W^{--}_{--} - 2W^{--}_{00}))\sin 2\Theta \\ &+\frac{3}{\sqrt{2}}(\rho_{++}W^{++}_{0t} + \rho_{--}W^{--}_{0t})\sin\Theta \\ &-\frac{3}{2}\rho_{+-}(W^{+-}_{0+} - W^{+-}_{-0})\cos 2\Theta \cos\phi \end{aligned}$$

$$-\frac{3}{2}\rho_{+-}(W_{0+}^{+-}+W_{-0}^{+-}+W_{t+}^{+-}-W_{-t}^{+-})\cos\Theta$$
$$\cos\phi-\frac{3}{2}\rho_{+-}(W_{t+}^{+-}+W_{-t}^{+-})\cos\phi\Big] \quad (30)$$

Note that the 14^{th} structure function combination $(W_{t+}^{+-}+W_{-t}^{+-})$ that was missing from the rate expressions in the main text makes its first appearance in the transverse normal polarization. As (30) shows, Λ_b-polarization as well as a determination of the τ-lepton's transverse polarization is necessary for a determination of the complete set of 14 structure functions.

The unnormalized transverse normal component is given by the combination $i(W_{+-}-W_{-+})$. One obtains

$$\{W\}^y = i(\{W(\Theta,\phi,\Theta_P)\}_{+-}-\{W(\Theta,\phi,\Theta_P)\}_{-+})$$
$$=\frac{m_\tau}{\sqrt{2q^2}}(q^2-m_\tau^2)$$
$$\times\Big[+\frac{3}{2}\rho_{+-}(W_{0+}^{+-}+W_{-0}^{+-}+W_{t+}^{+-}-W_{-t}^{+-})\sin\phi$$
$$+\frac{3}{2}\rho_{+-}(W_{0+}^{+-}-W_{-0}^{+-}+W_{t+}^{+-}+W_{-t}^{+-})\cos\Theta\sin\phi\Big] \quad (31)$$

Note again the contribution from the 14^{th} structure function combination $(W_{t+}^{+-}+W_{-t}^{+-})$. It is quite remarkable that the two combinations of helicity structure functions appearing in the transverse normal polarization of the τ in (31) can be seen to be entirely determined by the nonperturbative K_b contribution. We have no simple explanation of this fact.

The q_0-integration of the relevant density matrix elements (27) can easily be done as described in the main text. In fact, the relevant q_0-integration of the 14^{th} structure function combination is listed in Appendix A. The remaining q^2-integration is then done numerically. The numerical result is presented for the case $b\to c$ using the same numerical values as in Sect. III. We obtain

$$\frac{\mathrm{d}\Gamma^x}{\mathrm{d}\cos\Theta\mathrm{d}\cos\Theta_P\mathrm{d}\phi}$$
$$=\frac{\Gamma_b}{4\pi}\,[-0.03777\sin\Theta+0.00725\sin2\Theta$$
$$-P\cos\Theta_P(0.01519\sin\Theta+0.01907\sin2\Theta)$$
$$+P\sin\Theta_P\cos\phi(0.01682-0.00091\cos\Theta$$
$$+0.01714\cos2\Theta)] \quad (32)$$
$$\frac{\mathrm{d}\Gamma^y}{\mathrm{d}\cos\Theta\mathrm{d}\cos\Theta_P\mathrm{d}\phi}$$
$$=\frac{\Gamma_b}{4\pi}(+0.000908+0.000323\cos\Theta)P\sin\Theta_P\sin\phi \quad (33)$$

The transverse perpendicular polarization is dominated by the zeroth order parton contribution with angular coefficients comparable to the entries in Tables 1 and 2. The transverse normal angular coefficients are quite small as expected since they are proportional to K_b.

As mentioned before the transverse normal polarization is not affected by the boost from the lab frame to the $(\tau^-\bar\nu_\tau)$ c.m. frame. In order to compare our results with the corresponding results in [9] we integrate the transverse normal polarization with respect to $\cos\Theta$ and $\cos\Theta_P$ to obtain

$$\frac{\mathrm{d}\Gamma^y}{\mathrm{d}\hat q^2\mathrm{d}\phi}=+\frac{\Gamma_b}{2\pi}12\pi P\frac{(\hat q^2-\eta)^2}{\hat q^2}\sqrt{\frac{\eta}{2\hat q^2}} \quad (34)$$
$$\times I(W_{0+}^{+-}+W_{-0}^{+-}+W_{t+}^{+-}-W_{-t}^{+-})\sin\phi$$

As noted above, the linear combination $I(W_{0+}^{+-}+W_{-0}^{+-}+W_{t+}^{+-}-W_{-t}^{+-})$ is proportional to K_b only and has no zeroth order partonic contribution. The $\hat q^2$-integration is easily done and one finds

$$\frac{\mathrm{d}\Gamma^y}{\mathrm{d}\phi}=\frac{\Gamma_b}{2\pi}PA\sin\phi \quad (35)$$

where

$$A=-2\pi K_b\sqrt{\eta}\,\Big[\sqrt{R}(-2-5\eta+\eta^2-5\rho+\rho^2+10\eta\rho)/3$$
$$-4\eta(1-2\rho+\eta\rho+\rho^2)\ln\left(\frac{1+\eta-\rho-\sqrt{R}}{2\sqrt{\eta}}\right)$$
$$-4\rho(1-2\eta+\eta\rho+\eta^2)\ln\left(\frac{1-\eta+\rho-\sqrt{R}}{2\sqrt{\rho}}\right)\Big] \quad (36)$$

This result agrees with the result in [9] when their corresponding y-distribution is integrated with respect to y. Numerically one has

$$A=+0.11K_b \quad (37)$$

The results on the τ polarization discussed in the last two sections refer to the $(\tau,\bar\nu_\tau)$ rest frame which, because of the elusiveness of the neutrino $\bar\nu_\tau$, may not always be easy to construct. For some applications it may be preferable to avail of the longitudinal polarization of the τ in the Λ_b rest frame. This is the subject of the next section.

5 Polarization correlations in the Λ_b rest frame

In this section we determine the polarization of the τ in the Λ_b rest frame using a calculational technique originally proposed in [26] which involves an averaging over the azimuthal angle ϕ (see Fig. 1). In this way we determine the polarization components of the τ in the plane spanned by the τ and the polarization vector of the Λ_b.

The method of [26] is based on applying the unitarity relation to the amplitude for forward $\bar\tau\Lambda_b\to\bar\tau\Lambda_b$ scattering (see Fig. 2)

$$T(s)= \quad (38)$$
$$i\langle\Lambda_b(v,s)\bar\tau(p_\tau)|\int\mathrm{d}x\mathrm{T}\mathcal{H}_W^\dagger(x)\mathcal{H}_W(0)|\Lambda_b(v,s)\bar\tau(p_\tau)\rangle\,,$$

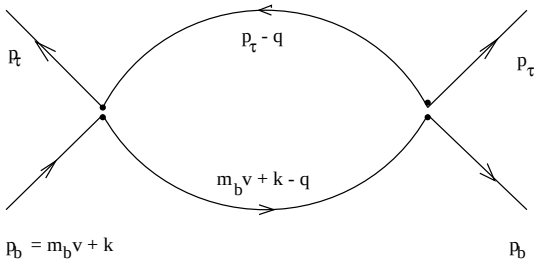

Fig. 2. Lowest-order graph contributing to the forward scattering amplitude $T(s)$

where

$$\mathcal{H}_W = 2\sqrt{2}V_{jb}G_F\,[\bar{j}\gamma_\mu P_L b][\bar{e}\gamma^\mu P_L \nu_e] \qquad (P_L = \frac{1}{2}(1-\gamma_5)) \tag{39}$$

is the interaction Hamiltonian responsible for the semileptonic decay $b \to j\tau^-\bar{\nu}_\tau$ and $s = (m_{\Lambda_b}v + p_\tau)^2$. By inserting a complete set of states in (38) one obtains

$$\begin{aligned}\text{Im } T(s) = \frac{1}{2}\sum_{X_j,\nu_\tau}\int \mathrm{d}\mu(X_j)\mathrm{d}\mu(\nu_\tau) \\ (2\pi)^4\delta^4(m_{\Lambda_b}v + p_\tau - p_{X_j} - p_{\nu_\tau}) \\ \times |\langle X_j\bar{\nu}_\tau|\mathcal{H}_W(0)|\Lambda_b(v,s)\bar{\tau}(p_\tau)\rangle|^2\,.\end{aligned} \tag{40}$$

The phase-space volume element is $\mathrm{d}\mu = \frac{d^3p}{(2\pi)^3 2E}$.

Comparing (40) with the inclusive semileptonic decay rate

$$\begin{aligned}\mathrm{d}\Gamma = |\langle X_j\bar{\nu}_\tau(p_{\nu_\tau})\tau(p_\tau)|\mathcal{H}_W|\Lambda_b(v,s)\rangle|^2 \\ (2\pi)^4\delta^4(m_{\Lambda_b}v - p_\tau - p_{X_j} - p_{\nu_\tau})\mathrm{d}\mu(X_j)\mathrm{d}\mu(\nu_\tau)\mathrm{d}\mu(\tau)\end{aligned} \tag{41}$$

one obtains the final formula relating (38) to quantities of experimental interest:

$$\mathrm{d}\Gamma = \frac{1}{(2\pi)^3}|\mathbf{p}_\tau|\mathrm{d}E_\tau\mathrm{d}\Omega_\tau\,\text{Im } T(s)\,. \tag{42}$$

Our problem is thus reduced to computing the function Im $T(s)$. This can be done with the help of an operator-product expansion combined with the heavy mass expansion as discussed in Sect. II. First the heavy hadron is replaced with a heavy quark with momentum $p_b = m_b v + k$. From the 1-loop diagram in Fig. 3 one reads off the lowest-order expression for $T(s)$

$$\begin{aligned}T(s) = 8G_F^2|V_{jb}|^2\,[\bar{u}(p_\tau)\gamma_\mu\gamma^\alpha\gamma_\nu P_L u(p_\tau)] \\ \times\left[\bar{u}(v)\gamma^\nu\gamma^\beta\gamma^\mu P_L u(v)\right] I_{\alpha\beta}(k)\end{aligned} \tag{43}$$

where we have defined

$$I_{\alpha\beta}(k) = \int\frac{\mathrm{d}^n q}{(2\pi)^n}\frac{(p_\tau - q)_\alpha(m_b v + k - q)_\beta}{[(p_\tau - q)^2 + i\epsilon][(m_b v + k - q)^2 - m_c^2 + i\epsilon]}\,. \tag{44}$$

We will be interested in polarized τ leptons in the final state. Thus the leptonic amplitude in (43) should be replaced by

$$\begin{aligned}\bar{u}(p_\tau)\gamma_\mu\gamma^\alpha\gamma_\nu P_L u(p_\tau) \to \\ \text{Tr}\,\left\{\frac{1}{2m_\tau}(\not{p}_\tau + m_\tau)\frac{1}{2}(1+\gamma_5\not{s}_\tau)\gamma_\mu\gamma^\alpha\gamma_\nu P_L\right\},\end{aligned} \tag{45}$$

with s_τ the spin vector of the τ lepton.

The integral $I_{\alpha\beta}(k)$ in (44) can be calculated by combining the denominators with a Feynman parameter x. The result is a complex function with an imaginary part given by

$$\begin{aligned}\frac{1}{2\pi}\text{Im } I_{\alpha\beta}(k) \\ = \frac{1}{(4\pi)^2}\int_0^1 \mathrm{d}x\Big\{\frac{1}{2}g_{\alpha\beta}s - x(1-x) \\ \times[m_b v + k - p_\tau]_\alpha[m_b v + k - p_\tau]_\beta\Big\}\theta(x_1 - x)\end{aligned} \tag{46}$$

where

$$\begin{aligned}s = x\left\{-m_\tau^2(1-x) + m_c^2 - (m_b v + k)^2(1-x)\right. \\ \left.+2(1-x)p_\tau\cdot(m_b v + k)\right\}\,.\end{aligned} \tag{47}$$

In (46) we have denoted x_1 the root of the equation $s(x_1) = 0$. It is given by

$$\begin{aligned}x_1 = 1 - \frac{\rho}{(1+\eta-y)\left(1 + \dfrac{2\tilde{k}\cdot(v-\tilde{p}_\tau) + \tilde{k}^2}{1+\eta-y}\right)} \\ = x_0 + \frac{1-x_0}{1+\eta-y} \\ \times\left(2\tilde{k}\cdot(v-\tilde{p}_\tau) + \tilde{k}^2 - \frac{4[\tilde{k}\cdot(v-\tilde{p}_\tau)]^2}{1+\eta-y} + \cdots\right),\end{aligned} \tag{48}$$

where $x_0 = 1 - \rho/(1+\eta-y)$ is the value of x_1 for $k = 0$. We have introduced reduced momenta $\tilde{k} = k/m_b$, $\tilde{p}_\tau = p_\tau/m_b$ and have expanded in powers of $\tilde{k}$ up to second order.

The integration in (46) can easily be performed with the result

$$\begin{aligned}\text{Im } I_{\alpha\beta}(k) = \frac{1}{8\pi}\left\{\frac{1}{4}\rho x_1^2\right. \\ \left.+\frac{1}{2}\left(\frac{x_1^2}{2} - \frac{x_1^3}{3}\right)\left[-\eta - (v+\tilde{k})^2 + 2\tilde{p}_\tau\cdot(v+\tilde{k})\right]\right\}g_{\alpha\beta} \\ -\frac{1}{8\pi}\left(\frac{x_1^2}{2} - \frac{x_1^3}{3}\right)(v + \tilde{k} - \tilde{p}_\tau)_\alpha(v + \tilde{k} - \tilde{p}_\tau)_\beta\,,\end{aligned} \tag{49}$$

where x_1 has to be replaced with the expanded form of (48).

In physical applications we are interested in heavy hadron decay, rather than free heavy quark decay. One can obtain the corresponding scattering amplitude $T(s)$ by replacing the hadronic spinor expression in (43) with expectation values of the appropriate operators

$$\bar{u}(v)f(\tilde{k}_\alpha)u(v) \to \langle\Lambda_b(v,s)|\bar{b}f\left(\frac{iD_\alpha}{m_b}\right)b|\Lambda_b(v,s)\rangle\,. \tag{50}$$

These matrix elements can in turn be expanded in powers of $1/m_b$ with the help of heavy quark effective theory (HQET) methods as discussed earlier. Referring to [13,10] for calculational details, we only give the final substitution rules needed for computing Im $T(s)$.

a) terms of order $\tilde{k}^0$

$$\gamma_\mu \to v_\mu \tag{51}$$

$$\gamma_\mu\gamma_5 \to s_\mu(1+\epsilon_b) \tag{52}$$

b) terms of order $\tilde{k}^1$

$$\tilde{k}_\mu\gamma_\nu \to \frac{1}{3}K_b(-2g_{\mu\nu}+5v_\mu v_\nu) \tag{53}$$

$$\tilde{k}_\mu\gamma_\nu\gamma_5 \to K_b(v_\mu s_\nu + \frac{2}{3}v_\nu s_\mu) \tag{54}$$

c) terms of order $\tilde{k}^2$

$$\tilde{k}_\mu\tilde{k}_\nu\gamma_\alpha \to -\frac{2}{3}K_b(g_{\mu\nu}-v_\mu v_\nu)v_\alpha \tag{55}$$

$$\tilde{k}_\mu\tilde{k}_\nu\gamma_\alpha\gamma_5 \to -\frac{2}{3}K_b(g_{\mu\nu}-v_\mu v_\nu)s_\alpha \tag{56}$$

It is now a simple matter to combine (48,49) and (51-56) into (42) and extract the decay rate. It can conveniently be written by splitting the rate into two terms

$$\mathrm{d}\Gamma = \mathrm{d}\Gamma + \mathrm{d}\Gamma^p = \frac{1}{2}\left(\mathrm{d}\Gamma(s_\tau)+\mathrm{d}\Gamma(-s_\tau)\right) + \frac{1}{2}\left(\mathrm{d}\Gamma(s_\tau)-\mathrm{d}\Gamma(-s_\tau)\right) . \tag{57}$$

$\mathrm{d}\Gamma$ and $\mathrm{d}\Gamma^p$ represent the decay rates into unpolarized and polarized leptons. For the unpolarized rate we obtain

$$\frac{1}{\Gamma_b}\frac{\mathrm{d}\Gamma}{\mathrm{d}y\mathrm{d}\cos\theta_\tau} = \sqrt{y^2-4\eta}\left(A(y)+(\tilde{p}_\tau\cdot s)B(y)\right) , \tag{58}$$

where

$$\begin{aligned}
A(y) &= x_0^2(-3y^2+6y(1+\eta)-12\eta) \\
&+x_0^3(y^2-3y(1+\eta)+8\eta) \\
&+K_b\Big\{2[(1+\eta)^2-y^2] \\
&-4x_0[y^2-y(1+\eta)+2(1+\eta^2)] \\
&+x_0^2[4y^2-8y(1+\eta)+16\eta+10(1+\eta^2)] \\
&+x_0^3[-\frac{4}{3}y^2+4y(1+\eta)-\frac{32}{3}\eta-4(1+\eta^2)] \\
&-4\frac{(1-\eta)^2(1+\eta)(1-x_0)^2(1-3x_0)}{1+\eta-y} \\
&+2\frac{(1-\eta)^4(1-x_0)^2(1-4x_0)}{(1+\eta-y)^2}\Big\}
\end{aligned} \tag{59}$$

$$\begin{aligned}
B(y) = \Big\{&(1+\epsilon_b)[6x_0^2(y-2\eta)-2x_0^3(1+y-3\eta) \\
&+K_b\left[4(y+2)-8x_0(2+\eta-y)\right. \\
&+8x_0^2(1+2\eta-y)+\frac{8}{3}x_0^3(y-3\eta) \\
&-4\frac{(1-\eta)(1-x_0)^2[3+\eta-2x_0(\eta+2)]}{1+\eta-y} \\
&\left.+4\frac{(1-\eta)^3(1-x_0)^2(1-4x_0)}{(1+\eta-y)^2}\right]\Big\} .
\end{aligned}$$

We have defined θ_τ as the angle between the Λ_b spin and the τ lepton momentum direction. For the decay rate into polarized leptons $\mathrm{d}\Gamma^p$ we obtain

$$2\frac{1}{\Gamma_b}\frac{\mathrm{d}\Gamma^p}{\mathrm{d}y\mathrm{d}\cos\theta_\tau} \tag{60}$$

$$\begin{aligned}
&= \sqrt{y^2-4\eta}\,\sqrt{\eta}\left\{(v\cdot s_\tau)(\tilde{p}_\tau\cdot s)A^p(y)\right. \\
&\left.+(v\cdot s_\tau)B^p(y)+(s\cdot s_\tau)C^p(y)\right\} ,
\end{aligned} \tag{61}$$

where

$$\begin{aligned}
A^p(y) = &-24(1+\epsilon_b)\left(\frac{x_0^2}{2}-\frac{x_0^3}{3}\right) \\
&-4K_b\Big[2+2x_0-5x_0^2+\frac{8}{3}x_0^3 \\
&-2\frac{(1+\eta)(1-x_0)^2(2-x_0)}{1+\eta-y} \\
&+2\frac{(1-\eta)^2(1-x_0)^2(1-4x_0)}{(1+\eta-y)^2}\Big]
\end{aligned} \tag{62}$$

$$\begin{aligned}
B^p(y) = &-6x_0^2(2-y)+2x_0^3(3-y-\eta) \\
&-4K_b\Big[-y-2\eta+2x_0(1+2\eta-y) \\
&+2x_0^2(-2-\eta+y)+\frac{2}{3}x_0^3(3-y) \\
&+\frac{(1-\eta)(1-x_0)^2[-1-3\eta+2x_0(1+2\eta)]}{1+\eta-y} \\
&+\frac{(1-\eta)^3(1-x_0)^2(1-4x_0)}{(1+\eta-y)^2}\Big]
\end{aligned} \tag{63}$$

$$\begin{aligned}
C^p(y) = &2x_0^3(1+\epsilon_b)(1+\eta-y) \\
&-2K_b\Big[2x_0(1+\eta+y)+x_0^2(-4-4\eta+y) \\
&+x_0^3(2+2\eta-\frac{4}{3}y) \\
&-\frac{2x_0(1-\eta)^2(1-x_0)^2}{1+\eta-y}\Big] .
\end{aligned} \tag{64}$$

(65)

We have checked that this formula agrees with the various particular cases presented in the literature. Thus, [17] compute the longitudinal polarization asymmetry corresponding to an unpolarized decaying baryon and [9] give the full results for various lepton polarizations from the decay of polarized Λ_b baryons. The method presented here has the advantage of performing the integration over the neutrino phase space automatically. Unfortunately, this aspect can prove to be also a limitation: because of integrating over all possible neutrino momenta, all information about the decay plane is lost. Therefore this method can only be applied for obtaining lepton polarization asymmetries averaged over the position of the decay plane, i.e. after azimuthal averaging. The azimuthal averaging does not affect the longitudinal polarization component and thus our results can directly be compared to corresponding results in the literature [9,17]. We have also checked that we agree with [9] on the transverse polarization component in the plane spanned by the vectors $\mathbf{p}_\tau$

and **P** which can be obtained from [9] with the appropriate azimuthal averaging. The transverse polarization component normal to this plane averages out when doing the azimuthal averaging. Our results for the transverse normal component of the τ-polarization have been presented in Sect. IV.

6 Summary and conclusions

We have analyzed the inclusive semileptonic decays of polarized Λ_b baryons into polarized τ-leptons. We discussed spin-spin, spin-momentum and momentum-momentum correlations between the spins of the Λ_b and the τ, and the momenta of the virtual W (or recoil momenta $\mathbf{p}_X$) and the τ. Using helicity techniques we presented detailed results on the above angular correlations involving a threefold angular decay distribution in the three helicity angles that can be defined for the process. By taking suitable combinations of helicity structure functions we identified observables that are directly proportional to the contributions of the $O(1/m_b^2)$ nonperturbative matrix elements. In the helicity method one determines the τ-polarization in the (τ^-, ν_τ) rest frame. We give also results on the τ-polarization in the Λ_b-rest frame using an elegant loop calculation.

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Appendix A: q_0-integrated structure functions

In this appendix we list the results of integrating the structure functions in (11) with respect to q_0. Our results are given in terms of the integrals

$$m_b \int \sqrt{\hat{q}_0^2 - \hat{q}^2} W d\hat{q}_0 = I(W)$$

where W stands for any of the linear combinations of the helicity structure functions in (11). One has

$$\begin{aligned}
&I(W_{++}^{++} + W_{--}^{++} + W_{++}^{--} + W_{--}^{--}) \\
&\quad = (1 - K_b)\hat{p}(-\hat{q}^2 + \rho + 1)/2 + 4K_b\hat{p}/3 \\
&\quad + G_b(\hat{p}^2(15(\hat{q}^2 - \rho) - 11) \\
&+ \hat{q}^2(3\hat{q}^2 - 3\rho) - 7) - 4\rho + 4))/(6\hat{p}) \\
&I(W_{++}^{++} + W_{--}^{++} - W_{++}^{--} - W_{--}^{--}) \\
&\quad = (1 + \epsilon_b)\hat{p}^2 + 2K_b(-2\hat{p}^2 - q^2)/3) \\
&I(W_{00}^{++} + W_{00}^{--}) \\
&\quad = (1 - K_b)\hat{p}(4\hat{p}^2 - \hat{q}^4 + \hat{q}^2\rho + \hat{q}^2)/(4\hat{q}^2) - 4K_b\hat{p}/3 \\
&\quad + G_b\left(\hat{p}^2(15(-4\hat{p}^2 + \hat{q}^4 - \hat{q}^2\rho) - 59\hat{q}^2 - 12\rho + 12)\right. \\
&\quad \left. + \hat{q}^2(\hat{q}^2(3(\hat{q}^2 - \rho) - 7) - 4\rho + 4)\right)/(12\hat{p}\hat{q}^2) \\
&I(W_{00}^{++} - W_{00}^{--}) = (1 + \epsilon_b)\hat{p}^2(\rho - 1)/(2\hat{q}^2) \\
&\quad + K_b(-2\hat{p}^2 - \rho + 1)/3 \\
&I(W_{++}^{++} - W_{--}^{++} + W_{++}^{--} - W_{--}^{--}) \\
&\quad = -\hat{p}^2 + 2K_b(2\hat{p}^2 + \hat{q}^2)/3 \\
&\quad + G_b(10\hat{p}^2 + 2\hat{q}^2 + 3\rho - 3)/3 \\
&I(W_{++}^{++} - W_{--}^{++} - W_{++}^{--} + W_{--}^{--}) \\
&\quad = (1 + \epsilon_b)\hat{p}(\hat{q}^2 - \rho - 1)/2 \\
&\quad + K_b\hat{p}(-3\hat{q}^2 + 3\rho - 5)/6 \\
&I(W_{-0}^{+-} + W_{0+}^{+-}) = (1 + \epsilon_b)\hat{p}(\hat{q}^2 + \rho - 1)/(2\sqrt{2}\sqrt{\hat{q}^2}) \\
&\quad + K_b\hat{p}(\hat{q}^2 - \rho + 1)/(3\sqrt{2}\sqrt{\hat{q}^2}) \\
&I(W_{-0}^{+-} - W_{0+}^{+-}) = -(1 + \epsilon_b)\hat{p}^2/(\sqrt{2}\sqrt{\hat{q}^2}) \\
&\quad + K_b(-\hat{p}^2 + 2\hat{q}^2)/(3\sqrt{2}\sqrt{\hat{q}^2}) \\
&I(W_{tt}^{++} + W_{tt}^{--}) \\
&\quad = (1 - K_b)\hat{p}(4\hat{p}^2 - \hat{q}^4 + \hat{q}^2\rho + \hat{q}^2)/(4\hat{q}^2) \\
&\quad + G_b\left(\hat{p}^2(5(-4\hat{p}^2 + \hat{q}^4 - 5\hat{q}^2\rho) - 9\hat{q}^2 - 4\rho + 4)\right. \\
&\quad \left. + \hat{q}^4(\hat{q}^2 - \rho - 1)\right)/(4\hat{p}\hat{q}^2) \\
&I(W_{tt}^{++} - W_{tt}^{--}) = (1 + \epsilon_b)\hat{p}^2(\rho - 1)/(2\hat{q}^2) \\
&\quad + K_b(-2\hat{p}^2\rho + 2\hat{p}^2 - \hat{q}^2\rho + \hat{q}^2)/(3\hat{q}^2) \\
&I(W_{0t}^{++} + W_{0t}^{--}) = \hat{p}^2(-\rho + 1)/(2\hat{q}^2) \\
&\quad + K_b(2\hat{p}^2\rho - 2\hat{p}^2 + \hat{q}^2\rho - \hat{q}^2)/(3\hat{q}^2) \\
&\quad + G_b(10\hat{p}^2\rho - 2\hat{p}^2 + 5\hat{q}^2\rho - \hat{q}^2)/(6\hat{q}^2) \\
&I(W_{0t}^{++} - W_{0t}^{--}) = (1 + \epsilon_b)\hat{p}(-4\hat{p}^2 + \hat{q}^4 - \hat{q}^2\rho - \hat{q}^2)/(4\hat{q}^2) \\
&\quad + K_b\hat{p}(12\hat{p}^2 - 3\hat{q}^4 + 3\hat{q}^2\rho + 11\hat{q}^2)/(12\hat{q}^2) \\
&I(W_{-t}^{+-} - W_{t+}^{+-}) = (1 + \epsilon_b)\hat{p}(\hat{q}^2 + \rho - 1)/(2\sqrt{2}\sqrt{\hat{q}^2}) \\
&\quad + K_b\hat{p}(-\hat{q}^2 + \rho - 1)/(3\sqrt{2}\sqrt{\hat{q}^2}) \\
&I(W_{-t}^{+-} + W_{t+}^{+-}) = (1 + \epsilon_b)(-\hat{p}^2)/(\sqrt{2}\sqrt{\hat{q}^2}) \\
&\quad + K_b(3\hat{p}^2 + 2\hat{q}^2)/(3\sqrt{2}\sqrt{\hat{q}^2})
\end{aligned}$$

where $\hat{p} = \frac{1}{2}\sqrt{(1 - \rho + \hat{q}^2)^2 - 4\hat{q}^2}$. For the sake of completeness we have retained the contribution of the chromomagnetic interaction G_b contribution, although it vanishes for Λ_b-decays. For applications of the helicity formalism to the mesonic sector one has to set $P = 0$ in the rate formulae and replace $(W_{\lambda_W\lambda'_W}^{++} + W_{\lambda_W\lambda'_W}^{--})$ by $W_{\lambda_W\lambda'_W}$.

Appendix B: Fully integrated rate functions

In this Appendix we list our fully integrated results for those structure function components that determine the angular decay distribution for unpolarized Λ_b-decay, i.e. for the helicity rates $\hat{\Gamma}_U^-$, $\hat{\Gamma}_L^-$, $\hat{\Gamma}_F^-$, $\hat{\Gamma}_U^+$, $\hat{\Gamma}_L^+$, $\hat{\Gamma}_S^+$ and $\hat{\Gamma}_{SL}^+$.

$$\begin{aligned}
\hat{\Gamma}_U^- = (1 - K_b)&\frac{\sqrt{R}}{3}(1 - 7\eta - 7\eta^2 + \eta^3 - 7\rho \\
&+ 12\eta\rho - 7\eta^2\rho - 7\rho^2 - 7\eta\rho^2 + \rho^3) \\
&+ 16K_b\frac{\sqrt{R}}{9}(1 - 5\eta - 2\eta^2 + 10\rho - 5\eta\rho + \rho^2) \\
&+ (1 - K_b)8\left[\rho^2(\eta^2 - 1)\ln\left(\frac{1 - \eta + \rho - \sqrt{R}}{2\sqrt{\rho}}\right)\right.
\end{aligned}$$

$$
\begin{aligned}
&+\eta^2(\rho^2-1)\ln\left(\frac{1+\eta-\rho-\sqrt{R}}{2\sqrt{\eta}}\right)\Bigg]\\
&+K_b\left[\frac{64\rho}{3}(1-2\eta+\eta^2+\rho)\ln\left(\frac{1-\eta+\rho-\sqrt{R}}{2\sqrt{\rho}}\right)\right.\\
&\left.+\frac{64\eta^2}{3}(\rho-1)\ln\left(\frac{1+\eta-\rho-\sqrt{R}}{2\sqrt{\eta}}\right)\right] \qquad (66)
\end{aligned}
$$

$$
\begin{aligned}
\hat{\Gamma}_L^- = &(1-K_b)\frac{2\sqrt{R}}{3}(1+10\eta+\eta^2-7\rho\\
&-10\eta\rho+\eta^2\rho-7\rho^2\\
&+10\eta\rho^2+\rho^3)\\
&-K_b\frac{16\sqrt{R}}{9}(1-5\eta-2\eta^2+10\rho-5\eta\rho+\rho^2)\\
&+(1-K_b)8\rho^2(-2+3\eta-\eta^2-\eta\rho)\\
&\times\ln\left(\frac{1-\eta+\rho-\sqrt{R}}{2\sqrt{\rho}}\right)\\
&+K_b\frac{64\rho}{3}(-1+2\eta-\eta^2)\ln\left(\frac{1-\eta+\rho-\sqrt{R}}{2\sqrt{\rho}}\right)\\
&+(1-K_b)8\eta(1-\rho)(1+\eta-2\rho+\eta\rho+\rho^2)\\
&\times\ln\left(\frac{1+\eta-\rho-\sqrt{R}}{2\sqrt{\eta}}\right)\\
&+K_b\frac{64\eta^2}{3}(1-\rho)\ln\left(\frac{1+\eta-\rho-\sqrt{R}}{2\sqrt{\eta}}\right) \qquad (67)
\end{aligned}
$$

$$
\begin{aligned}
\hat{\Gamma}_F^- = &(-1+\eta+2\sqrt{\rho}-\rho)(1-7\eta-7\eta^2+\eta^3\\
&+2\sqrt{\rho}-12\eta\sqrt{\rho}-2\eta^2\sqrt{\rho}-17\rho+38\eta\rho\\
&-7\eta^2\rho+28\sqrt{\rho}^3-12\eta\sqrt{\rho}^3\\
&-17\rho^2-7\eta\rho^2+2\sqrt{\rho}^5+\rho^3)/3\\
&+4\eta^2(1-\rho)^2\ln\left(\frac{\eta}{(1-\sqrt{\rho})^2}\right)\\
&+K_b\left[\frac{4}{9}(1-\eta-2\sqrt{\rho}+\rho)(9-23\eta\right.\\
&+\eta^2+\eta^3-30\sqrt{\rho}+20\eta\sqrt{\rho}-2\eta^2\sqrt{\rho}\\
&+31\rho+22\eta\rho-7\eta^2\rho-4\sqrt{\rho}^3-12\eta\sqrt{\rho}^3\\
&-9\rho^2-7\eta\rho^2+2\sqrt{\rho}^5+\rho^3)\\
&\left.-\frac{16\eta^2}{3}(1-\rho)^2\ln\left(\frac{\eta}{(1-\sqrt{\rho})^2}\right)\right] \qquad (68)
\end{aligned}
$$

$$
\begin{aligned}
\hat{\Gamma}_U^+ = &(1-K_b)\frac{2\eta\sqrt{R}}{3}(1+10\eta+\eta^2-2\rho+10\eta\rho+\rho^2)\\
&+K_b\frac{8\eta\sqrt{R}}{3}(1+5\eta+\rho)\\
&+(1-K_b)\frac{8\eta^2\rho^2}{1-\rho}(-1+\eta+\rho)\\
&\times\ln\left(\frac{1-\eta+\rho-\sqrt{R}}{2\sqrt{\rho}}\right)\\
&+K_b\frac{32\eta\rho}{3(1-\rho)}(1-2\eta+\eta^2-\rho+2\eta\rho)\\
&\times\ln\left(\frac{1-\eta+\rho-\sqrt{R}}{2\sqrt{\rho}}\right)\\
&+(1-K_b)\frac{8\eta^2}{1-\rho}(1-\rho-\rho^2+\rho^3+\eta+\eta\rho^2)\\
&\times\ln\left(\frac{1+\eta-\rho-\sqrt{R}}{2\sqrt{\eta}}\right)\\
&+K_b\frac{32\eta^2}{3(1-\rho)}(2+\eta-4\rho+\eta\rho+2\rho^2)\\
&\times\ln\left(\frac{1+\eta-\rho-\sqrt{R}}{2\sqrt{\eta}}\right) \qquad (69)
\end{aligned}
$$

$$
\begin{aligned}
\hat{\Gamma}_L^+ = &(1-K_b)\eta\sqrt{R}(-3-3\eta+4\rho-3\eta\rho-3\rho^2)\\
&-K_b\frac{8\eta\sqrt{R}}{3}(1+5\eta+\rho)\\
&+(1-K_b)\frac{2\eta\rho^2}{1-\rho}(-3+4\eta-\eta^2\\
&+4\rho-4\eta\rho-\rho^2)\ln\left(\frac{1-\eta+\rho-\sqrt{R}}{2\sqrt{\rho}}\right)\\
&+K_b\frac{32\eta\rho}{3(1-\rho)}(-1+2\eta-\eta^2+\rho-2\eta\rho)\\
&\times\ln\left(\frac{1-\eta+\rho-\sqrt{R}}{2\sqrt{\rho}}\right)\\
&+(1-K_b)2\eta(-4\eta(1-\rho^2)\\
&+\frac{-1-\eta^2+4\rho-6\rho^2-\eta^2\rho^2+4\rho^3-\rho^4}{1-\rho})\\
&\times\ln\left(\frac{1+\eta-\rho-\sqrt{R}}{2\sqrt{\eta}}\right)\\
&+K_b\frac{32\eta^2}{3(1-\rho)}(-2-\eta+4\rho-\eta\rho-2\rho^2)\\
&\times\ln\left(\frac{1+\eta-\rho-\sqrt{R}}{2\sqrt{\eta}}\right) \qquad (70)
\end{aligned}
$$

$$
\begin{aligned}
\hat{\Gamma}_S^+ = &(1-K_b)\left[\eta\sqrt{R}(-3-3\eta+4\rho-3\eta\rho-3\rho^2)\right.\\
&+2\eta(-4\eta(1-\rho^2)\\
&+\frac{-1-\eta^2+4\rho-6\rho^2-\eta^2\rho^2+4\rho^3-\rho^4}{1-\rho})\\
&\times\ln\left(\frac{1+\eta-\rho-\sqrt{R}}{2\sqrt{\eta}}\right)\\
&-\frac{2\eta\rho^2}{1-\rho}(3-4\eta+\eta^2-4\rho+4\eta\rho+\rho^2)\\
&\left.\times\ln\left(\frac{1-\eta+\rho-\sqrt{R}}{2\sqrt{\rho}}\right)\right] \qquad (71)
\end{aligned}
$$

$$
\hat{\Gamma}_{SL}^+ = \eta\frac{1+\sqrt{\rho}}{\sqrt{\rho}-1}(1-\eta-2\sqrt{\rho}+\rho)
$$

$$
\begin{aligned}
&\times(3+3\eta-2\sqrt{\rho}-4\eta\sqrt{\rho}-2\rho \\
&+3\eta\rho-2\sqrt{\rho}^3+3\rho^2) \\
&-\eta(1+4\eta+\eta^2-3\rho-\eta^2\rho+3\rho^2-4\eta\rho^2-\rho^3) \\
&\times\ln\left(\frac{\eta}{(1-\sqrt{\rho})^2}\right) \\
&+K_b\left[\frac{4\eta}{3}(1+\sqrt{\rho})(-1+\eta+2\sqrt{\rho}-\rho)\right. \\
&\times(-1-\eta-3\sqrt{\rho}+3\eta\sqrt{\rho}+\rho+3\sqrt{\rho}^3) \\
&+\frac{4\eta}{3}(\rho-1)(-1-\eta^2+2\rho-4\eta\rho-\rho^2) \\
&\left.\times\ln\left(\frac{\eta}{(1-\sqrt{\rho})^2}\right)\right] \qquad (72)
\end{aligned}
$$

where R is related to the maximal momentum of the τ in the Λ_b rest frame and is given by

$$R=1-2\eta+\eta^2-2\rho-2\eta\rho+\rho^2$$

As a necessary check we reproduce the total rate formula [6–9]:

$$
\begin{aligned}
\Gamma &= \Gamma_b\left(\hat{\Gamma}_U^- + \hat{\Gamma}_U^+ + \hat{\Gamma}_L^- + \hat{\Gamma}_L^+ + 3\hat{\Gamma}_S^+\right) \\
&= \Gamma_b\,(1-K_b)\left[\sqrt{R}(1-7\eta\right. \\
&\quad -7\eta^2+\eta^3-7\rho+12\eta\rho-7\eta^2\rho-7\rho^2-7\eta\rho^2+\rho^3) \\
&\quad +24\eta^2(\rho^2-1)\ln\left(\frac{1+\eta-\rho-\sqrt{R}}{2\sqrt{\eta}}\right) \\
&\quad \left.+24\rho^2(\eta^2-1)\ln\left(\frac{1-\eta+\rho-\sqrt{R}}{2\sqrt{\rho}}\right)\right] \qquad (73)
\end{aligned}
$$

We mention that we have double checked all our analytic results by comparing them to a numerical evaluation.

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