

Nonlinear k_\perp -factorization: a new paradigm for hard processes in a nuclear environment

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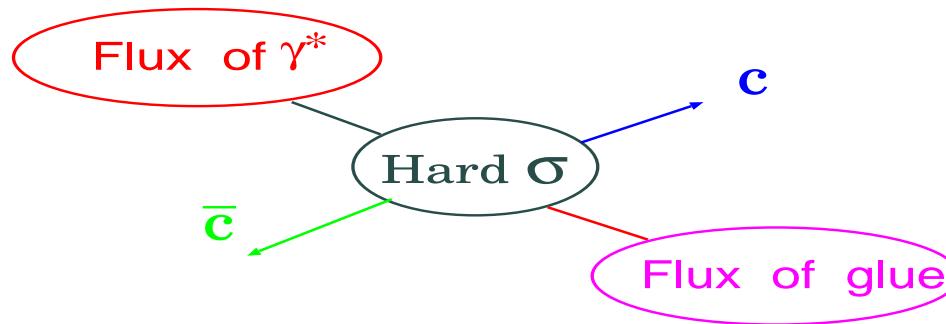
Ustav Casticove a Jaderne Fyziky, Prague, December 1, 2005

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pQCD factorization theorems

Example: open charm in $e p \rightarrow c \bar{c} X$



- Forward dijets: $x_\gamma = z_+ + z_- \approx 1$, jet-jet decorrelation momentum $\Delta = \mathbf{p}_+ + \mathbf{p}_-$

$$\frac{d\sigma_N(\gamma^* \rightarrow c\bar{c})}{dz d^2\mathbf{p}_+ d^2\Delta} = \frac{\alpha_S(\mathbf{p}^2)}{2(2\pi)^2} f(\Delta) |\Psi(z, \mathbf{p}_+) - \Psi(z, \mathbf{p}_+ - \Delta)|^2 .$$

$$f(\kappa) = \frac{4\pi}{N_c} \cdot \frac{1}{\kappa^4} \cdot \frac{\partial G_N(x, \kappa)}{\partial \log \kappa^2}$$

$$\sigma_0(x) = \int d^2\kappa f(\kappa) = \sigma(x, \mathbf{r})|_{r \rightarrow \infty}$$

- ★ A linear functional of the unintegrated glue.
- ★ The dijet momentum Δ probes the gluon momentum.

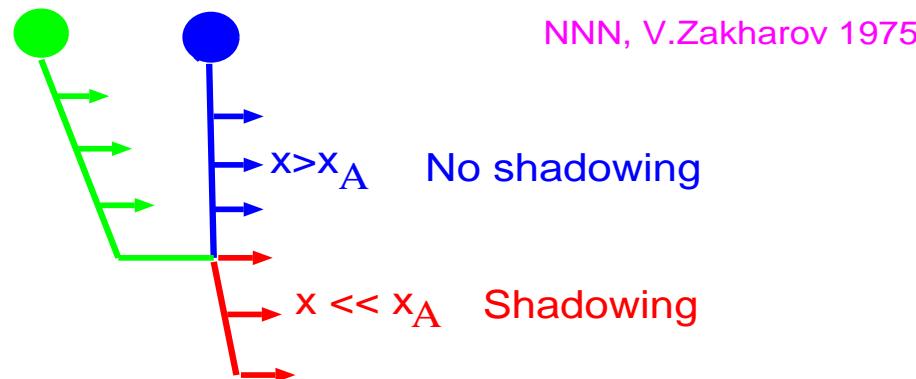
- ★ Back to 1973-74: DIS at $x \ll 1$ in the Breit frame
- ★ Lorentz-contracted ultrarelativistic nucleus:

$$R_A \rightarrow R_A \frac{m_N}{p_N} < \lambda = \frac{1}{k_z} = \frac{1}{x p_N} .$$

- ★ Spatial overlap of partons from many nucleons if

$$x \lesssim x_A = 1/R_A m_N$$

⇒ FUSION & NUCLEAR SHADOWING.



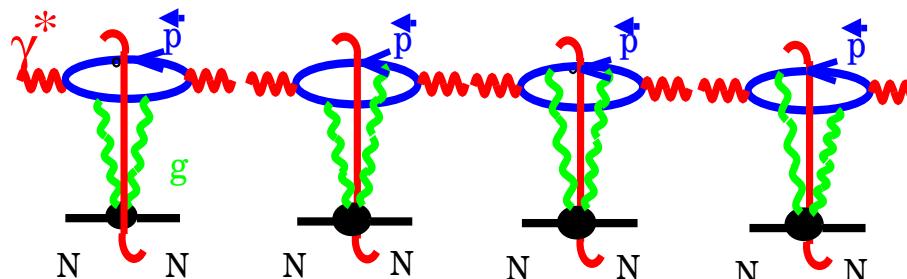
- ★ Nuclear parton density (if it can be meaningfully defined!) is a nonlinear functional of the free nucleon parton density: the same sea is shared by many nucleons.
- ★ Must describe all nuclear observables!
- ★ Major strategy of this talk: shadowing from unitarity for dipole amplitudes.

Coherent diffractive and truly inelastic DIS

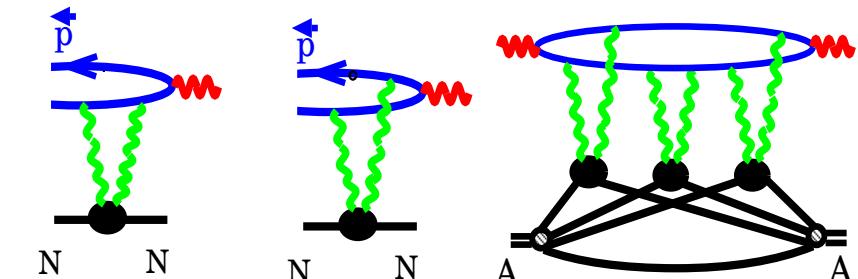
Color dipole is coherent over whole nucleus for $x \lesssim x_A$: \Rightarrow Glauber–Gribov formalism (NNN, Zakharov (91)):

$$\sigma_A(\mathbf{r}) = 2 \int d^2\mathbf{b} \Gamma_A(\mathbf{b}, \mathbf{r}) = 2 \int d^2\mathbf{b} [1 - \exp(-\frac{1}{2} \sigma(\mathbf{r}) T(\mathbf{b}))]$$

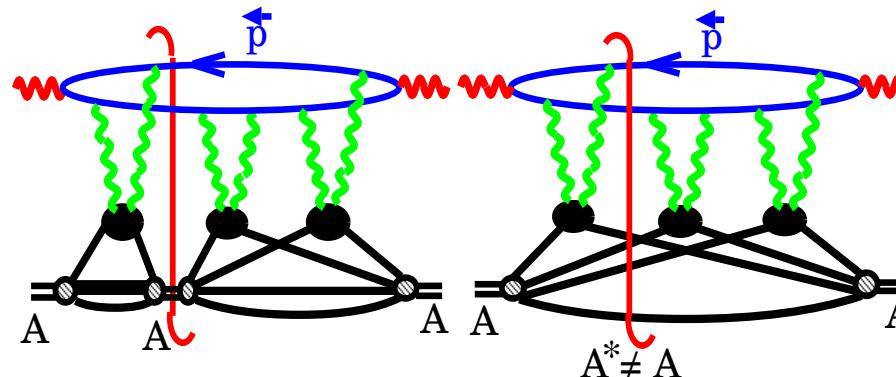
* The unitarity content of DIS



Unitarity cuts for DIS off free nucleons

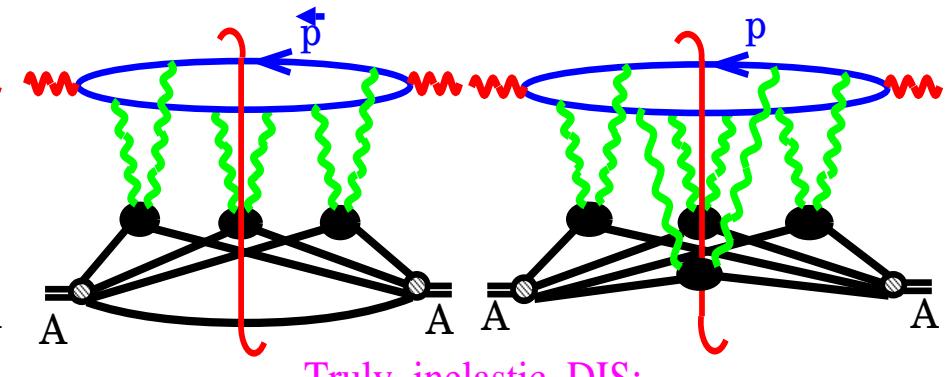


Amplitude of diffractive DIS $\gamma^* A$ Compton amplitude



Coherent diffractive cut

Incoherent diffractive cut

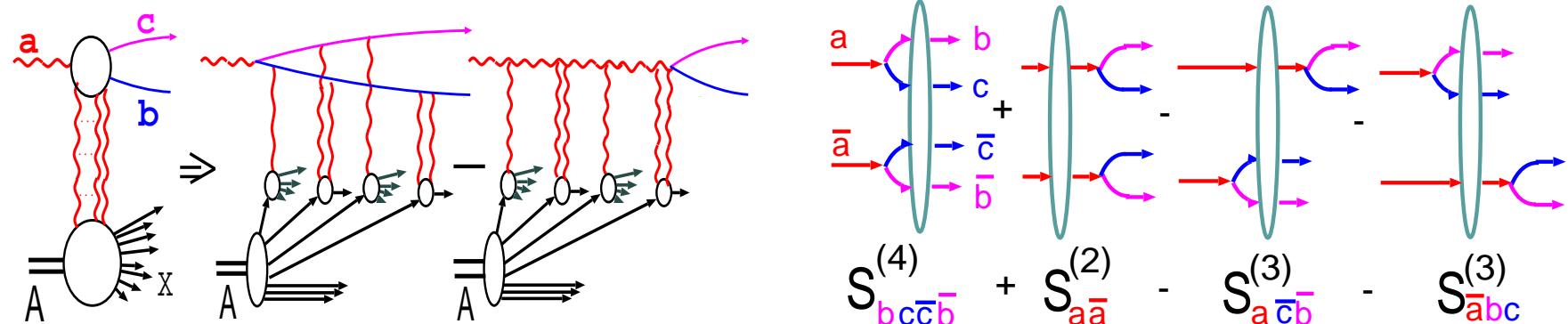


Truly inelastic DIS:
Single color excitation

Multiple color excitation

Production processes as excitation of beam Fock states $a \rightarrow bc$

Zakharov (87), NNN, Zakharov, Piller (95), NNN, Schäfer, Zakharov, Zoller (03)



- ★ Interactions with the nucleus **after** and **before** the virtual decay interfere destructively.
- ★ Apply closure over the nucleon & nucleus excitations
- ★ Hermitian conjugated S -matrix = S -matrix for an antiparticle!

$$S_a S_b^\dagger = S_{a\bar{b}}$$

- ★ Partial cross sections with **color-excitation** of ν nucleons (ν cut pomerons in the Abramovsky-Gribov-Kancheli language)
- ★ Requires evaluation of specific intermediate states in $S^{(n)}$: **well developed technique is available** (NNN, Schafer, Zakharov (05))

Non-Abelian coupled-channel evolution and master formula for dijets

NNN, Zakharov, Piller (95), NNN, Schäfer, Zakharov, Zoller (03)

$$\frac{d\sigma(a^* \rightarrow bc)}{dz_b d^2\mathbf{p}_b d^2\mathbf{p}_c} = \frac{1}{(2\pi)^4} \int d^2\mathbf{b}_b d^2\mathbf{b}_c d^2\mathbf{b}'_b d^2\mathbf{b}'_c \times \exp[-i\mathbf{p}_b(\mathbf{b}_b - \mathbf{b}'_b) - i\mathbf{p}_c(\mathbf{b}_c - \mathbf{b}'_c)]$$

$$\Psi(z_b, \mathbf{b}_b - \mathbf{b}_c) \times \Psi^*(z_b, \mathbf{b}'_b - \mathbf{b}'_c)$$

$$\{S_{\bar{b}\bar{c}cb}^{(4)}(\mathbf{b}'_b, \mathbf{b}'_c, \mathbf{b}_b, \mathbf{b}_c) + S_{\bar{a}a}^{(2)}(\mathbf{b}', \mathbf{b}) - S_{\bar{b}\bar{c}a}^{(3)}(\mathbf{b}, \mathbf{b}'_b, \mathbf{b}'_c) - S_{\bar{a}bc}^{(3)}(\mathbf{b}', \mathbf{b}_b, \mathbf{b}_c)\}.$$

★ Coupled-channel non-Abelian evolution:

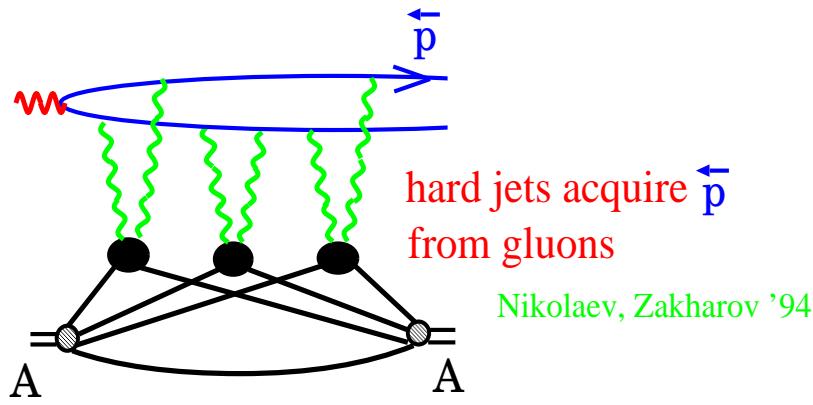
- DIS: $\gamma^* \rightarrow q\bar{q}$: $\Rightarrow \underbrace{1}_{1} + \underbrace{8}_{N_c^2}$

- Open charm: $g \rightarrow c\bar{c}$: $\Rightarrow \underbrace{1}_{1 \text{ (}} (N_c \text{ suppressed}) \underbrace{8}_{N_c^2}$

- Forward dijets: $q \rightarrow qg$: $\Rightarrow \underbrace{3}_{N_c} + \underbrace{6+15}_{N_c \times N_c^2}$

- Central dijets: $g \rightarrow gg$: $\Rightarrow \underbrace{1}_{1 \text{ (}} (N_c \text{ suppressed}) + \underbrace{8_A + 8_S}_{N_c^2} + \underbrace{10 + \overline{10} + 27 + R_7}_{N_c^2 \times N_c^2}$

★ Universality classes depending on color excitation



Diffractive DIS off nuclei defines collective nuclear glue

- ★ Diffractive hard dijets from pions: $\pi N \rightarrow Jet_1 + Jet_2, \quad \mathbf{p}_{Jet_2} = -\mathbf{p}_{Jet_1} \gg \frac{1}{R_N}$:

$$M_{diff,N}(\mathbf{p}) \propto \int d^2\mathbf{r} \sigma(\mathbf{r}) \exp(i\mathbf{p} \cdot \mathbf{r}) = -f(\mathbf{p})$$

- ★ Diffraction off nuclei (NNN, Shäfer, Schwiete '01):

$$M_A(\mathbf{p}) \propto \int d^2\mathbf{r} \Gamma_A(\mathbf{b}, \mathbf{r}) \exp(i\mathbf{p} \cdot \mathbf{r})$$

- ★ Nuclear profile function (partial amplitude)

$$\Gamma_A(\mathbf{b}, \mathbf{r}) = 1 - \exp\left[-\frac{1}{2}\sigma(\mathbf{r})T(\mathbf{b})\right] = \int d^2\kappa \phi(\mathbf{b}, \kappa) \{1 - \exp[i\kappa \mathbf{r}]\}$$

- ★ Optical thickness $T(\mathbf{b}) = \int dz n_A(\mathbf{b}, z)$ - a new large dimensional scale.

- ★ Collective glue is defined through the physical observable: $M_{diff,A}(\mathbf{p}) \propto \phi(\mathbf{b}, \mathbf{p})$

- Nuclear glue per unit area in the impact parameter space

$$\phi(\mathbf{b}, \boldsymbol{\kappa}) = \sum_{j=1}^{\infty} w_j(\mathbf{b}) f^{(j)}(\boldsymbol{\kappa})$$

- Probability to find j overlapping nucleons

$$w_j(\mathbf{b}) = \frac{\nu_A^j(\mathbf{b})}{j!} \exp[-\nu_A(\mathbf{b})], \quad \nu_A(\mathbf{b}) = \frac{1}{2}\sigma_0 T(\mathbf{b})$$

- Collective glue of j overlapping nucleons:

$$f^{(j)}(\boldsymbol{\kappa}) = \int \prod_i^j d^2 \boldsymbol{\kappa}_i f(\boldsymbol{\kappa}_i) \delta(\boldsymbol{\kappa} - \sum_i^j \boldsymbol{\kappa}_i), \quad f^{(0)}(\boldsymbol{\kappa}) \equiv \delta(\boldsymbol{\kappa})$$

- Nuclear S -matrix for the dipole: $S_A(\mathbf{b}, \mathbf{r}) = \exp[-\frac{1}{2}\sigma(\mathbf{r})T(\mathbf{b})]$

$$\Phi(\mathbf{b}, \boldsymbol{\kappa}) = \frac{1}{(2\pi)^2} \int d^2 \mathbf{r} S_A(\mathbf{b}, \mathbf{r}) \exp(-i \mathbf{r} \boldsymbol{\kappa}) = \phi(\mathbf{b}, \boldsymbol{\kappa}) + w_0(\mathbf{b}) \delta(\boldsymbol{\kappa})$$

- Antishadowing of hard, $\kappa^2 \gtrsim Q_A^2$, glue per bound nucleon
(NNN, Schäfer, Schwiete '00):

$$\begin{aligned} f_A(\mathbf{b}, x, \kappa) &= \frac{\phi(\mathbf{b}, \kappa)}{\nu_A(\mathbf{b})} \\ &= f(x, \kappa) \left[1 + \frac{\gamma^2}{2} \cdot \frac{\alpha_S(\kappa^2) G(x, \kappa^2)}{\alpha_S(Q_A^2) G(x, Q_A^2)} \cdot \frac{Q_A^2(\mathbf{b})}{\kappa^2} \right]. \end{aligned}$$

- γ = exponent of the large- κ^2 tail

$$f(\kappa) \sim \alpha_S(\kappa^2) / (\kappa^2)^\gamma$$

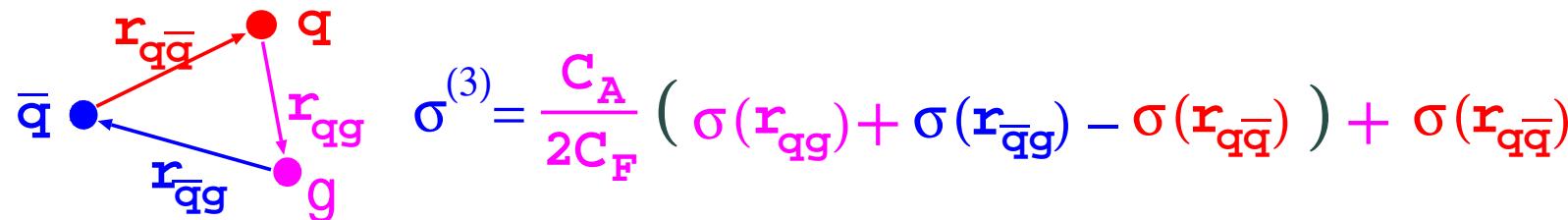
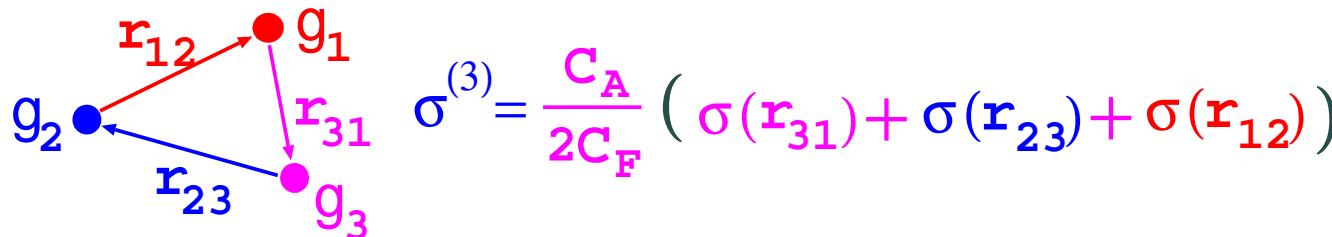
- Antishadowing \implies the Cronin effect.
- Plateau for softer collective glue

$$\phi(\mathbf{b}, \kappa) \approx \frac{1}{\pi} \frac{Q_A^2(\mathbf{b})}{(\kappa^2 + Q_A^2(\mathbf{b}))^2},$$

- Width of the plateau (saturation & higher twist scale, independent of auxiliary soft $\sigma_0(x)$)

$$Q_A^2(\mathbf{b}, x) \approx \frac{4\pi^2}{N_c} \alpha_S(Q_A^2) G(x, Q_A^2) T(\mathbf{b}).$$

The origin, and inevitability of the nonlinear k_\perp -factorization



- * Glauber-Gribov multiple scattering theory for the dilute-gas nucleus:

$$S_i(\mathbf{b}_c', \mathbf{b}_b', \mathbf{b}_c, \mathbf{b}_b) = \exp\left\{-\frac{1}{2}\Sigma_i(\mathbf{b}_c', \mathbf{b}_b', \mathbf{b}_c, \mathbf{b}_b)T(\mathbf{b})\right\}$$

*

$$\begin{aligned} S_{123} &= \exp\left\{-\frac{1}{2} \cdot \frac{C_A}{2C_F} \sigma(r_{12}) T(\mathbf{b})\right\} \exp\left\{-\frac{1}{2} \cdot \frac{C_A}{2C_F} \sigma(r_{13}) T(\mathbf{b})\right\} \exp\left\{-\frac{1}{2} \cdot \frac{C_A}{2C_F} \sigma(r_{23}) T(\mathbf{b})\right\} \\ &= \int d^2\kappa_1 d^2\kappa_2 d^2\kappa_3 \Phi(\mathbf{b}, \kappa_1) \Phi(\mathbf{b}, \kappa_2) \Phi(\mathbf{b}, \kappa_3) \exp(i\kappa_1 r_{12} + i\kappa_2 r_{13} + i\kappa_3 r_{23}) \end{aligned}$$

- * The multiparton S -matrix is a **nonlinear** functional of the **collective nuclear glue**!

Exceptional case of single-quark spectrum in DIS: Abelianization of evolution

$$\frac{d\sigma_{in}}{d^2\mathbf{b} d^2\mathbf{p}_+ dz} = \frac{1}{(2\pi)^2} \times \left\{ \int d^2\kappa \phi(\kappa) |\langle \gamma^* | z, \mathbf{p} \rangle - \langle \gamma^* | z, \mathbf{p} - \kappa \rangle|^2 \right. \\ \left. - \underbrace{\left| \int d^2\kappa \phi(\kappa) (\langle \gamma^* | z, \mathbf{p} \rangle - \langle \gamma^* | z, \mathbf{p} - \kappa \rangle) \right|^2}_{Nonlinear} \right\}$$

Coherent diffraction = 50 per cent of total DIS for heavy nucleus (NNN, Zakharov, Zoller '94).

$$\frac{d\sigma_D}{d^2\mathbf{b} d^2\mathbf{p} dz} = \frac{1}{(2\pi)^2} \times \underbrace{\left| \int d^2\kappa \phi(\kappa) (\langle \gamma^* | z, \mathbf{p} \rangle - \langle \gamma^* | z, \mathbf{p} - \kappa \rangle) \right|^2}_{Nonlinear}.$$

$$\frac{d[\sigma_D + \sigma_{in}]}{d^2\mathbf{b} d^2\mathbf{p} dz} == \frac{1}{(2\pi)^2} \int d^2\kappa \phi(\kappa) |\langle \gamma^* | z, \mathbf{p} \rangle - \langle \gamma^* | z, \mathbf{p} - \kappa \rangle|^2$$

- ★ Exceptional case of linear k_\perp -factorization: FSI and ISI are fully reabsorbed into collective nuclear glue!
- ★ Doesn't hold for the two-particle and all other single-particle spectra

Dijets: Universality class of coherent diffraction

- ★ Coherent distortion of dipole WF in slice $[0, \beta]$ of the nucleus:

$$\Psi(\beta; z, \mathbf{p}) = \int d^2\kappa \Phi(\beta; \mathbf{b}, x, \kappa) \Psi(z, \mathbf{p} + \kappa) \quad (1)$$

$$\exp \left[-\frac{1}{2} \beta \sigma(x, \mathbf{r}) T(\mathbf{b}) \right] = \int d^2\kappa \Phi(\beta; \mathbf{b}, x, \kappa) \exp(i\kappa \mathbf{r}) \quad (2)$$

- ★ Diffractive DIS:

$$\frac{(2\pi)^2 d\sigma_A(\gamma^* \rightarrow Q\bar{Q})}{d^2\mathbf{b} dz d^2\mathbf{p} d^2\Delta} = \delta^{(2)}(\Delta) |\Psi(1; z_g, \mathbf{p}) - \Psi(z_g, \mathbf{p})|^2,$$

- ★ Exactly back-to-back dijets
- ★ $q \rightarrow qg$: net color charge of the incident parton

$$\frac{(2\pi)^2 d\sigma_A(q^* \rightarrow qg)}{d^2\mathbf{b} dz d^2\mathbf{p}_g d^2\Delta} = \delta^{(2)}(\Delta) S_{abs}(2\nu_A(\mathbf{b})) |\Psi(1; z_g, \mathbf{p}_g) - \Psi(z_g, \mathbf{p}_g)|^2. \quad (3)$$

- ★ Intranuclear attenuation of the incident quark wave:

$$S_{abs}(2\nu_A(\mathbf{b})) = \exp[-2\nu_A(\mathbf{b})]$$

Dijets: Universality class of dijet in higher color representation from partons in lower representation: $q \rightarrow qg|_{6+15}$

$$\begin{aligned}
& \left. \frac{d\sigma(q^* \rightarrow qg)}{d^2\mathbf{b} dz d^2\Delta d^2\mathbf{p}} \right|_{6+15} = \frac{1}{(2\pi)^2} T(\mathbf{b}) \int_0^1 d\beta \\
& \times \int d^2\kappa d^2\kappa_1 d^2\kappa_2 d^2\kappa_3 \delta(\kappa + \kappa_1 + \kappa_2 + \kappa_3 - \Delta) \\
& \times \underbrace{\Phi(\beta; \mathbf{b}, \kappa_3)}_{\text{Quark ISI}} \underbrace{f(\kappa) |\Psi(\beta; z, \mathbf{p} - \kappa_1) - \Psi(\beta; z, \mathbf{p} - \kappa_1 - \kappa)|^2}_{\text{Hard Excitation}} \\
& \times \underbrace{\Phi(1 - \beta; \mathbf{b}, \kappa_1)}_{\text{Quark FSI}} \underbrace{\Phi\left(\frac{C_A}{C_F}(1 - \beta); \mathbf{b}, \kappa_2\right)}_{\text{Gluon FSI}}
\end{aligned}$$

- ★ $\gamma^* \rightarrow q\bar{q}|_8$: the same as $q \rightarrow qg|_{6+15}$ modified for vanishing ISI
- ★ $g \rightarrow gg|_{10+\overline{10}+27+R_7}$: the same as $q \rightarrow qg$ subject to two modifications:
 - Quark FSI/ISI \Rightarrow Gluon FSI/ISI
 - C_A/C_F : collective glue is different!

Dijets: Universality class of dijets in the same lower color representation as the beam parton: $q \rightarrow qg|_3$

$$\left. \frac{d\sigma(q^*A \rightarrow qg)}{d^2\mathbf{b} dz d^2\Delta d^2\mathbf{p}} \right|_3 = \frac{1}{(2\pi)^2} \phi(\mathbf{b}, \Delta) |\Psi(1; z, \mathbf{p} - \Delta) - \Psi(z, \mathbf{p} - z\Delta)|^2$$

- ★ $\Psi(z, \mathbf{p} - z\Delta)$ = probability amplitude for the qg state in physical quark - driving term of quark jet fragmentation
- ★ Color triplet dijets: fragments of the multiply-scattered quark
- ★ Coherent nuclear-distorted $\Psi(1; z, \mathbf{p} - \Delta)$:

$$|\underbrace{\Psi(z, \mathbf{p} - \Delta)}_{in-vacuum} - \Psi(z, \mathbf{p} - z\Delta)|^2 \implies |\underbrace{\Psi(1; z, \mathbf{p} - \Delta)}_{in-nucleus} - \Psi(z, \mathbf{p} - z\Delta)|^2$$

Interpretation: nuclear modification of the fragmentation function

- ★ More universality classes: $g \rightarrow q\bar{q}|_8$, $g \rightarrow gg|_{8_A+8_S}$, $g \rightarrow gg|_{8_S}$
- ★ Different collective nuclear glue - density matrix in the space of color representations, not a single function.

Fixed multiplicity of color-excited nucleons: Unitarity cuts and AGK rules

- ★ Multiple-scattering theory for final states with j color-excited nucleons = j cut pomerons in the AGK language
- ★ Hadronic activity in the nucleus hemisphere

$$\eta_{Lab} \lesssim \log(R_A m_\perp)$$

- ★ Manifest unitarity at the level of fully differential cross sections:

$$\sum_j d\sigma_j = d\sigma_{inclusive}$$

- ★ Unitarity rules depend on the universality class
- ★ Example of AGK rule: coherent diffractive mechanism:

$$d\sigma_j = \delta_{j0} d\sigma_D$$

- ★ Coherent diffraction = 50% of total DIS (NNN, Zakharov, Zoller (95))
- ★ Persists in all processes, albeit suppressed by nuclear attenuation

Example: AGK rules for $q \rightarrow qg|_3$

$$\frac{d\sigma_j(q^*A \rightarrow qg)}{d^2\mathbf{b} dz d^2\Delta d^2\mathbf{p}} \Big|_3 = \frac{1}{(2\pi)^2} w_j(\nu_A(\mathbf{b})) \frac{f^{(j)}(\Delta)}{\sigma_0^j} |\Psi(1; z, \mathbf{p} - \Delta) - \Psi(z, \mathbf{p} - z\Delta)|^2$$

- ★ Quark-nucleon quasielastic scattering $qN \rightarrow q'N^*$:

$$\frac{d\sigma_{qN}}{d^2\kappa} = \frac{1}{2} f(\kappa)$$

The target debris N^* in the color-excited state.

- ★ j -fold quasielastic scattering $\Rightarrow j$ cut pomerons :

$$\frac{d\sigma^{(j)}}{d^2\kappa} = \frac{1}{2} \frac{f^{(j)}(\kappa)}{\sigma_0^{j-1}}$$

- ★ Multiple uncut (elastic) pomeron exchanges: the unitarity sum rule

$$\sum_{j=0} w_j(\nu_A(\mathbf{b})) = 1$$

- ★ Multiple uncut pomeron exchanges in $\Psi(1; z, \mathbf{p} - \Delta)$
- ★ Scattered quark fragments independent of j

Example: AGK rules for $q \rightarrow qg|_{6+15}$

$$\begin{aligned}
& \frac{d\sigma_j(q^* \rightarrow qg)}{d^2\mathbf{b} dz d^2\Delta d^2\mathbf{p}} \Big|_{6+15} = \frac{1}{(2\pi)^2} T(\mathbf{b}) \int_0^1 d\beta \\
& \times \int d^2\kappa d^2\kappa_1 d^2\kappa_2 d^2\kappa_3 \delta(\kappa + \kappa_1 + \kappa_2 + \kappa_3 - \Delta) \sum_{n,k,m} \delta(j - n - k - m - 1) \\
& \times \underbrace{w_m(\beta \nu_A(\mathbf{b})) \frac{f^{(m)}(\kappa_3)}{\sigma_0^m}}_{\text{Quark ISI}} \underbrace{f(\kappa) |\Psi(\beta; z, \mathbf{p} - \kappa_1) - \Psi(\beta; z, \mathbf{p} - \kappa_1 - \kappa)|^2}_{\text{Hard excitation}} \\
& \times \underbrace{w_k\left(\frac{C_A}{C_F}(1 - \beta)\nu_A(\mathbf{b})\right) \frac{f^{(k)}(\kappa_1)}{\sigma_0^k}}_{\text{Gluon FSI}} \times \underbrace{w_n((1 - \beta)\nu_A(\mathbf{b})) \frac{f^{(n)}(\kappa_1)}{\sigma_0^n}}_{\text{Quark FSI}}
\end{aligned}$$

- ★ 1 cut pomeron for hard excitation at the depth β
- ★ m cut pomerons between incident quark and nucleons in the slice $[0, \beta]$
- ★ k cut pomerons between final-state gluon and nucleons in the slice $[\beta, 1]$
- ★ n cut pomerons between final-state quark and nucleons in the slice $[\beta, 1]$
- ★ DIS: the same **minus ISI**

Summary and further applications:

- Nonlinear k_\perp -factorization: explicit quadratures in terms of the collective glue defined by coherent diffraction
- Expansion of nuclear unintegrated glue in terms of *collective glue of overlapping nucleons*.
- *A non-abelian* intranuclear evolution of color dipoles.
- Non-trivial interplay of coherent and incoherent nuclear effects
- Universality classes for nonlinear k_\perp -factorization.
- Explicit quadratures are available for single-jet and dijet spectra from all pQCD subprocesses
- Excitation of nuclei: universality class-dependent AGK rules
- Application of AGK rules: energy loss for color-excitation of nucleons \Rightarrow quenchung of forward jets