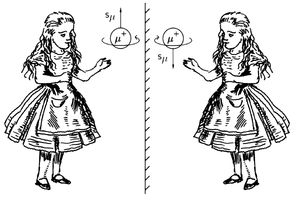


Computer Simulation of Negative and Positive Muon Track Dynamics

A.S. Baturin,
V.N. Gorelkin,
V.R. Soloviev

Moscow Institute of Physics and Technology,
Dolgoprudny, Russia

Talk plan

- Accounting for self-consistent electric field
- Sphere region dynamics at the end of negative muon track
- Dynamics of positive muon linear track
- External electric field influence on track dynamics

Governing Equations

$$\frac{\partial n_i}{\partial t} + div\mathbf{J}_i = -\alpha_r n_e n_i, \quad \mathbf{J}_i = -D_i \nabla n_i + n_i K_i \mathbf{E},$$

$$\frac{\partial n_e}{\partial t} + div\mathbf{J}_e = -\alpha_r n_e n_i, \quad \mathbf{J}_e = -D_e \nabla n_e - n_e K_e \mathbf{E}$$

$$\mathbf{E} = -\nabla \varphi$$

$$\Delta \varphi = -4\pi e(n_i - n_e)$$

$$\frac{\partial \varepsilon}{\partial t} = -\frac{2m}{M} \nu_e(\varepsilon)\left(\varepsilon - \frac{3}{2}T\right)$$

for $\frac{eE}{NQ_m} << \frac{2m}{M}(T_e - T)$ electron heating by E-field is negligible

Boundary Conditions

For concentrations

$$\left.\frac{\partial n_e}{\partial r}\right|_{r=0} = 0, \quad \left.\frac{\partial n_i}{\partial r}\right|_{r=0} = 0, \quad \left.\frac{\partial \varphi}{\partial r}\right|_{r=0} = -E_r = 0$$

$$J_{ez} = J_{iz} = 0, \quad z \to \pm\infty$$

$$J_{er} = J_{ir} = 0, \quad r \to \infty$$

For electric field

$$E = E_{out}, \quad z \to \pm\infty, r \to \infty$$

Sphere at the end of μ^- track

Initial electron-ion distribution

$$n_{e(i)}(z,r) = n_0 \exp\left(-\frac{z^2+r^2}{\Delta_0}\right)$$

$$\Delta_0 = \langle r^2 \rangle = \frac{4}{3} Z_{ei} \lambda_i^2$$

$$n_0 = \left(\frac{3}{\pi}\right)^{3/2} \frac{1}{(2\lambda_i)^3 \sqrt{Z_{ei}}}$$

Definitions for current distribution

Dispersion

$$\Delta = 2\langle z^2 \rangle = \frac{2}{Z_{ie}} \int \frac{z^2+r^2}{3} n_e(z,r,t) 2\pi r dr dz$$

for $l_e / r_0 \approx Q_i / Q_m Z_{ei} \approx 1/Z_{ei} << 1$

Continuum media approach is applicable

Characteristic Lengths

$$r_D(t) = \sqrt{\frac{\varepsilon(t)}{6\pi e^2 n_e(0,0,t)}}$$

Debay radius

$$e\varphi(r_{onz}, t) = \frac{3}{2}T$$

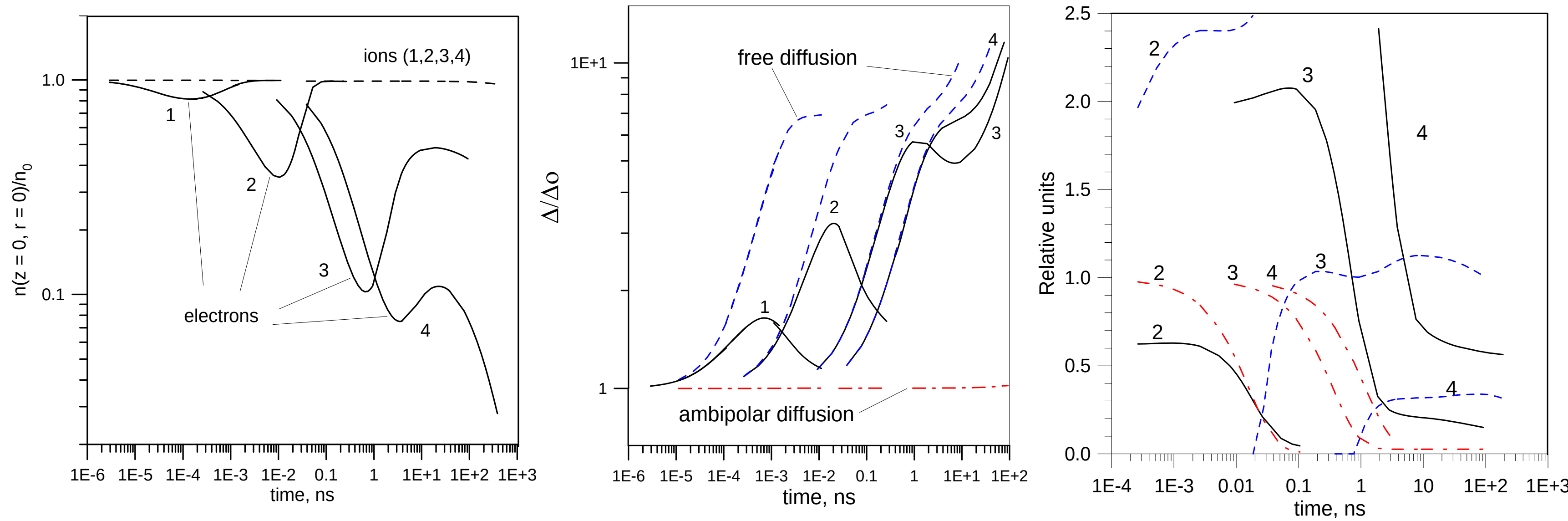
Equation for Onzager radius

$$r_t = \sqrt{\frac{\Delta(t)}{2}}$$

Characteristic linear size of the cloud

Different possible regimes of electron cloud dynamics

$\nu_e = const$ approximation in Ne with no recombination



1,2,3,4 – decreasing density from liquid to gas phase

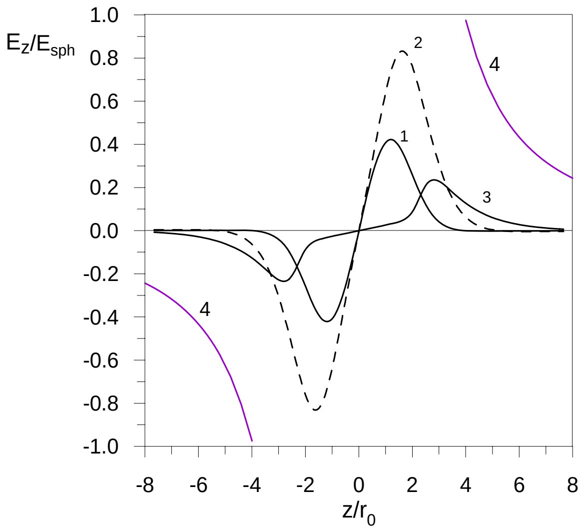
$$\frac{r_{onz}(t_T)}{r_t(t_T)} > 1$$ electron coming back condition

r_D/r_t **solid line**

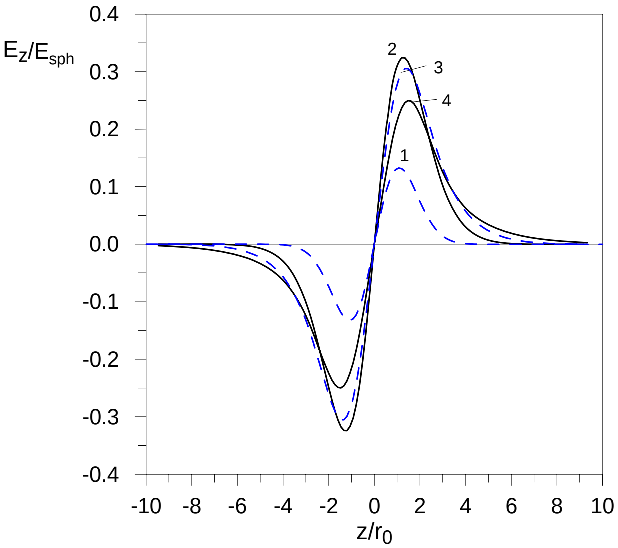
r_{onz}/r_t **dash line**

Internal E-field Is of the Order of Debay Charge Separation Field

Electron coming back case;
$N = 6.5\times10^{22}$ cm^{-3}



Electron running away case;
$N = 1\times10^{21}$ cm^{-3}

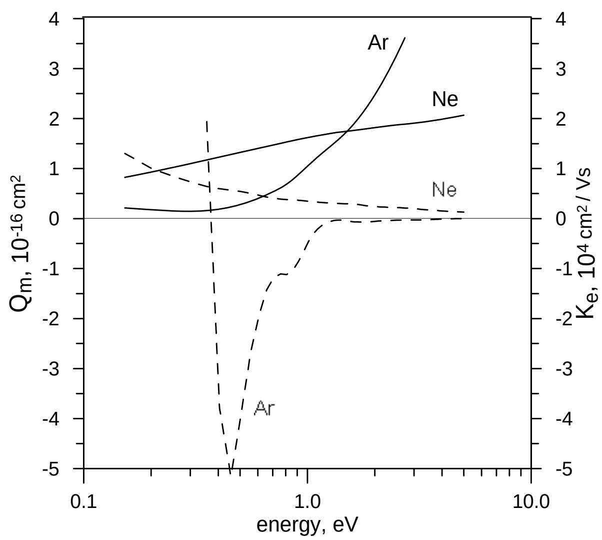


$$E_{sph} = \frac{4}{3}\pi e n_0 r_D$$

Negative Mobility in Gaseous Argon

$$K_e(\varepsilon) = \frac{2}{3}\frac{e}{m\nu_e(\varepsilon)}\left(1 - \frac{\varepsilon}{Q_m(\varepsilon)}\frac{\partial Q_m}{\partial \varepsilon}\right)$$

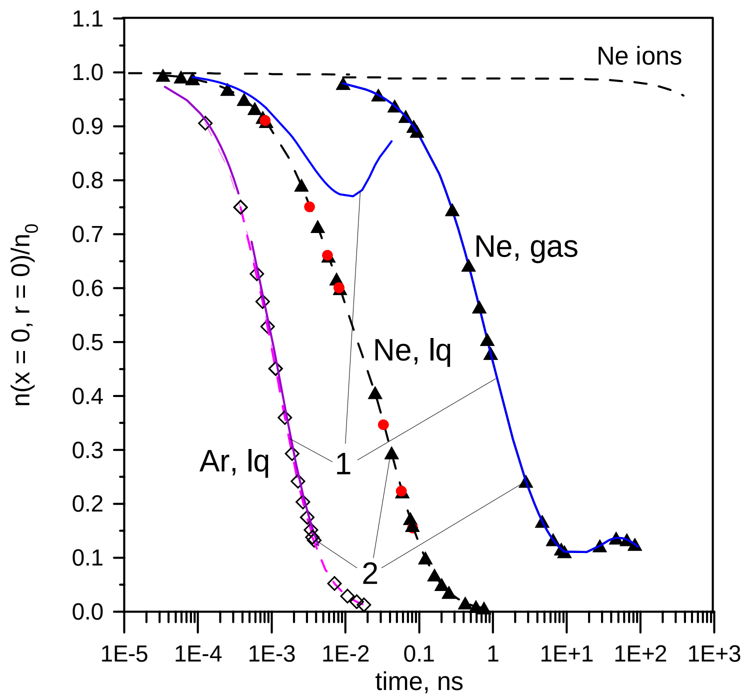
$$D_e(\varepsilon) = \frac{\sqrt{2\varepsilon}}{3\sqrt{m}NQ_m(\varepsilon)}$$



Electron transport cross section – solid line

Mobility – dash line

Recombination changes electron dynamics in dense Ne and do not change in Ar



1 – with no recombination

2 – with recombination, E_{out}=0

Red symbols - E_{out}=80kV/cm

External electric field does not influence the result

The rest muon polarization in Ar is due to negative electron mobility

$$P_i(t) = 1 - \frac{1}{n_0}\int_0^t \alpha_r n_e(0,t) n_i(0,t)\,dt$$

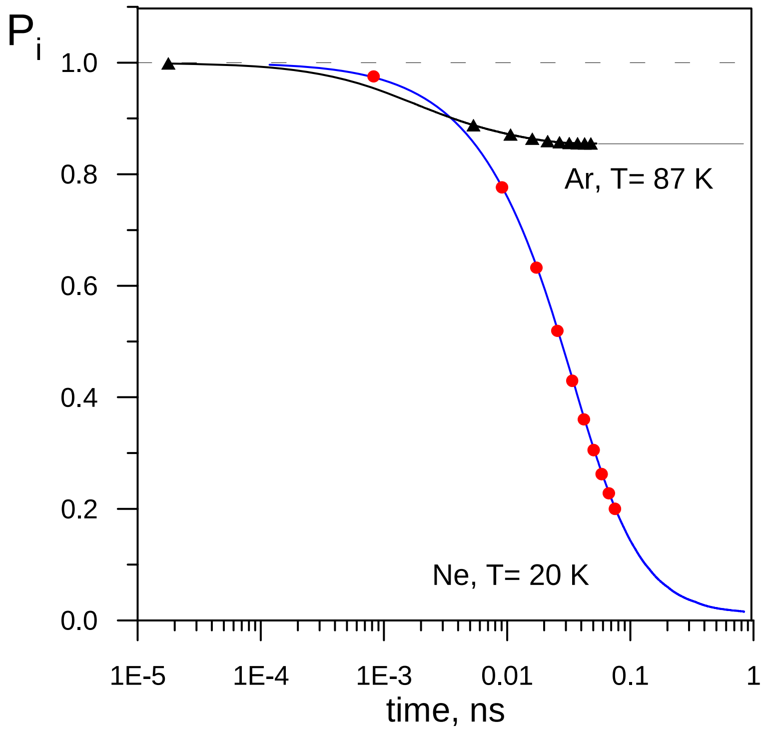
$$\alpha_{ra} = \frac{32\sqrt{2\pi}}{3}\frac{e^6 N}{T_e^{7/2}}\frac{T\sqrt{m}Q_m(T_e)}{M}$$

$$\alpha_{re} = \frac{8\pi\sqrt{2\pi}}{9\sqrt{m}}\frac{e^{10} n_e \bar{\Lambda}}{T_e^{9/2}}$$

$$\alpha_{rdis} = 2\cdot 10^{-7}\sqrt{\frac{300}{T_e(K)}}\left[1+5\left(1-\exp\left(-0.01\frac{N}{N_0}\right)\right)\right]\frac{\text{cm}^3}{\text{s}}$$

$$\alpha_{rL} = \frac{4\pi e^2}{m\nu_e}$$

$$\alpha_r = \alpha_{ra} + \alpha_{re} + \alpha_{rdis} > \alpha_{rL}$$

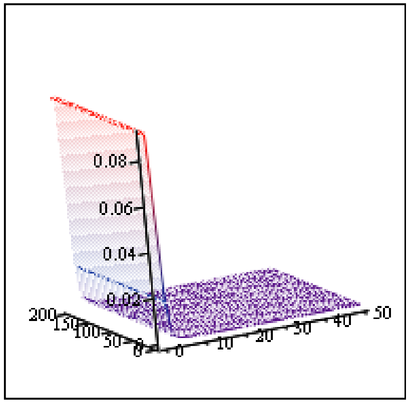


Summary for μ^- spherical region

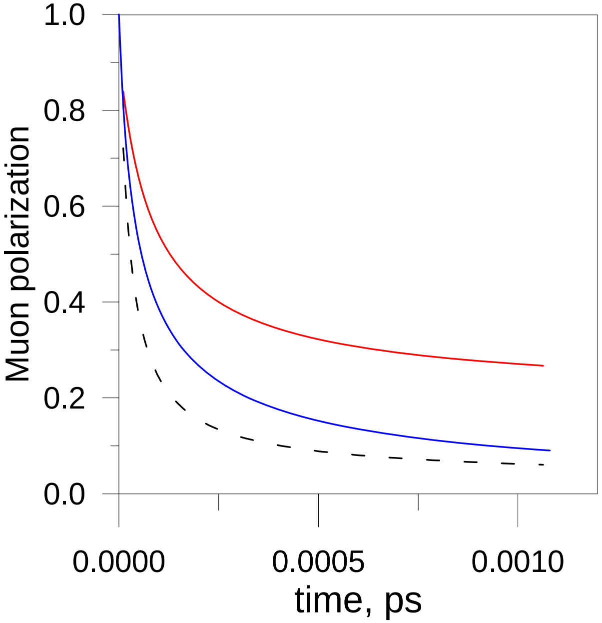
- Electron cloud dynamics is complex and depends on medium density, initial electron energy and electron-ion pair number
- External electric field ~50kV/cm should not affect muon’s polarization
- The rest muon’s polarization could be seen in dense gaseous Ar due to negative electron mobility

Analytical results for cylindrical track are close to numerical ones

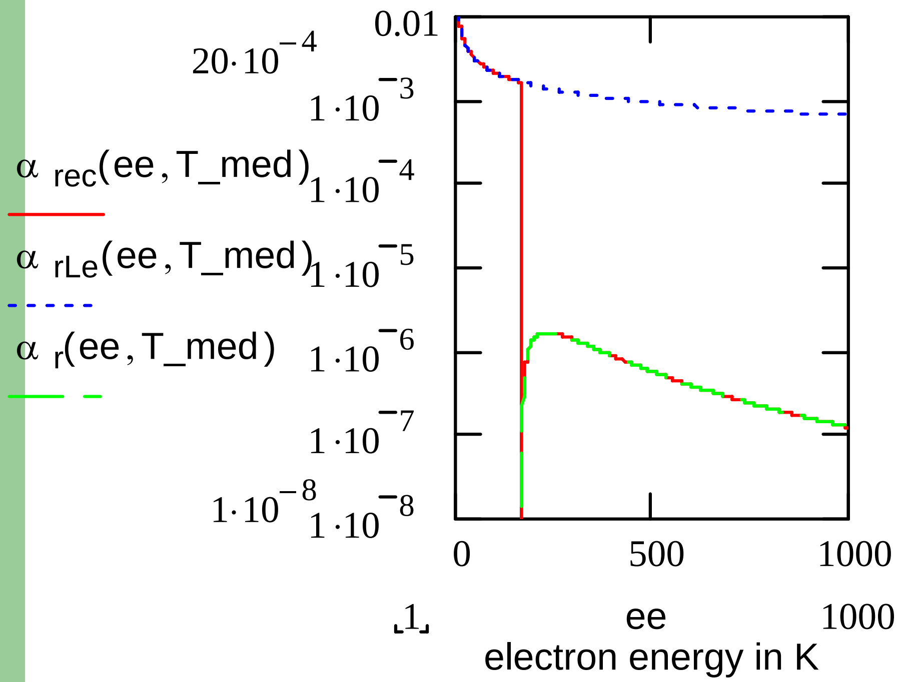
- Infinite cylindrical track in solid Ar
- Langeven recombination is assumed



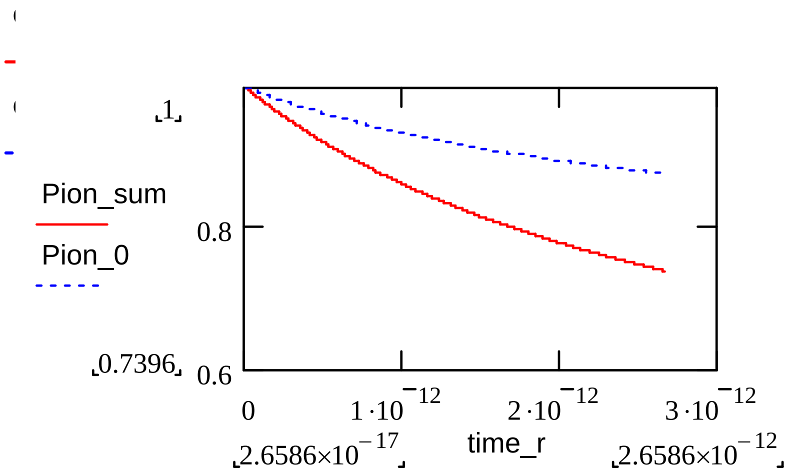
ne_fin



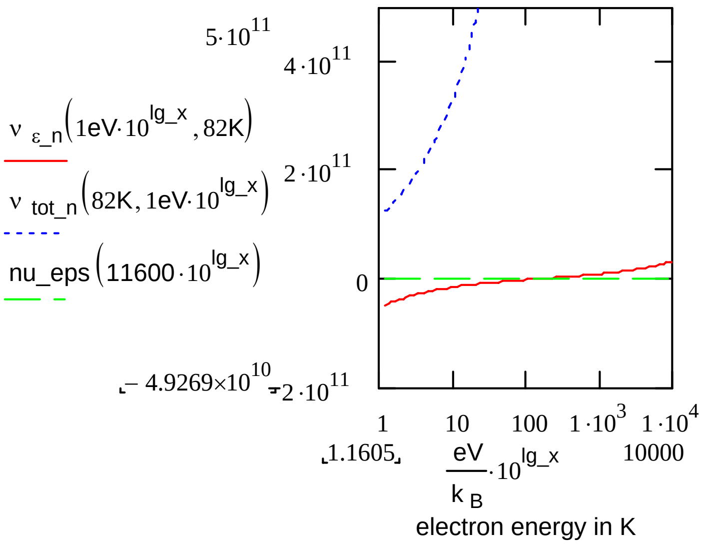
Realistic recombination rate is much less then Langeven’s limit

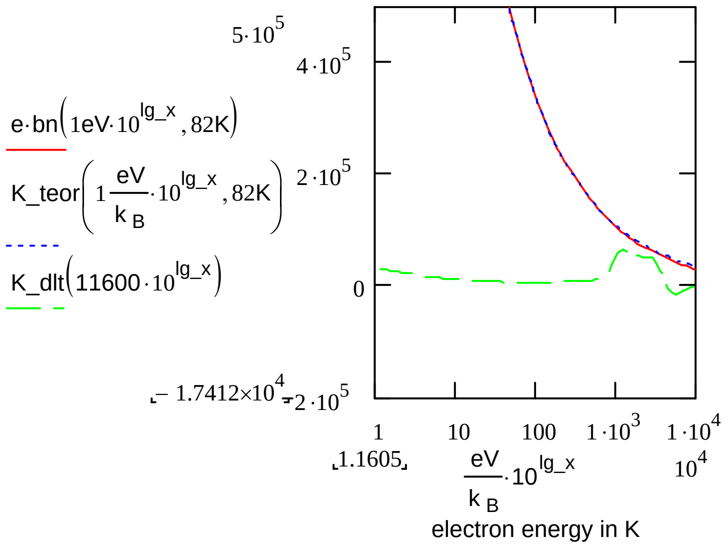


Polarization decrease is much less for realistic recombination coefficient

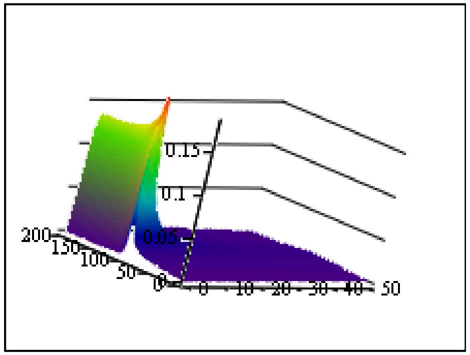


Mobility and energy loss frequency due to electron-phonon interaction in solid Ar

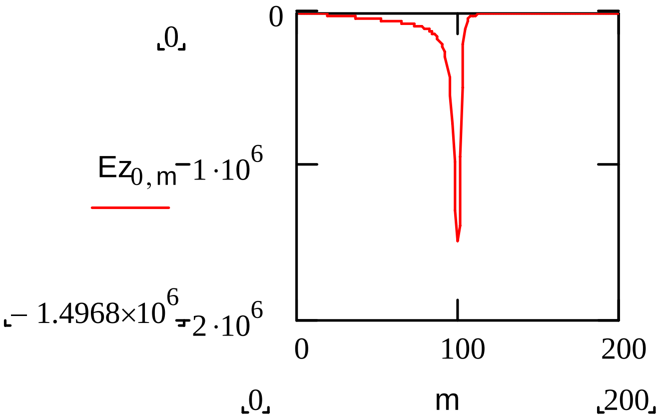


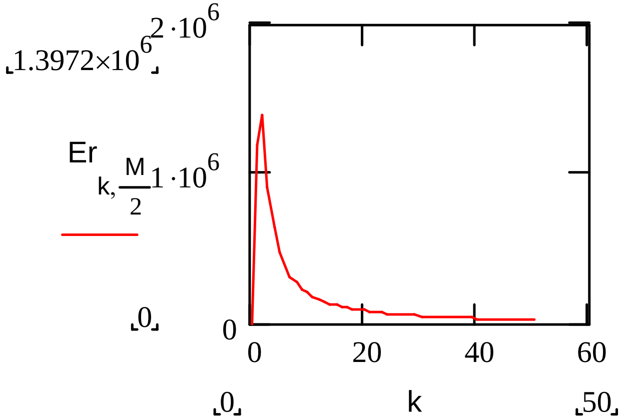


Electric Field at the End of Cylindrical Track Is Much Higher Then 2kV/cm



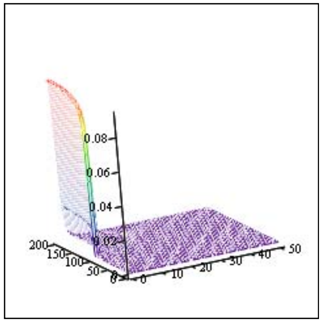
E2





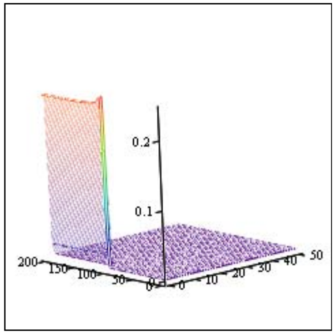
Electrons Should Return to Track in Solid Ar (preliminary result)

Electrons

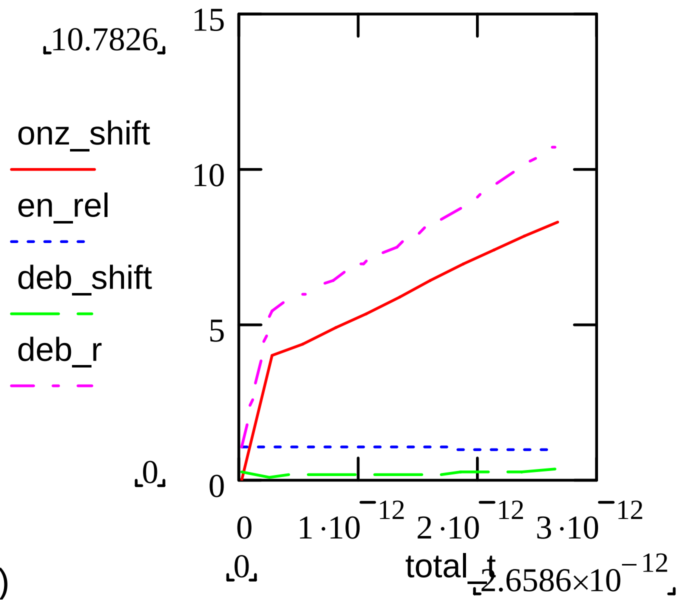


Re(Frames_cp1)

Ions



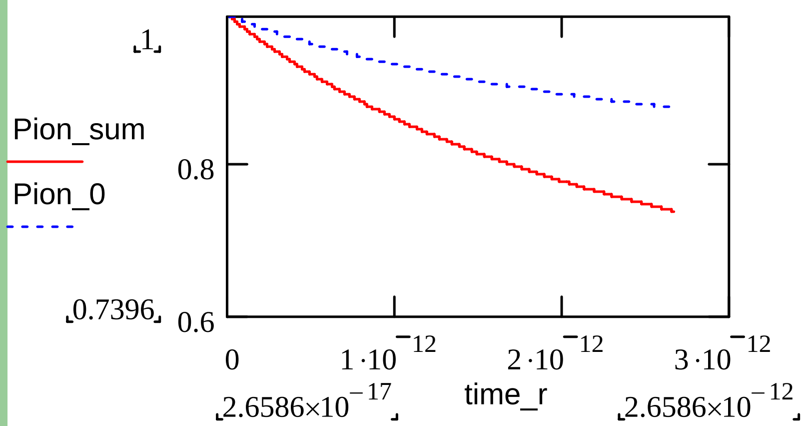
Im(Frames_cp1)

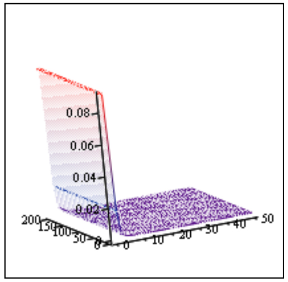


o2)

Muon Polarization Decrease Is ~2 Times Less for Track End Then for Infinite Track

For infinite track





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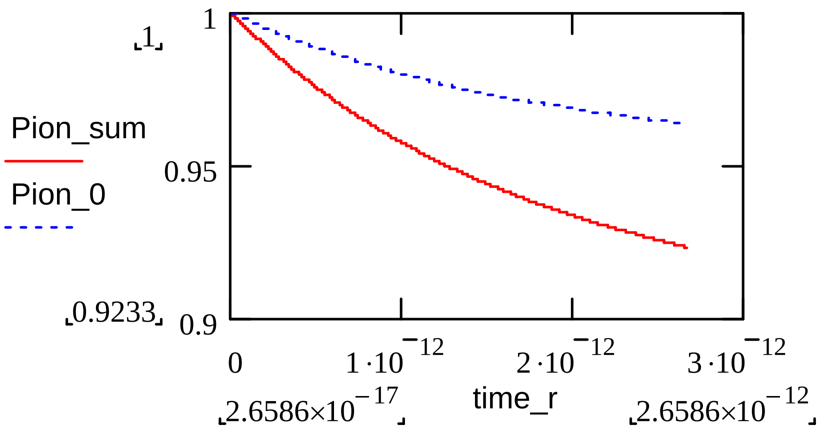
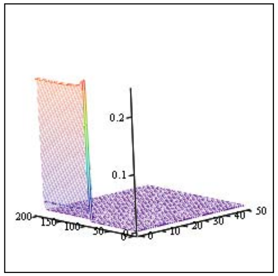
For end of the track





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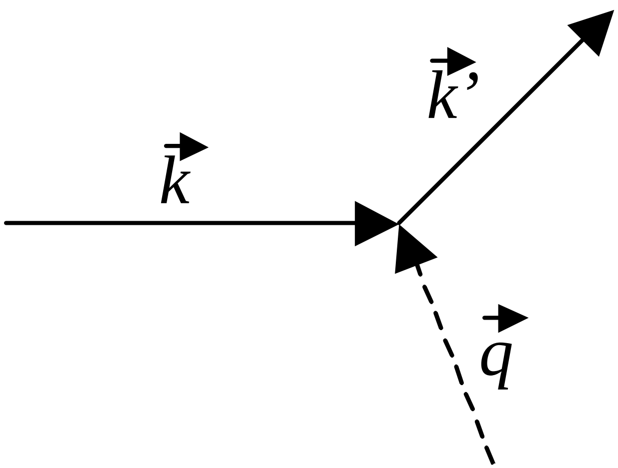
Summary for cylindrical track

- Applied E-fields can not change the dynamics of electrons in solid Ar
- Additional calculations for greater times are necessary to predict positive muon polarization behavior

Phonon capturing



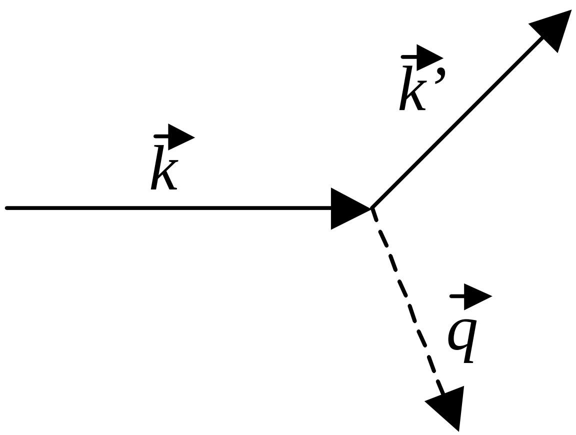
$$\vec{k} + \vec{q} = \vec{k}'$$

$$\frac{\hbar^2 k^2}{2m} + \hbar c q = \frac{\hbar^2 k'^2}{2m}$$

$$\delta\left(\frac{\hbar^2 \left(\vec{k}\vec{q}\right)}{m} + \frac{\hbar^2 q^2}{2m} - \hbar c q \right)$$

$$\Omega^- = \frac{2\pi}{\hbar} \frac{\hbar \Xi^2 q^2}{2M\omega_\lambda} \bar{n}_q \delta\left(\frac{\hbar^2 q^2}{2m} - \hbar c q + \frac{\hbar^2 \left(\vec{k}\vec{q}\right)}{m} \right) = \pi \frac{\Xi^2 q}{Mc} \bar{n}_q \delta\left(\frac{\hbar^2 q^2}{2m} - \hbar c q + \frac{\hbar^2 \left(\vec{k}\vec{q}\right)}{m} \right)$$

Phonon generation



$$\vec{k} = \vec{k}' + \vec{q}$$

$$\frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 k'^2}{2m} + \hbar c q$$

$$\delta\left(\frac{\hbar^2 q^2}{2m} + \hbar c q - \frac{\hbar^2 \left(\vec{k}\vec{q}\right)}{m}\right)$$

$$\Omega^{+}\left(\vec{k},\vec{q}\right) = \pi \frac{\Xi^2 q}{Mc}\left(\bar{n}_q + 1\right)\delta\left(\frac{\hbar^2 q^2}{2m} + \hbar c q - \frac{\hbar^2\left(\vec{k}\vec{q}\right)}{m}\right)$$

Total and momentum collision rate and energy loss rate

$$\nu_{tot}(k)=\frac{\Xi^2}{4\pi\rho c}\frac{m}{\hbar^2 k}\left[\int_0^{\max\left(0,2k-\frac{2mc}{\hbar}\right)}\left(\bar{n}_q+1\right)q^2dq+\int_{\max\left(0,\frac{2mc}{\hbar}-2k\right)}^{2k+\frac{2mc}{\hbar}}\bar{n}_q q^2 dq\right]$$

$$\nu_d(k)=\frac{\Xi^2}{4\pi\rho c}\frac{m}{\hbar^2 k^3}\left[\int_0^{\max\left(0,2k-\frac{2mc}{\hbar}\right)}\left(\bar{n}_q+1\right)q^2\left(\frac{mcq}{\hbar}+\frac{q^2}{2}\right)dq-\int_{\max\left(0,\frac{2mc}{\hbar}-2k\right)}^{2k+\frac{2mc}{\hbar}}\bar{n}_q q^2\left(\frac{mcq}{\hbar}-\frac{q^2}{2}\right)dq\right]$$

$$\frac{d\varepsilon}{dt}(\varepsilon)=\frac{\Xi^2}{4\pi\hbar\rho}\sqrt{\frac{m}{2\varepsilon}}\left[\int_{\max\left(0,\frac{2mc}{\hbar}-\frac{2\sqrt{2m\varepsilon}}{\hbar}\right)}^{\frac{2\sqrt{2m\varepsilon}}{\hbar}+\frac{2mc}{\hbar}}\bar{n}_q q^3dq-\int_0^{\max\left(0,\frac{2\sqrt{2m\varepsilon}}{\hbar}-\frac{2mc}{\hbar}\right)}\left(\bar{n}_q+1\right)q^3dq\right]$$