MUON SPIN DEPOLARIZATION IN SOLID He-3

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Some history

First experiments with solid Hydrogen - evidence of relaxation in para- and ortho-hydrogen [E.P. Krasnoperov group, 1984] Explanation – a molecular ion $H_2\mu^+$ formation
1) a "frozen" ion – gives relaxation in para-hydrogen, bad agreement with experiments,
2) rotational diffusion – incorrect temperature dependence,
3) non-thermalized ion – spin-rotation interaction exists in ortho- and para-ions, relaxation induced by interactions of vibrations and rotations of the ion with lattice (spin-phonon interaction). The ion forms large clusters.

Suggestion of experiments with solid Helium for evidence of similar molecular ions $He_2\mu^+$ and $He\mu^+$.

Muonium formation and temperature dependence of a relaxation rate in liquid Helium (^{4}He) was observed [E.P. Krasnoperov's group, 1985] and only diamagnetic component without relaxation in solid ^{4}He was observed. If ions $He_2\mu^+$ or $He\mu^+$ form a relaxation mechanism due to spin-rotation interaction similar to that in solid Hydrogen must exist.

Question: do molecular ions form in Helium?

If not – what is the reason? What is the difference from Hydrogen?

Molecular ions $He_2\mu^+$ and $He\mu^+$ have sufficiently large binding energy 1.9 eV and 0.47 eV respectively.

New direction – interaction with track and its role for Muonium formation.

Evidence of a strong temperature dependence of a diamagnetic component relaxation in solid ^{3}He for T<1 K [E.P.Krasnoperov et.al., 1994]

Λ, MHz

0.2

0.1

0.0

0.0   0.5   1.0   1.5 T, K

○ — 1
△ — 2
⊞ — 3
• — 4

Temperature dependence of a relaxation rate of a diamagnetic component in solid ^{3}He [E.P.Krasnoperov et.al., 1996] for different specific volume (curves 1, 2 and 4) and in pure ^{4}He (curve 3).
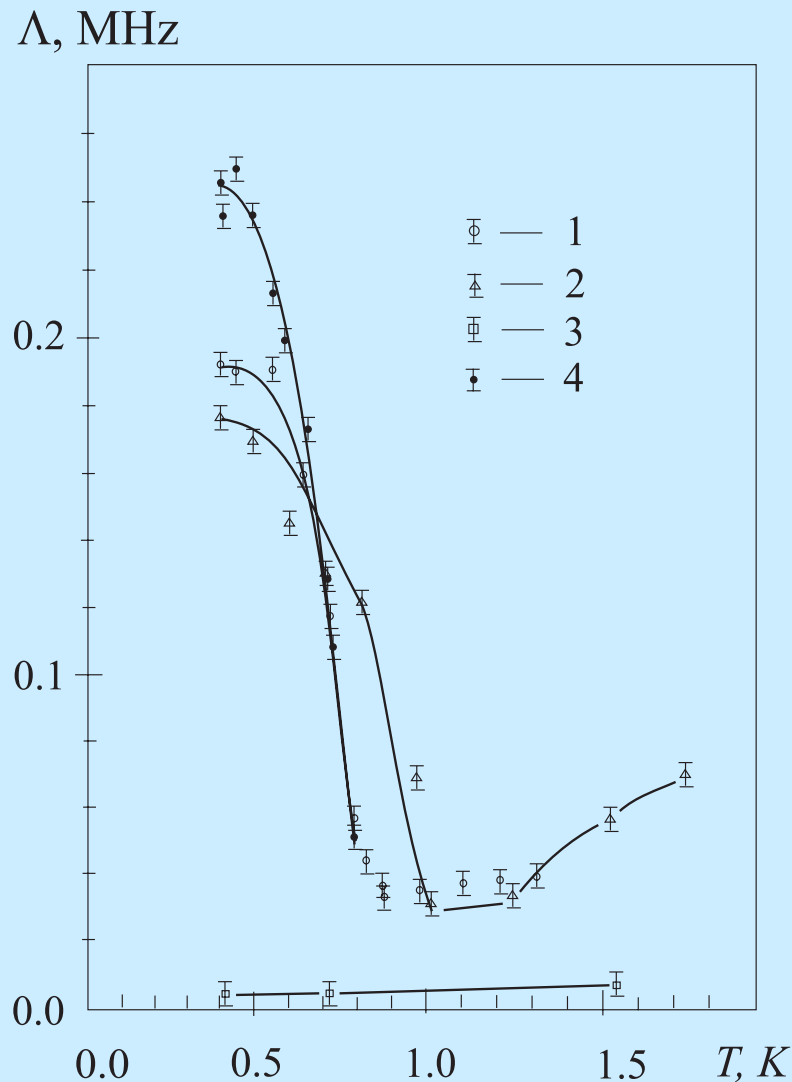
Maximum value is more higher and minimum is more smaller than the second moment for inhomogeneous static dipole-dipole interaction
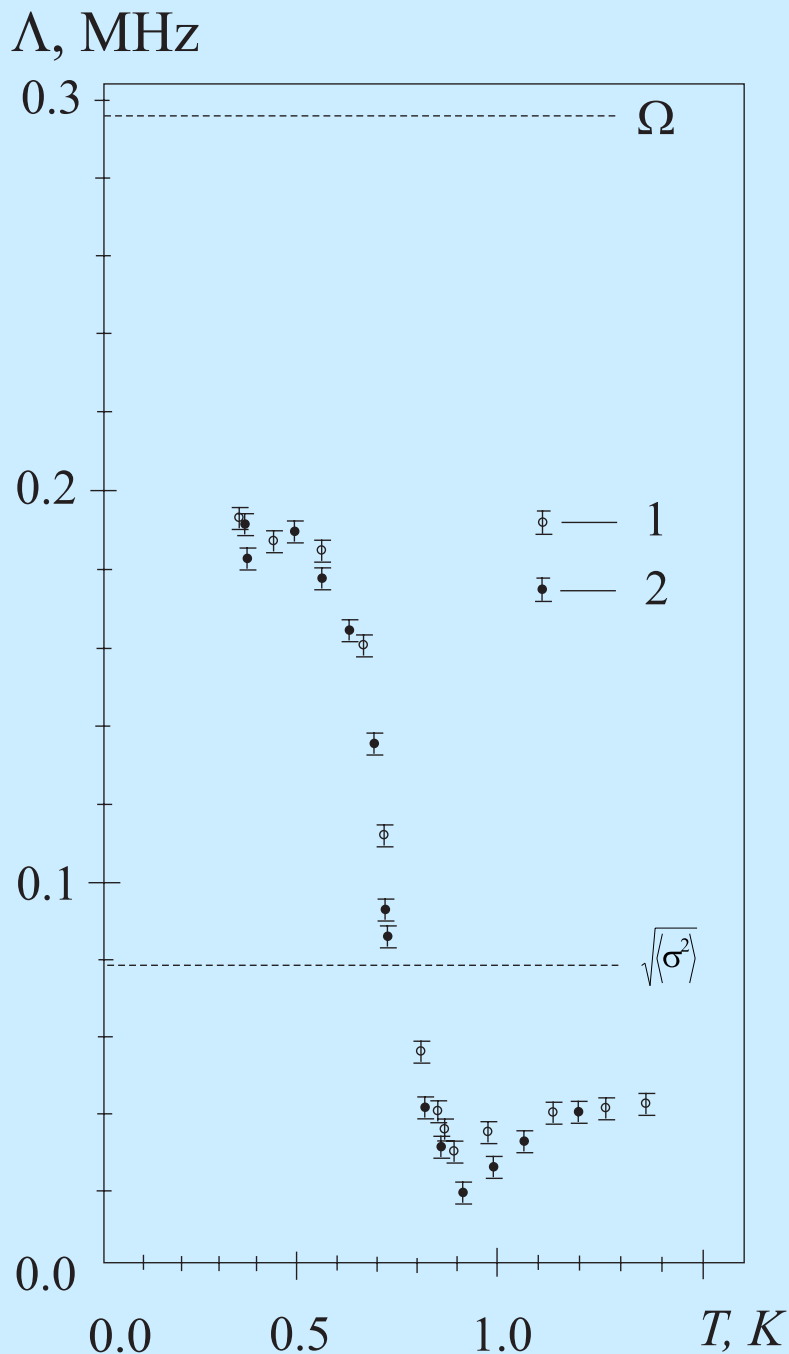
Λ, MHz

0.3 — Ω

0.2

$\boxed{\underline{\text{ }}}$ — 1

$\boxed{\underline{\text{ }}}$ — 2

0.1

$\sqrt{\langle \sigma^2 \rangle}$

0.0

0.0   0.5   1.0 T, K

Temperature dependence of a relaxation rate of a diamagnetic component in solid ^{3}He [E.P.Krasnoperov et.al., 1997] for different concentration of ^{4}He.

The value of dipole-dipole interaction in the ion He$_2\mu^+$ Ω is of the order of the maximum value of a relaxation rate.

Molecular ion He$_2\mu^+$ or Heμ^+ must be formed in solid Helium.

First suggestion – the molecular ion He$_2\mu^+$ is "frozen" and spin-rotation interaction doesn't exist.

It gives incorrect dependence of a relaxation rate on an external magnetic field.

If the molecular ions formed they are not "frozen".
Are they thermalized?

Coulomb interactions of ions with the crystal lattice

The binding energy of the ions $He_2\mu^+$ and $He\mu^+$ is too large and just after formation the ions must be at high levels of a vibrational-rotational spectrum with the excitation energy $\varepsilon \sim 1$ eV. This means that the ions have large vibrational v and rotational K numbers at the initial state. There are no interactions exceeding this value in quantum crystal. Therefore, freezing of the ions is not obvious.

Thermalization of the excited ions is possible owing to interactions of vibrational and rotational motions with a lattice, when a phonon is radiated. This process is possible as a result of radial phonon radiation, because an energy of acoustic phonons is too small. Interaction of an ion with lattice is determined by an energy of its electric multipole moments with inhomogeneous electric field created by polarized Helium atoms. We consider that an expansion of the interaction on multipoles is correct.

Let's examine at first the ion $He\mu^+$, which possesses both quadrupole and dipole moments relatively to its center of masses. The dipole moment is negligibly small: $d \approx 0.3 \cdot 10^{-2} e R_0$, where $R_0 \approx 0.93$ Å is a linear dimension of the ion. Components of the quadrupole moment are equal to $Q_{\parallel} = -2Q_{\perp} = Q = e R_0^2$ respectively. The ion $He_2\mu^+$ possesses only quadrupole moment two times more than the ion $He\mu^+$. The characteristic value of the interaction energy of a quadrupole moment is equal to

$$ V_Q \sim \frac{6\alpha e Q}{R_1^6} \sim 3 \cdot 10^{-3} eV. \tag{1} $$

Here $\alpha = 1.383 a_0^3$ is the Helium atom polarizability and a_0 is the Bohr radius.

This value is less than a quantum of a rotation energy of the ion $He\mu^+$ and two order less than its quantum of a vibration energy. The value (1) is of the order of a quantum of a rotation energy of the ion $He_2\mu^+$ and two order less of its quantum of a vibration energy. Thus we can see, that the interaction of the electric quadrupole moment with the lattice couldn't freeze the exited ions both $He\mu^+$ and $He_2\mu^+$. Nevertheless, this interaction is very important for the rotation levels with $K = 1$.

The interaction energy of the quadrupole moment of an ion with the inhomogeneous electric field induced by a polarized Helium atom depends on the orientation of the ion and is equal to

$$V_Q = \frac{2\alpha e}{r^6} Q_{\alpha\beta} \left(3n_\alpha n_\beta - \delta_{\alpha\beta} \right). \tag{2}$$

We took into account, that polarized Helium atoms of a lattice are oriented along the radius-vector and the property of the tensor $Q_{\alpha\alpha} = 0$.

Relaxation of rotational states

Let's consider at first relaxation of rotational states. Well known, that the interaction (2) is correspondent to the operator describing an interaction of a rotation moment with electrostatic field of a crystal (see e.g. [10]):

$$\widehat{V}_Q = -\frac{1}{(2K-1)(2K+3)} \frac{2\alpha e}{r^6} Q_{\alpha\beta} \left(\frac{3}{2} \left(\widehat{K}_\alpha \widehat{K}_\beta + \widehat{K}_\beta \widehat{K}_\alpha \right) - \delta_{\alpha\beta} K(K+1) \right). \tag{3}$$

The operator (3) doesn't depend on time, conserves energy of the system and cannot give rise a relaxation. But we can derive from it an operator describing interactions of an ions rotation with phonons. Previously it was shown [5, 9], that positively charged ion strongly distorts a lattice and hence a phonon spectrum. As a result, in the crystal only radial phonons could be excited. We can get the operator describing interactions of rotations of an ion with radial phonons if a partial derivative on a radial variable was taken:

$$\widehat{V}_{K-ph} = \frac{1}{(2K-1)(2K+3)} \frac{12\alpha e}{r^7} Q_{\alpha\beta} \left(\frac{3}{2} \left(\widehat{K}_\alpha \widehat{K}_\beta + \widehat{K}_\beta \widehat{K}_\alpha \right) - \delta_{\alpha\beta} K(K+1) \right) \hat{u}. \tag{4}$$

Here the operator $\hat{u}$ describes radial shifts of lattice atoms and is equal to:

$$\hat{u} = \frac{1}{r} \sum_k \sqrt{\frac{\hbar}{2\rho\omega(k)R_B}} e^{-ikr} (b_k^+ + b_k). \tag{5}$$

Here ρ is the crystal specific mass, $\omega(k)$ is the frequency of a phonon with a wave-vector k, the creation and annihilation operators $\hat{b}_k^+$ and $\hat{b}_k$ respectively satisfy commutation relations $\left[\hat{b}_{k'}, \hat{b}_k^+ \right] = \delta_{k',k}$ and the parameter $R_B \to \infty$.

In the long-wave limit the radial-phonon frequency is equal to [5, 9]:

$$\omega = \omega_0 \sqrt{1 + \frac{c_\parallel^2}{\omega_0^2} k^2} \approx \omega_0 + \mathcal{B}k^2, \tag{6}$$

where $\omega_0 = c_\parallel \sqrt{A/3\pi R_1^6} \sim 10^{12} \text{ c}^{-1}$, $A = 5\alpha e^2 n/c_\parallel^2 \rho$, $c_\parallel$ is a longitudinal velocity of a sound, n is the crystal density, $\mathcal{B} = c_\parallel^2/2\omega_0 \sim 10^{-3} \text{ cm}^2/\text{c}$.

The interaction operator can change a rotational moment and emit one radial phonon. Note, that for the ion $He_2\mu^+$ the only possible changing is $\Delta K = 2$ and for the ion $He\mu^+$ $\Delta K = 1, 2$ is possible.

We shall take the well-known formula from perturbation theory for transitions into a continuous spectrum (see, e.g., [11]) to find the probability of the transition of the ion from the initial state $|i\rangle = |K, M\rangle$ to the final state $|f\rangle = |K', M'\rangle$:

$$\mathrm{d}w_{if} = \frac{2\pi}{\hbar} \sum_{M'} \left| \langle n_{ph} + 1 | \langle K', M' | \widehat{V}_{K-ph} | K, M \rangle | n_{ph} \rangle \right|^2 \delta \left(E_i - E_f \right) \mathrm{d}\nu_f. \quad (7)$$

We took into account, that only one phonon could be emitted, $\mathrm{d}\nu_f$ is the number of radial phonon states with the wave-vector k,

$$E_i - E_f = \hbar\omega_r K(K+1) - \hbar\omega_r K'(K'+1) - \hbar\omega(k) = 2\hbar\omega_r(2K-1) - \hbar\omega(k),$$

$\hbar\omega_r = B_r$ is the quantum of rotational energy. We suppose, that the rotational number changes as $K' = K - 2$.

Detail calculations of matrix elements are very complicated in a general form. But it is not necessary to carry out them for evaluating interesting probabilities. It is sufficient to calculate the simplest case when matrix elements are diagonal on a projection of the rotational number. Omitting complex calculations for lack of space, we give the expression for the probability of the transition $K' = K - 2$ in a unit of time:

$$w_{K \to K-2} = \frac{N_1}{R_1^{16}} \frac{\mathcal{C}^2 B(K)}{2B_r \rho} \left(n \left(2B_r (2K-1) \right) + 1 \right) \left(\frac{d\omega(k)}{dk} \right)^{-1} \Bigg|_{\omega = 2\omega_r(2K-1)}, \quad (8)$$

where R_1 is the radius of the first coordination sphere, $\mathcal{C} = 12\alpha eQ$,

$$B(K) = \frac{9K(K-1)^2}{(2K+1)(2K-1)^2(2K-3)} \left(K^2 - \frac{14}{15}(K-2) + \frac{5K}{2K-3} \right),$$

$n \left(2B_r(2K-1) \right)$, $d\omega/dk$ are a distribution function and a spectrum density for radial phonons.

We get finally that the rate transition doesn't depend on temperature and could be represented in this way:

$$w_{K \to K-2}\Big|_{K \gg 1} \sim 10^9 \sqrt{K}. \tag{9}$$

Thus, our estimates show, that a transition time from upper to the lowest rotational levels of energy with $K = 1$ for the ortho- and $K = 0$ for the para-ion is of the order of

$$\tau_{\text{rot}} \sim 10^{-8} - 10^{-9} \text{c}.$$

Relaxation of vibrational states

Now we shall evaluate a transition rate between energy levels of a vibrational spectrum. Firstly note, that for the ion $\text{He}_2\mu^+$ two types of vibrations are exist: symmetric and antisymmetric one. Frequencies of antisymmetric vibrations are approximately one order more then the same for symmetric vibrations. Therefore, we shall consider transitions only between states of symmetric vibrations. Let's direct z-axis along the ion axis and keep only part linearly dependent on nucleus displacements. In this case the operator of the phonon-vibrational interaction is equal to:

$$\widehat{V}_{v-ph} = \frac{12\alpha e}{r^7} Q_0 \frac{a_0}{R_0} \left(\hat{a}^+ + \hat{a}\right) \hat{u} \left(3\cos^2\theta - 1\right), \tag{10}$$

where $\hat{a}^+$ and $\hat{a}$ are increasing and decreasing operators respectively in the vibrational part of the ion's Hamiltonian, $a_0 = \sqrt{\hbar/M_{\text{He}}\omega_A}$ is the characteristic unit of length of the ion vibrations and ω_A is the frequency of symmetric vibrations.

The transition probability between vibrational states is calculated by the same way as in the case of transitions between rotational states, taking the well-known formula from perturbation theory for transitions into a continuous spectrum. Omitting complex calculations for lack of space, we give the expression for the probability of the transition $v' = v - 1$ in a unit of time:

$$w_{v \to v-1} = N_1 \mathcal{C}^2 \left(\frac{a_0}{R_0} \right)^2 \frac{v}{5 \rho \omega(k) R_1^{16}} \left(n(\omega_A) + 1 \right) \left(\frac{d\omega}{dk} \right)^{-1}_{\omega = \omega_A}. \tag{11}$$

The formulas (8) and (11) allow us to compare transition rates between vibrational and rotational states:

$$\frac{w_{v \to v-1}}{w_{K \to K-2}} = \frac{2}{5} \left(\frac{a_0}{R_0} \right)^2 \frac{v \omega_r}{B(K) \omega_A} \sqrt{\frac{2\omega_r(2K-1) - \omega_0}{\omega_A - \omega_0}} \frac{n(\omega(k)) + 1}{n(2\omega_r(2K-1)) + 1}. \tag{12}$$

In the limit of large values of the rotational number K the ratio (12) could be simplified:

$$\frac{w_{\text{vibr}}}{w_{\text{rot}}} \sim \frac{v}{\sqrt{K}} \left(\frac{\omega_r}{\omega_A} \right)^{3/2}. \tag{13}$$

Thus, we can see, that the rates (8) and (11) are of the same order and approximately equal to each other for the lowest energy levels.

The calculations, that were carried out, show that the molecular ion $He_2\mu^+$ is thermalized during quite short time $\tau \sim 10^{-8} - 10^{-9}$c. A thermalization time for the molecular ion $He\mu^+$ is approximately for the one order less. Really, according to the formulas (8) and (11), we have $(\omega_r/\omega_A)^{3/2} \sim (M_{He}/m_\mu)^{3/4} \sim 10$. If we suppose that at the initial moment of the ion formation all spin states of the helium nuclear are equiprobable, we shall get that with the probability of $1/4$ the ion $He_2\mu^+$ is in the para-state ($I = 0$) with $K = 0$ and with the probability of $3/4$ is in the ortho-state ($I = 1$) with $K = 1$. The molecular ion $He\mu^+$ is completely thermalized and lives in the ground state with $K = 0$ and $v = 0$.

Removing degeneration on M

In spite of the ion $He_2\mu^+$ after a thermalization process in the ortho-state possesses non-zero rotational number $K = 1$, a "usual" rotation could not exist, because of very strong interaction of its quadrupole moment with un-homogenous electric field of the crystal lattice. This interaction gives rise to the effect that could be interpreted as a "slowing down" rotation.

Really, the interaction operator (3) removes a degeneration of rotational states on the projection M of the rotational number K. After taking into account this interaction, rotational energy levels depend on both K and a modulus of $|M|$: $E_{K,|M|}$. The value of the interaction (2) is of the order of the quantum of rotational energy, nevertheless, the perturbation theory is valid. Naturally, the energy interval for the following permitted level with $K = 3$ is $\Delta E = E_3 - E_1 = 10B_r \gg \hbar\omega_0$. Thus, the interaction (3) splits the triple-degenerated rotational level with $K = 1$ on two levels: single state with $M = 0$ and a double degenerated level with $M = \pm 1$. The energy level splitting could be found as a diagonal matrix element:

$$\varepsilon^{(1)} = \langle K, M | \widehat{V}_Q | K, M \rangle. \tag{14}$$

After some calculations we get the splitting for the level with $K = 1$:

$$\Delta\varepsilon = \frac{4}{5} \frac{\alpha e Q}{R_1^6}. \tag{15}$$

We can write eigen states in the form:

$$|1, 0\rangle = |K = 1, M = 0\rangle, \tag{16}$$

$$|1, \pm\rangle = \frac{1}{\sqrt{2}} \Big(|K = 1, M = +1\rangle \pm |K = 1, M = -1\rangle \Big), \tag{17}$$

The results are very important when deriving spin-hamiltonian of the molecular ion.

Conclusions I

In the conclusion we emphasize that the molecular ions both $He_2\mu^+$ and $He\mu^+$, if they form, are thermalized very quickly respectively to the muon life-time. If a formation of the ion $He\mu^+$ was preferably, no relaxation of the muon spin polarization could be observed. Thus, we may approve that a formation of the molecular ion $He_2\mu^+$ is preferably and the ion $He\mu^+$ could be formed in the less part of events. In a pure solid ^4He the formed molecular ion $^4He_2\mu^+$ is in the ground state with $K = 0$. Any hyperfine interactions are absent for this state, and no depolarization could be observed. Three types of molecular ions could be formed in a mixture ^3He-^4He: $^3He_2\mu^+$, $^4He_2\mu^+$ and $^4He^3He\mu^+$. The ground states for the two last types of the ions have the rotational number $K = 0$, and no depolarization could be observed in these cases. A muon spin depolarization could be observed only for the molecular ion $^3He_2\mu^+$.

Spin-hamiltonian of thermalized ion $\mathbf{He_2\mu^+}$

For a thermalized ion ${}^3\mathrm{He}_2\mu^+$ at the external magnetic field $\mathbf{B}$ the Hamiltonian of the hyperfine interactions is in general case of the form:

$$\widehat{H}_{hfs} = -2\mu_\mu \mathbf{B}(\hat{\mathbf{s}} + \zeta\widehat{\mathbf{I}}) + (\omega_1\hat{\mathbf{s}} + \omega_2\widehat{\mathbf{I}})\widehat{\mathbf{K}} + \omega_N(\widehat{\mathbf{K}}\mathbf{B}) +$$

$$+ \Omega g_K\left\{ K(K+1)\hat{\mathbf{s}}\widehat{\mathbf{I}} - \frac{3}{2}\left[(\widehat{\mathbf{K}}\widehat{\mathbf{I}})(\widehat{\mathbf{K}}\hat{\mathbf{s}}) + (\widehat{\mathbf{K}}\hat{\mathbf{s}})(\widehat{\mathbf{K}}\widehat{\mathbf{I}})\right] \right\}, \qquad (1)$$

where μ_μ is the magnetic moment of a muon,

$$\Omega = \frac{\zeta\mu_\mu^2}{\hbar R_0^3} \approx 3\cdot 10^6 \mathrm{c}^{-1},$$

$$g_K = -\frac{2}{(2K-1)(2K+3)}$$

is the reduced matrix element, ζ is the ratio of the magnetic moments of ${}^3\mathrm{He}$ and μ^+, $R_0 \approx 0.93\text{Å}$ is a linear dimension of an ion, ω_1 and ω_2 are constants describing spin-rotation interactions of magnetic moments of a muon and nuclei respectively, ω_N is a precession frequency of an ion rotational moment.

One can easy see that hyperfine interactions are equal to zero for a para-ion with a total nuclear spin $I = 0$. So we consider only an ortho-ion when a total nuclear spin $I = 1$.

By the definition the spin-hamiltonian is a diagonal matrix element on the eigen states of the orbital motion (see e.g. [9, 10]. The eigen states for the ion $He_2\mu^+$ at the rotational state with $K = 1$ are of the form [6]:

$$|0\rangle = |K = 1, M = 0\rangle, \tag{2}$$

$$|1, \pm\rangle = \frac{1}{\sqrt{2}}\left(|K = 1, M = +1\rangle \pm |K = 1, M = -1\rangle\right). \tag{3}$$

We can see that a spin-rotation interaction is equal to zero in all states

Let us direct the z-axes along the external magnetic field $\mathbf{B}$. In this case the spin-hamiltonian for the states (2)-(3) could be written in the form:

$$\widehat{H}_0 = -\omega(\widehat{s}_z + \zeta\widehat{I}_z) - \frac{4}{5}\Omega\left(\widehat{s}_z\widehat{I}_z - \frac{1}{4}(\widehat{s}_+\widehat{I}_- + \widehat{s}_-\widehat{I}_+)\right); \tag{4}$$

$$\widehat{H}_{1,\pm} = -\omega(\widehat{s}_z + \zeta\widehat{I}_z) + \frac{2}{5}\Omega\left(\widehat{s}_z\widehat{I}_z - \frac{1}{4}(\widehat{s}_+\widehat{I}_- + \widehat{s}_-\widehat{I}_+)\right) \pm \frac{3}{10}\Omega(\widehat{s}_+\widehat{I}_+ + \widehat{s}_-\widehat{I}_-). \tag{5}$$

Here ω is the precession frequency of a muon magnetic moment at the external magnetic field.

The formulas (4) and (5) show that the spin-hamiltonian has very anisotropic view and constants differs both in a value and a sign. In particular it means that depolarization rates will be differ for different states. It is obvious that in a zero external magnetic field the Hamiltonians (4) and (5) have the same spectrum: three double degenerated energy levels:

$$\varepsilon_{1,2} = \frac{1+\sqrt{3}}{5}\Omega, \quad \varepsilon_{3,4} = \frac{1-\sqrt{3}}{5}\Omega, \quad \varepsilon_{5,6} = -\frac{2}{5}\Omega. \tag{6}$$

A simple analytical solution for a nonzero magnetic field exists only for the Hamiltonian (4):

$$\varepsilon_{1,2} = \frac{1}{5}\Omega \mp \zeta\omega + \sqrt{3\left(\frac{\Omega}{5}\right)^2 \pm \frac{2}{5}\Omega(1-\zeta)\omega + (1-\zeta)^2\omega^2},$$

$$\varepsilon_{3,4} = \frac{1}{5}\Omega \pm \zeta\omega - \sqrt{3\left(\frac{\Omega}{5}\right)^2 \mp \frac{2}{5}\Omega(1-\zeta)\omega + (1-\zeta)^2\omega^2},$$

$$\varepsilon_{5,6} = -\frac{2}{5}\Omega \pm (1+2\zeta)\omega. \tag{7}$$

Depolarization rate

The spin flip rate frequency ν for the nuclear ^3He in an ion can be change in a wide range in dependence of temperature and other parameters [5]. It is necessary to solve relaxation equations for describing depolarization process. Relaxation rates rates in our case could be calculated in frames of the Wangsness-Bloch equations because they are applicable for fluctuated magnetic fields (see, e.g. [10, 11]). For the molecular ion $\text{He}_2\mu^+$ they have the form [10]:

$$\frac{\partial\rho}{\partial t} + \frac{\mathrm{i}}{\hbar}\left[H_{hfs}, \rho\right] = 2\nu\left(\mathbf{i}_1\rho\mathbf{i}_1 + \mathbf{i}_2\rho\mathbf{i}_2 - \frac{3}{2}\rho\right). \tag{8}$$

Here ρ is a spin-density matrix of the ion, H_{hfs} is the spin-hamiltonian (4) or (5), $\mathbf{i}_{1,2}$ are spin operators of Helium nuclei in the ion. A direct mechanism of the muon spin depolarization is neglected.

In a general case of an arbitrary relation between external magnetic field $\mathbf{B}$, energy of magnetic dipole-dipole interaction and a spin flip frequency ν an analytical solution of the equations (8) doesn't exist. If a spin flip frequency $\nu \ll \Omega$ a depolarization rate is simply equal to ν. A behaviour of a muon spin polarization is determined a complex many-frequency picture which effectively observed as a fast depolarization [12]. In this case a depolarization rate doesn't depend on temperature and sufficiently exceeds experimentally observed one. This is the upper limit of a depolarization rate and it can exist at very low temperature that was not achieved at experiments [1, 2, 3, 4].

Let us consider a limit of the strong external magnetic field $\omega \gg \Omega$ when rather simple analytical formulas could be got. In this case we may keep in the Hamiltonian only a secular part of the magnetic dipole-dipole interaction:

$$\widehat{H}_{hfs} = -\omega(\widehat{s}_z + \zeta \widehat{I}_z) - \widetilde{\Omega}\widehat{s}_z\widehat{I}_z, \tag{9}$$

where

$$\widetilde{\Omega} = \begin{cases} -4\Omega/5 & \text{for the state (2),} \\ 2\Omega/5 & \text{for the states (3).} \end{cases} \tag{10}$$

In the secular approximation (9) a longitudinal component of a muon spin polarization is conserved. Let define a total polarization of nuclear spins in an ion as

$$\mathbf{P}^I = \operatorname{Tr} \widehat{\mathbf{I}}\rho, \quad \text{or} \quad P^I_z = \operatorname{Tr} \widehat{I}_z\rho, \quad P^I_\pm = \operatorname{Tr} \widehat{I}_\pm\rho. \tag{11}$$

A longitudinal component P^I_z of the total polarization of nuclear spins in an ion relaxes with a rate 2ν in this case.

For a transverse component of a muon spin polarization we get a system of five connected equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} P_+ \\ P^I_+ \\ Q_1 \\ Q_2 \\ T \end{pmatrix} = \begin{pmatrix} -\mathrm{i}\omega & 0 & \mathrm{i}2\widetilde{\Omega} & 0 & 0 \\ 0 & -\mathrm{i}\zeta\omega - 2\nu & 0 & \mathrm{i}\widetilde{\Omega} & 0 \\ 0 & 0 & -\mathrm{i}\omega - 2\nu & 0 & \mathrm{i}\widetilde{\Omega} \\ 0 & \mathrm{i}\widetilde{\Omega}/4 & 0 & -\mathrm{i}\zeta\omega - 2\nu & 0 \\ 2\nu & 0 & \mathrm{i}\omega & 0 & -\mathrm{i}\omega - 2\nu \end{pmatrix}\begin{pmatrix} P_+ \\ P^I_+ \\ Q_1 \\ Q_2 \\ T \end{pmatrix}, \tag{12}$$

where

$$Q_1 \equiv Q_{+z} = \operatorname{Tr} \widehat{s}_+ \widehat{I}_z\rho, \quad Q_2 \equiv Q_{+z} = \operatorname{Tr} \widehat{s}_Z \widehat{I}_+\rho, \quad T \equiv T_{+z} = \operatorname{Tr} \widehat{s}_+ \widehat{I}_z^2\rho, \tag{13}$$

The equation system (12) gives four rapidly relaxed solutions with very small amplitudes (roots of the characteristic equation are $\lambda_{1,2} = -2\nu - \mathrm{i}(\zeta\omega \pm \widetilde{\Omega}/2)$, $\lambda_3 = 2\lambda_4 \approx -2\nu$) and one slowly down solution. An amplitude of this solution is approximately equal to 1 with a neglecting parts of the order $\widetilde{\Omega}/\nu$. Thus the time dependence of a transverse component of a muon spin polarization in a strong external magnetic field is determined by a simple formula:

$$P_+(t) = \mathrm{e}^{-\Lambda t - \mathrm{i}\omega t} P_+(0), \tag{14}$$

where

$$\Lambda = \frac{4\nu\widetilde{\Omega}^2}{8\nu^2 + \omega\widetilde{\Omega}}. \tag{15}$$

The formulae (14)-(15) show that a depolarization rate of a transverse component of a muon spin polarization in state (2) is approximately four time exceeds that one in states (3).

Conclusions II

A polarization of an ensemble of muons must be written as a sum of polarizations in para- and ortho-ions:

$$\mathcal{P}_\perp(t) = \frac{1}{4}P_+^{(0)}(t) + \frac{3}{4}P_+^{(1)}(t), \tag{16}$$

where the first item describes the polarization behaviour for the para-ion and the second one for the ortho-ion. We assume that Helium nuclei are unpolarized. The component of a total polarization $P_+^{(0)}(t)$ conserves and doesn't depend on temperature. The component $P_+^{(\pm 1)}(t)$ obeys the formulae (14) and (15):

$$P_+^{(1)}(t) = w_0 P_+^{(1,0)}(t) + w_1 P_+^{(1,\pm 1)}(t)$$

If all the three states (2)-(3) were equiprobable, we can write for probabilities $w_0 = 1/3$ and $w_1 = 2/3$. The relaxation of this component was observed at the experiments.
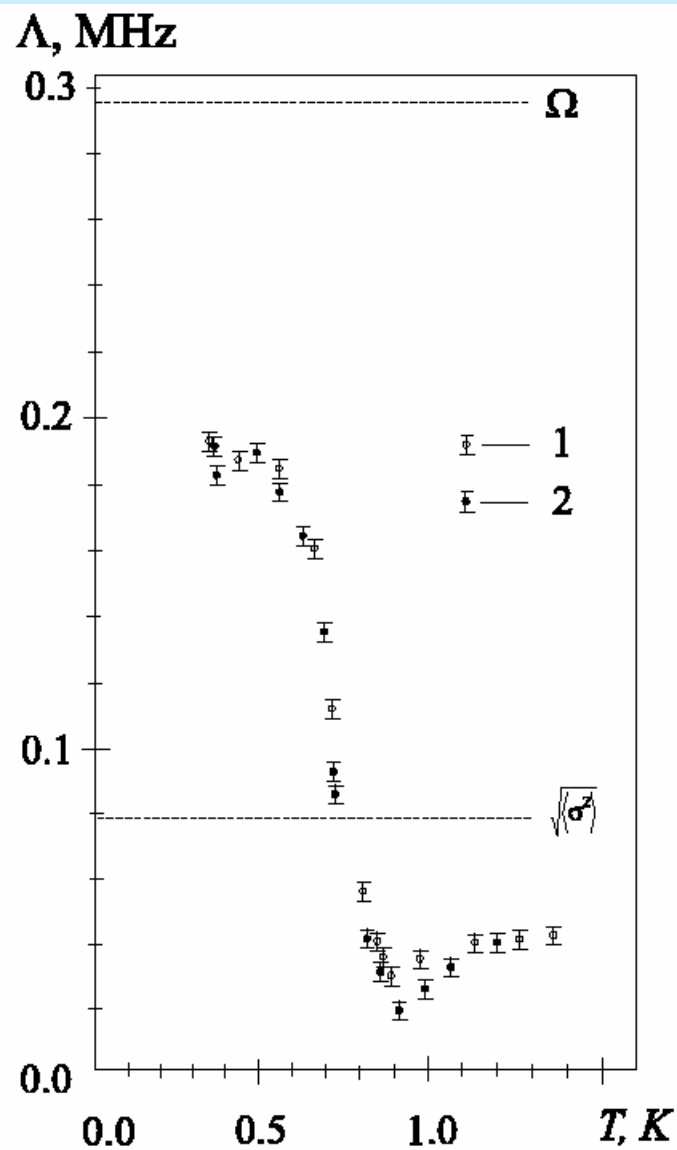
Figure 1: Temperature dependence of a depolarization rate in solid ^{3}He with different concentrations of ^{4}He: 1 - $5 \cdot 10^{-4}$ and 2 - $\sim 1 \cdot 10^{-6}$ at $v = 22.4$ cm^{-3} [4]

The above-given formulae explain the main experimental results (see fig. 1). Really, the relaxation rate (15) decreases when the magnetic field increases. The relaxation rate is effectively reciprocally proportional to ν, if the spin flip frequency was respectively large ($\nu \gg \omega, \Omega$). This ratio gives an exponential growth of a relaxation rate, when temperature decreases at $T < 1\mathrm{K}$. The equation (15) isn't valid when $\nu \sim \omega, \Omega$, but we can see that a relaxation rate has a maximum. The value of this maximum must be less than the characteristic parameter Ω. So we can say that:

$$\Lambda_{\mathrm{max}}\big|_{\nu \sim \Omega} \leq \Omega.$$

We can get the lowest estimation of the maximum value, if we extrapolate the equation (15) in the region $\nu \sim \Omega$. Simple calculations show, that the relaxation rate Λ has a maximum when $8\nu^2 = \omega\Omega$. For this value of spin flip frequency we get a maximum value for the relaxation rate as:

$$\Lambda_{\mathrm{max}} = \Omega\sqrt{\frac{\Omega}{2\omega}}.$$

For the external fields $B = 100$ Gs used in experiments [1]-[4] we obtain an estimation $\Lambda_{\mathrm{max}} \sim 1.5 \cdot 10^6 \ \mathrm{c}^{-1}$, that is in a good agreement with experimental data. When a spin flip frequency decreases to $\nu < \Omega$ many frequency precession with a relaxation proportional to ν must be observed. This limit was not achieved at experiments.

The value of a relaxation rate for the observed plateau at $T \geq 1\mathrm{K}$ corresponds the second moment for the direct magnetic dipole-dipole interactions of a muon with lattice nuclei:

$$\langle \sigma^2 \rangle = \frac{2}{15} \left(\hbar \gamma_\mu \gamma_I \right)^2 I(I+1) \sum_a r_a^{-6}.$$

For the un-deformed lattice the second moment $\sqrt{\langle \sigma^2 \rangle} \approx 0.9 \cdot 10^5 \ \mathrm{c}^{-1}$ is greater than experimental-observed minimum value. This difference could be explained by fluctuations of host nuclear magnetic moments slowing down the static value.

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Good by, ZOO!

Thank you for
the attention