

# Gribov Light Quark Confinement scenario

Объединенный семинар ОФВЭ и ОТФ ПИЯФ

Yu. Dokshitzer

01. 06. 2023

"Gribov Light Quark Confinement *scenario*", not yet "Gribov Light Quark Confinement *theory*"

V.N. Gribov became interested in non-Abelian fields, and started to study them in 1976.

As an apprentice, he made important discoveries right from the start.

*How come that Gribov pinpointed flaws in the non-Abelian field dynamics that no one saw before him?*

He had his own way: instead of learning the established rules of a new game, he tried to reconstruct the game himself, given the basic objective.

Gribov belongs to the generation of physicists who witnessed the birth of QFT, who felt the fragility of its bases and knew to look for cracks.

His two lectures at the 12th LNPI Winter School (February 1977) were to change forever the non-Abelian QFT landscape.

"Instability of non-Abelian gauge fields and impossibility of the choice of the Coulomb gauge."

"Quantization of non-Abelian gauge theories."

Raised the questions about formulation of QFT of Yang-Mills fields [*gluodynamics*] that remain unanswered to this day.

An outline of the intensive 20-year long pursuit in two statements :

Confinement of colour in the real world is driven by the presence of very light (practically massless) quarks.

Confinement by *perturbative* tools : quark and gluon Green functions.

The reason why Gribov's paper was initially rejected by a NPB referee

- As QCD apprentice (1976):
  - Physical explanation of Antiscreening in non-Abelian QFT
  - Gribov copies
  - Revision of the functional integral of QCD
- Vacuum currents and QFT anomalies
  - Currents in the Dirac sea in external fields
  - Local confinement of charge in massless QED (1982)
  - Quantum anomalies due to collective flow of negative energy fermions (1987)
  - Manifestation of UV momenta other than charge and mass renormalization
- Light quark confinement (1990):
  - Supercritical binding of light Coulomb-like interacting fermions
  - Gribov equation for the quark Green function
  - Critical coupling
  - Chiral symmetry breaking and pions
- Byproducts:
  - Higgs as a superbound  $t\bar{t}$  boson state
  - QED at short distances: Resolution of the “Landau pole” problem

*A brief **pre-history***

*and **autopsy***

*of **Asymptotic Freedom***

## a brief history of Asymptotic Freedom

1955

The polarization of **QED** vacuum makes the coupling run with virtuality  $\alpha \rightarrow \alpha(\mathbf{k}^2)$

Initial calculation of the fermion loop produced *a wrong sign* - a **QCD-ish**  $\beta$ -function.

The time spanned *before B.Ioffe and A.Galanin have pointed at the error* proved to be enough for *L.Landau and I.Pomeranchuk* to develop and enthusiastically discuss with their pupils a *beautiful physical picture* of what we know now under the name of “**asymptotic freedom**”.

“**Moscow Zero**”: vanishing of the physical interaction (*renormalized coupling*) in the limit of a *point-like bare interaction*  $\Lambda_{UV} \rightarrow \infty$

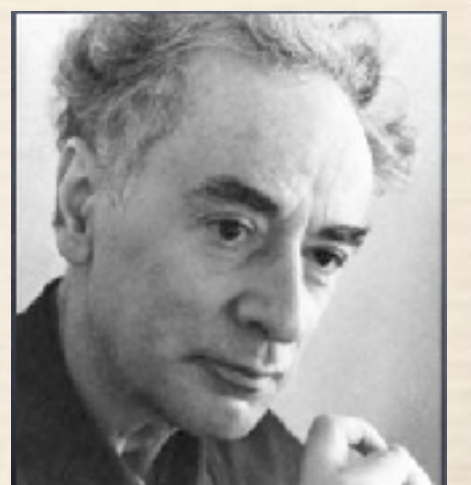
“*...nullification of the theory is tacitly accepted even by theoretical physicists who profess to dispute it.*” (Landau)

Looked as a general, inevitable property of a QFT... (*Pomeranchuk, 1955-58*)

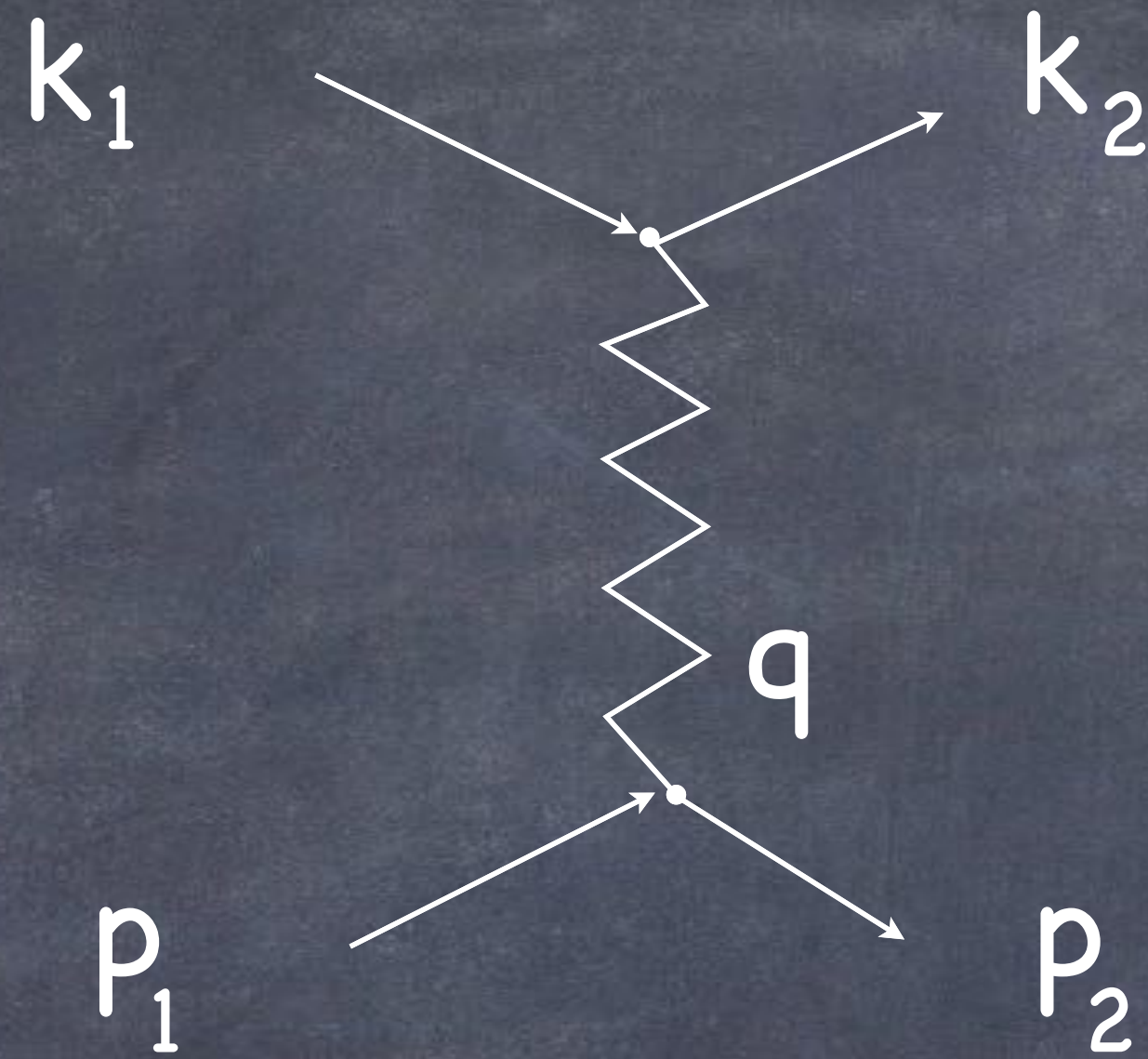
1958 Dyson : “*the correct meson theory will not be found in the next hundred years*”



1960 Landau : “*the Hamiltonian method for strong interactions is dead and must be buried, although of course with deserved honour*”



scattering  
annihilation



relativistic crossing

$$s = (p_1 + k_1)^2$$

invariant  
energy

$$t = (p_1 - p_2)^2$$

momentum  
transfer

one and the same amplitude as a function of its *invariants*  $A(s,t)$  describes three physically different processes related by *crossing*

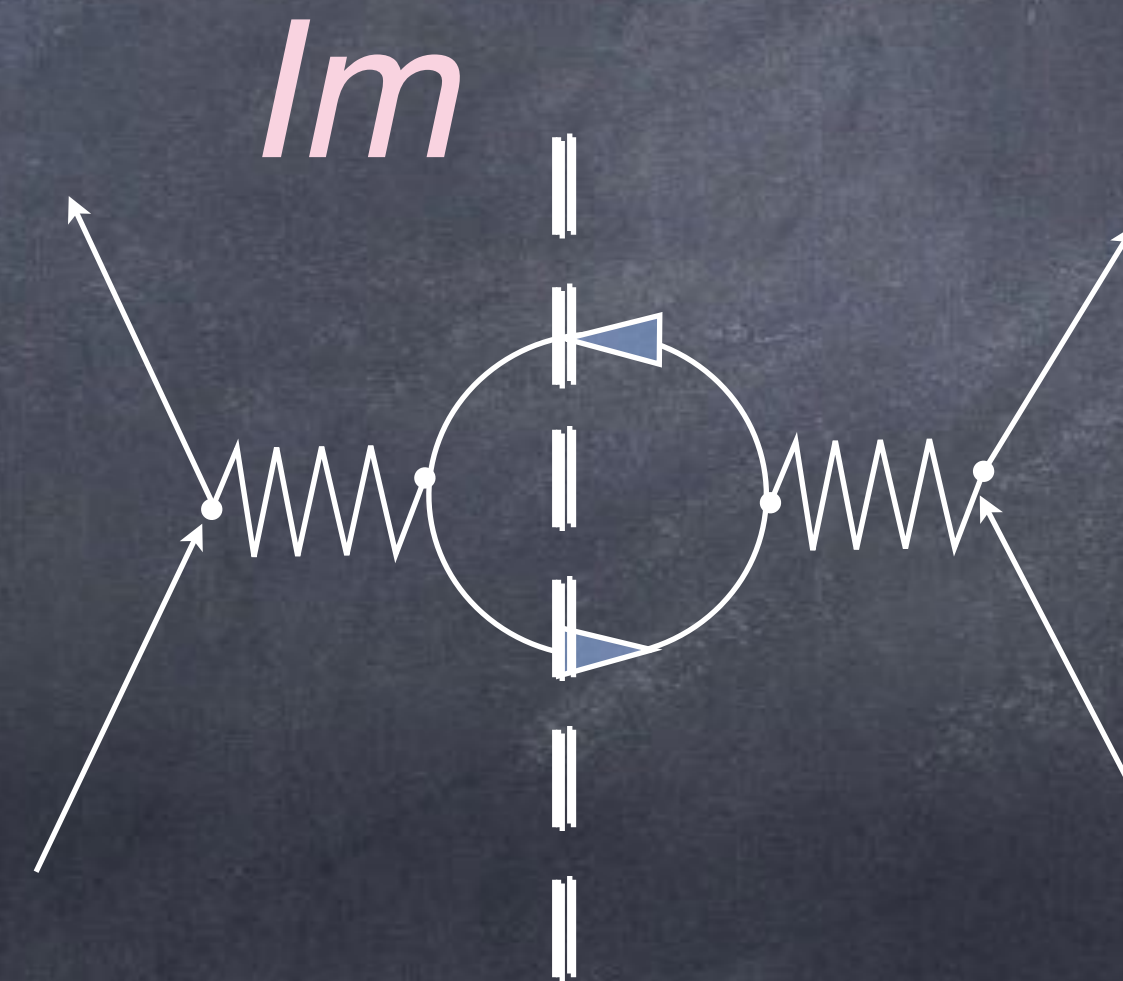
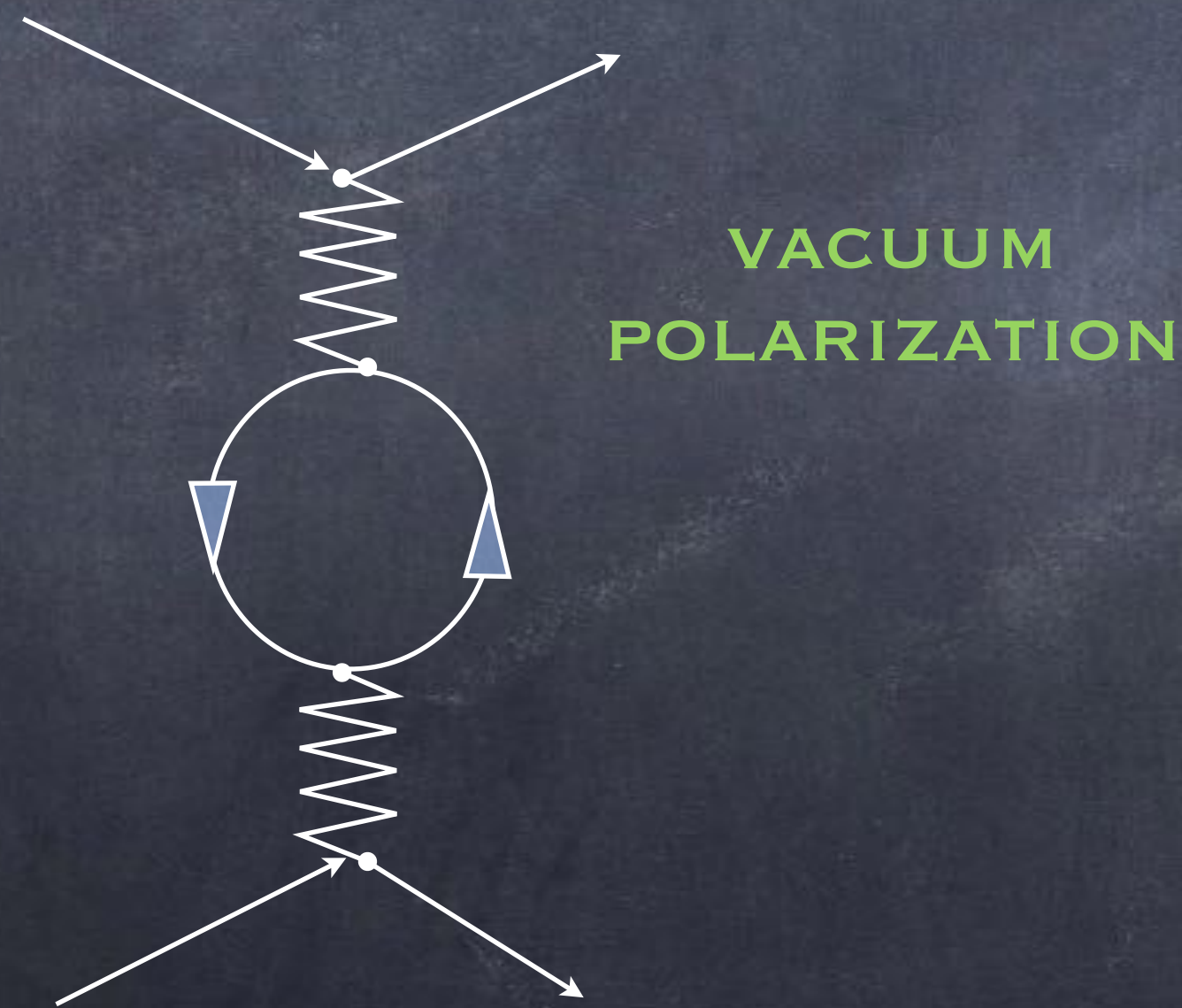
$A(s,t)$  is an **analytic function** of energy  $s$  (**causality**)  
and of the momentum transfer  $t$  (**crossing**)

whose **singularities** are determined by the **unitarity**

as any symmetry,  
the **crossing symmetry** has many a powerful,  
and sometimes dramatic, consequences

in particular, it is **crossing** and **unitarity** that made one think  
that the "asymptotically free" behavior of the effective coupling  
(*QCD*) is **impossible**

Indeed, as any QFT amplitude, the **vacuum polarization** loop is **analytic** in  $k^2$ .



$$Im A = BB^* > 0$$

In the *crossing* channel, the imaginary part of the loop amplitude is proportional to the cross section of pair production (*unitarity*). Thus, the “*zero-charge*” sign of the  $\beta$ -function inevitably follows from *positivity* of the decay cross section !

### 1969

I.Khriplovich : *the SU(2) Yang–Mills gauge theory coupling disrespects this wisdom !*

“ *Same-sign charges repulse; same-sign currents attract (gluon magnetic moment)...* ”

*This sort of qualitative incantations do not explain how does YM QFT manage to overpass the unitarity + crossing Landau-Pomeranchuk argument ...*

**Why then - and how - did this argument fail  
in the non-Abelian gauge field theory ?**

### 1977

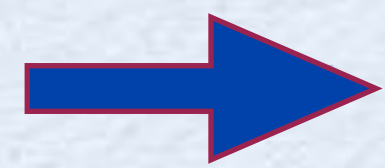
V.Gribov : *physics of “anti-screening” - statistical effect of “zero-fluctuations”*



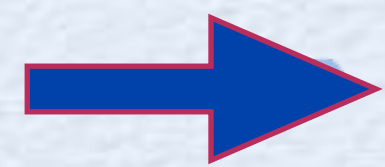
# AUTOPSY OF ASYMPTOTIC FREEDOM

- To address a question starting from *how* or *why* we better talk *physical degrees of freedom*; i.e. use the Hamiltonian language

- Then, we have gluons of *two sorts*:



two “physical” transversely polarized gluons and



Coulomb gluon field - the mediator of the *instantaneous interaction* between colour charges.

Consider Coulomb interaction between two heavy colour charges

**Instantaneous Coulomb interaction**

$= -N_c * \frac{1}{3} - n_f * \frac{2}{3}$

**Transverse gluons (and quarks)**

$\longleftrightarrow$  *screening*

**Instantaneous Coulomb interaction**

$= +N_c * 4$

**Vacuum fluctuations of transverse fields**

$\longleftrightarrow$  *ANTI-screening*

The origin of *antiscreening* — deepening of the ground state under the 2nd order perturbation in NQM:

Combine into the QCD  $\beta$ -function:

$$\beta(\alpha_s) = \frac{d}{d \ln Q^2} 4\pi\alpha_s^{-1}(Q^2) = \left[ 4 - \frac{1}{3} \right] * N_c - \frac{2}{3} * n_f$$

$$\Delta E_0 = \sum_n \frac{|\langle 0 | \delta V | n \rangle|^2}{E_0 - E_n} < 0.$$

# ***Gribov copies***

( *"Gribov horizon", "Gribov uncertainties"* )

The three-dimensional transversality condition  $(\nabla \cdot \mathbf{A}) \equiv \frac{\partial A_i}{\partial x_i} = 0, \quad i = 1, 2, 3$  is usually imposed on the field potential to describe massless vector particles

Being antisymmetric, the field strength tensor  $F_{\mu\nu}^a$  does not contain *time derivative* of the zero-component of the potential  $A_0^a$ .

$A_0^a$  - a *cyclic variable* which does not constitute a physical degree of freedom.

It can be eliminated contributing to the Hamiltonian that describes transverse gluons an additional term responsible for *Coulomb interaction* between "charges".

The Coulomb field "propagator"  $G(\mathbf{x} - \mathbf{y}) = - \left\langle \frac{1}{\mathbf{D}[\mathbf{A}_\perp] \cdot \nabla} \nabla^2 \frac{1}{\mathbf{D}[\mathbf{A}_\perp] \cdot \nabla} \right\rangle$

averaging over transverse gluon fields in the vacuum

Faddeev-Popov operator

Covariant derivative

$$\mathbf{D}[\mathbf{A}_\perp] \cdot = \nabla \cdot + ig_s [\mathbf{A}_\perp, \cdot]$$

Estimate of *non-linearity*:  $g_s \mathbf{A}_\perp / \nabla \sim g_s \cdot |\mathbf{A}_\perp| L \sim 1$  normally  $= \mathcal{O}(g_s \cdot 1) \ll 1$

However, when the field potential gets **large**, the operator  $\mathbf{D}[\mathbf{A}_\perp] \cdot \nabla$  may "vanish" causing singularity in the propagation of the Coulomb field...

$$(\mathbf{D}[\mathbf{A}_\perp] \cdot \nabla) C_0 = \nabla^2 C_0 + ig_s [\mathbf{A}_\perp, \nabla C_0] = 0$$

Appearance of **Zero Modes** of the operator  $\mathbf{D}[\mathbf{A}_\perp] \cdot \nabla$  signals

- failure of extracting physical d.o.f. of the gauge theory
- the "**Gribov horizon**" in the space of gluon fields, beyond which the gauge fixing condition acquires multiple solutions
- also in covariant gauges (e.g. Landau gauge) - the **ghost** becomes a **zombie**: starts walking
- one is forced to reformulate the theory by restricting the integration over gluon fields in the functional integral to the "fundamental domain" where the **Faddeev-Popov determinant** is strictly positive (before the first zero mode  $C_0$ )

Effective suppression of large gluon field results, semi-quantitatively, in an infrared singular polarization operator  $\Pi \propto k^{-2}$ ,  $D^{-1}(k) = k^2 + \Pi(k^2) \simeq k^2 + \frac{\sigma^2}{k^2}$ .

The **massless gluon** disappears,

$$D(k) \propto \frac{k^2}{k^4 + \sigma^2}$$

a sketch, not an answer (causality!)

Meanwhile, the **Coulomb** (ghost) propagator becomes infrared singular, corresponding to a linear increase of the interaction energy at large distances  $R = |\mathbf{x} - \mathbf{y}|$  between colour charges,  $V(R) \propto \sigma R$ .

$$G(k) \propto \frac{1}{N_c g_s^2} \cdot \frac{\sigma}{\mathbf{k}^4}$$

The idea of confinement emerging from dressed Coulomb exchange

Gribov proposal was pursued:

D. Zwanziger *Coulomb confinement* (1998)

J. Greensite and C. Thorn *The gluon chain model* (2002)  
 't Hooft G. Perturbative Confinement. [hep-th/0207179](https://arxiv.org/abs/hep-th/0207179) (2002)

Confinement in pure gluodynamics (?)

Returning to the task of constructing consistent QFT dynamics of non-Abelian gauge fields, we must conclude that, in spite of many attempts, the problem of *Gribov copies* (“Gribov horizon”, “Gribov uncertainties”) remains essentially open today.

By mid 80's Gribov decided to change direction and not pursue pure gluodynamics.

He convinced himself that the solution to the confinement problem lies not in the understanding of the interaction of "large gluon fields" but instead in the understanding of how the QCD dynamics can be arranged as to *prevent the non-Abelian fields from growing real big*.

- No mechanism for binding massless *bosons* (gluons) seems to exist in Quantum Field Theory (QFT), while the Pauli exclusion principle may provide means for binding together massless *fermions* (light quarks).

**fallout :** gluodynamics

[ fetching a 1M\$ jackpot won't help understanding hadron physics ]

- The** confinement in the real world (with 2 very light *u* and *d* quarks), rather than **a** confinement.

lattice calculations

[ large Compton wavelength fermions don't fit in ]

- The Feynman diagram technique has to be reconsidered in QCD if one goes beyond trivial perturbative correction effects. Feynman's famous *iε* prescription was designed for (and applies only to) the theories with *stable perturbative vacua*.

Euclidean rotation in general

[ QFT translation into a Stat. Phys. problem endangered ]

- The problem of ultraviolet regularization may be more than a technical trick in a QFT with apparently infrared-unstable dynamics: the *ultraviolet* and *infrared* regimes of the theory may be closely linked.

RG ideology and practice

[ **pion** example: a Goldstone boson (small distances) vs. quark-antiquark bound state (large distances) ]

To understand and describe a physical process in a *confining theory*, it is necessary to take into consideration the *response of the vacuum*, which leads to essential modifications of the quark and gluon Green functions.

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A known QFT example of such a violent response of the vacuum – screening of *super-charged ions* with  $Z > 137$ .

The expression for Dirac energy levels of an electron in a field created by the point-like electric charge  $Z$  contains

$$\epsilon \propto \sqrt{1 - (\alpha_{\text{e.m.}} Z)^2}.$$

For  $Z > 137$  the energy becomes *complex*. This means *instability*.

- Classically, the electron “*falls onto the centre*”.
- Quantum-mechanically, it also “*falls*”, but into the Dirac sea.

The super-charged ion [*vacuum*] becomes unstable and decays

$$A_Z \implies A_{Z-1} + e^+, \quad \text{for } Z > Z_{\text{crit.}}$$

(Pomeranchuk & Smorodinsky 1945)

# binding massless fermions

In the **QCD** context, the increase of the running quark-gluon coupling at large distances replaces the large **Z** of the **QED** problem.

Gribov generalized the problem of supercritical binding in the field of an *infinitely heavy source* to the case of two **massless fermions** interacting via **Coulomb-like** exchange.

He found that in this case the supercritical phenomenon develops much earlier: a pair of light fermions develops supercritical behaviour if the coupling hits a definite critical value

$$\frac{\alpha}{\pi} > \frac{\alpha_{\text{crit}}}{\pi} = 1 - \sqrt{\frac{2}{3}}$$

With account of the QCD color Casimir operator, **the value of the coupling** above which *restructuring of the perturbative vacuum* leads to **chiral symmetry breaking** and, likely, to **confinement**, translates into

Gribov scenario presumes that the QCD coupling exists in the small momentum domain. Such a coupling (**finite, analytic**) can be indeed designed, and its infrared behaviour can be linked with non-perturbative power suppressed contributions to various pQCD-calculable **CIS** (**C**ollinear-and-**I**nfrared-**S**afe) observables.

$$\frac{\alpha_{\text{crit}}}{\pi} = C_F^{-1} \left[ 1 - \sqrt{\frac{2}{3}} \right] \simeq 0.137$$

Yu.L. Dokshitzer, G. Marchesini and B.R. Webber, Nucl. Phys. **B 469** (1996) 93.

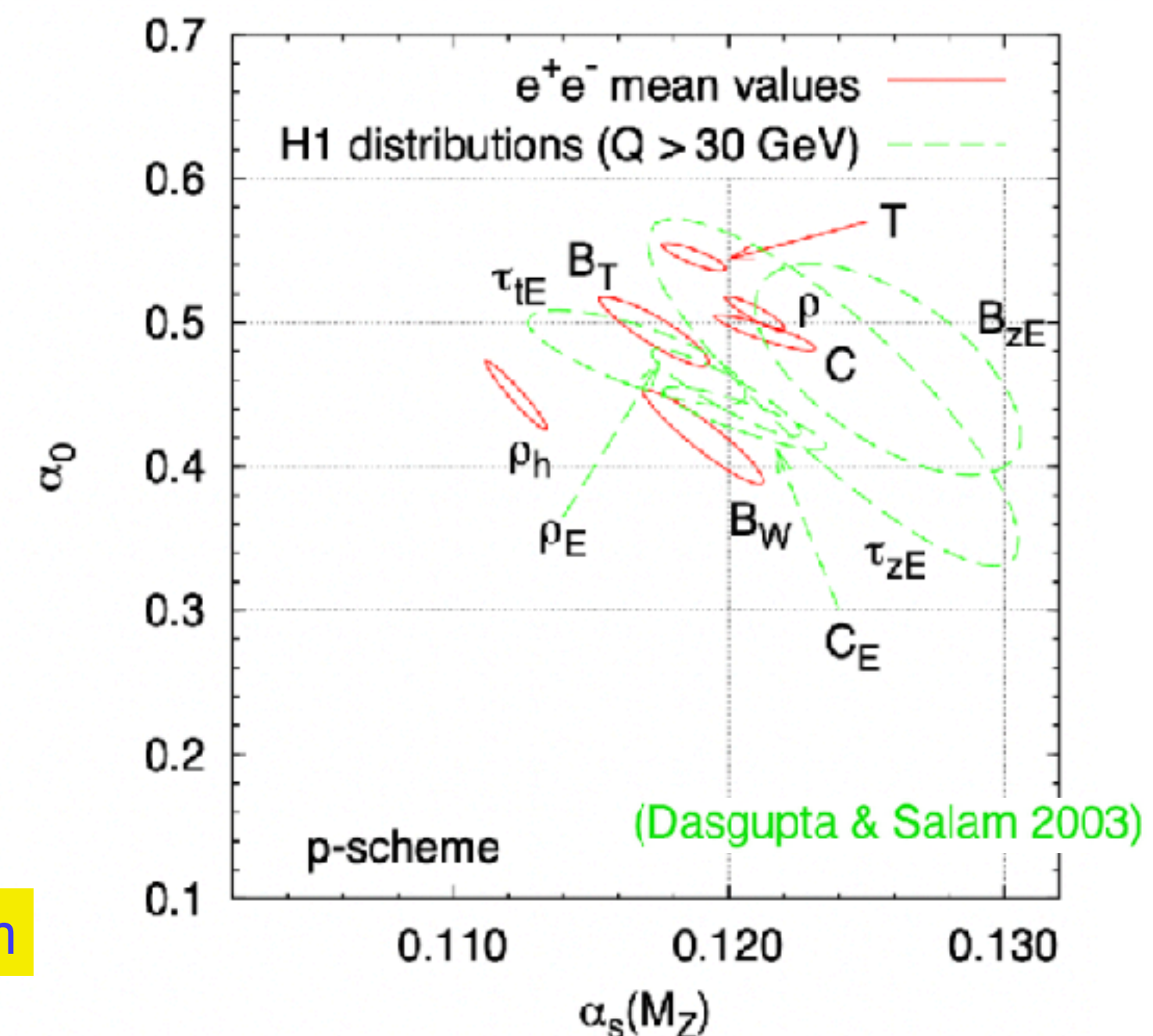
The **average value** of the QCD coupling in the region of small virtual momenta that emerged from the study of the **leading power-suppressed corrections** to **jet shapes** in **e+e-** annihilation and **DIS** turned out to be consistent with the Gribov limit

$$a_0 \equiv \langle \alpha_s(Q^2) \rangle = \frac{1}{2 \text{ GeV}} \int_0^{2 \text{ GeV}} dQ \alpha_s(Q^2) = 0.47 \pm 0.07$$

This value happens to be

- Universal
- Reasonably small
- Comfortably **above the** Gribov's **critical value** ( $\pi \cdot 0.137 \simeq 0.4$ )

Let us dive into some **math** (not without **phys!**) in order to see where the critical coupling came from



Gribov developed a new approximation to the **Schwinger–Dyson equation** for the fermion (quark) Green function which

- takes into account the most singular (logarithmically enhanced) **infrared** and **ultraviolet** renormalization effects,
- makes a smart use of the **gauge invariance**,
- is **local** in the momentum space,
- retains essential **non-linearity** due to quark-gluon interactions
- and possesses a rich **non-perturbative structure**.

Can be looked upon as a perturbative (“leading logarithmic”) approximation that allows one to penetrate into the region of **large anomalous dimensions**,  $\mathcal{O}(1)$ .

Gribov Equation is based on two simple but powerful observations, one “algebraic”, another — dynamical

Take the first order self-energy diagram  $\Sigma_1(q)$ : a fermion (quark/electron) with momentum  $q$  virtually decays into a quark (electron) with momentum  $q'$  and a massless vector boson (gluon/photon) with momentum  $k = q - q'$ :

$$\Sigma_1(q) = [C_F] \frac{\alpha}{\pi} \int \frac{d^4 q'}{4\pi^2 i} [\gamma_\mu G_0(q') \gamma_\mu] D_0(q - q'), \quad D_0(k) = \frac{1}{k^2 + i\epsilon}$$

$G$  and  $D$  the fermion and boson propagators, respectively.

The corresponding Feynman integral diverges linearly at  $q' \rightarrow \infty$ .

To kill the ultraviolet divergences (both linear and logarithmic), it suffices to *differentiate* it twice over the external momentum  $q$ .

The first Gribov’s observation was that  $1/k^2$  of the boson propagator happens to be *the Green function of the four-dimensional Laplace operator*,

$$\partial_\mu^2 \frac{1}{(q - q')^2 + i\epsilon} = -4\pi^2 i \delta(q - q')$$

$$\partial_\mu \equiv \frac{\partial}{\partial q_\mu}$$



- Graphically,
- differentiate twice over the external momentum,
- use the  $\delta$ -function property

$$\partial_\mu^2 \Sigma_1(q) = \text{diagram with dashed loop and } q, q' \text{ momenta} = \text{diagram with two vertical lines and } k=0 \text{ labels}$$

In *higher orders* the fermion Green function and the vertices get dressed:  $G_0(q) \implies G(q), \quad \gamma_\nu \implies \Gamma(q, q, 0)$

Invoking the Ward identity, we arrive at the non-linear algebraic equation

$$(g \equiv [C_F] \frac{\alpha}{\pi})$$

$$\partial_\mu^2 G^{-1}(q) = g \cdot (\partial_\nu G^{-1}(q)) G(q) (\partial_\nu G^{-1}(q)) + \dots \quad \text{with } \dots \text{ standing for less singular } \mathcal{O}\left(\left(\frac{\alpha}{\pi}\right)^2\right) \text{ terms}$$

The whole PT series expansion may be constructed for the right hand side in terms of exact Green functions (and their momentum derivatives).

In particular, with account of the first subleading terms one gets an integro-differential equation

$$\partial_\mu^2 G^{-1}(q) = \text{diagram 1} - \text{diagram 2} + 2 \cdot \text{diagram 3} + \dots$$

For  $g \rightarrow 0$  ( $|q^2| \rightarrow \infty$  in QCD) the solution of the *free equation*  $\partial^2 G^{-1} = 0$  is

$$G^{-1}(q) = Z_0^{-1} \left[ (m_0 - \hat{q}) + \frac{\nu_1^3}{q^2} + \frac{\nu_2^4 \hat{q}}{q^4} \right]$$

The new dimensional parameters can be directly linked with the famous non-perturbative vacuum condensates

$$\nu_1^3 \propto \langle \bar{\psi}\psi \rangle, \quad \nu_2^4 \propto \langle \alpha_s F_{\mu\nu}^a F_{\mu\nu}^a \rangle$$

(ITEP sum rules)

Unlike the standard renormalization group (RG) approach, the new equation is the second order (matrix) differential equation.

Therefore, two new integration constants ( $\nu_1, \nu_2$ ), in addition to the familiar bare mass  $m_0$  and the wave function renormalization constant  $Z_0$ .

$$G^{-1}(q) = Z_0^{-1} \left[ (m_0 - \hat{q}) + \frac{\nu_1^3}{q^2} + \frac{\nu_2^4 \hat{q}}{q^4} \right]$$

New terms are *singular* at  $q^2 \rightarrow 0$ . In QED we simply drop them, returning to the RG structure.

In so doing, however, we exploit the knowledge that nothing dramatic happens in the *infrared domain*, so that the real electron in the physical spectrum of the theory, whose propagation we seek to describe, is inherently that very same object that we put into the Lagrangian as the fundamental bare field.

Not so clear in an infrared unstable theory.

In QCD therefore we better keep all four terms, *wait and see*.

In the deep (Euclidean) region,  $|q^2| \gg m^2$ , the new term constitutes but a small power-suppressed correction, e.g.  $m(q) \simeq m_0 + \frac{\nu_1^3}{q^2} \rightarrow m_0$

For finite momenta,  $|q^2| \sim m^2$ , all terms are to be treated on the same footing.

Then, if in the infrared region the coupling exceeded the *critical value*, a *bifurcation* in the Gribov equation occurs, giving the *non-perturbative solution*.

It corresponds to the phase with *spontaneously broken chiral symmetry*.

This means that (given a supercritical coupling in the infrared) the quark Green function may possess a non-trivial mass operator even in the chiral limit of vanishingly small bare (ultraviolet) quark mass  $m_0 \rightarrow 0$ .

To see what happens to the *mass operator*, and how the *critical coupling* emerges, it suffices to carry out the following simple algebraic exercise

Spontaneous  
Chiral  
Symmetry  
Breaking

Substitute the standard representation for the fermion propagator in terms of the wave function renormalization factor  $Z(\xi)$  and the running mass  $m(\xi)$ ,

$$G^{-1}(q) = Z^{-1}(\xi) [m(\xi) - \hat{q}], \quad (\xi = \ln q \equiv \ln \sqrt{q^2})$$

and introduce *anomalous dimensions*

$$\Gamma = \dot{Z}/Z, \quad \Gamma_m = \dot{m}/m \quad (\dot{f} \equiv df/d\xi)$$

Then, in the region of relatively large momenta  $|q^2| \gg m^2$ , one derives a system of coupled differential equations for anomalous dimensions:

$$\dot{\Gamma} \simeq (1-g)(1-\Gamma)^2 + 2(1-\Gamma) - 3(1-g), \quad (1)$$

$$\dot{\Gamma}_m \simeq -(\Gamma_m^2 + 2) + 2(1-g)(1-\Gamma)(1-\Gamma_m). \quad (2)$$

(1) is self-contained and produces the *wave function renormalization*  $\Gamma$ .

(2) determines *mass renormalization* as driven by that of the *wave function*.

**NB:**

Correct one-loop perturbative Wave Function and Mass anomalous dimensions (in the Feynman gauge) derived *algebraically* without calculating integrals!

Its stable point  $\dot{\Gamma} = \dot{\Gamma}_m = 0$  determines the ultraviolet anomalous dimensions.

**Solution:**

$$\Gamma^*(g) = \frac{2 - \sqrt{3(1-g)^2 + 1} - g}{1-g}$$

$$= \frac{1}{2}g + \mathcal{O}(g^2)$$

$$\Gamma_m^*(g) = -(1-g)(1-\Gamma) + \sqrt{[(1-g)(1-\Gamma) + 1]^2 - 3} = 1 - \sqrt{3(1-g)^2 + 1} \pm \sqrt{3(1-g)^2 - 2}$$

$$= -\frac{3}{2}g + \mathcal{O}(g^2)$$

**Here comes the criticality.**

For  $g = [C_F] \frac{\alpha}{\pi} > 1 - \sqrt{\frac{2}{3}} \equiv g_{\text{crit}}$

the running mass becomes *complex*, signaling *instability*.

At the point  $g = g_{\text{crit}}$  the *two solutions* collide,  $\Gamma_{m\pm}^*(g = g_{\text{crit}}) = -(\sqrt{3} - 1)$ .

The mass operator  $m(\xi)$  (remaining real) starts to oscillate with  $\xi = \ln q$  producing a chiral symmetry breaking solution whose mass operator is regular at  $q = 0$  and decays fast in the ultraviolet,  $m(\xi) \propto \exp(-2\xi) \propto 1/q^2$ , corresponding to  $m_0 = 0$ .

$$\partial_\mu^2 G^{-1}(q) = g \cdot (\partial_\nu G^{-1}(q)) G(q) (\partial_\nu G^{-1}(q))$$

As far as *confinement* is concerned, the approximation described above turned out to be *insufficient*.

A numerical study of the Gribov Equation showed that the corresponding quark Green function does not possess an analytic structure that would correspond to a confined object.

(C. Ewerz, 2000)

The dynamical chiral symmetry breaking brings in *Goldstone pions*. They, in turn, affect the propagation of quarks.

Pion feed-back:

$$\partial_\mu^2 G^{-1}(q) = g \cdot \partial_\nu G^{-1}(q) G(q) \partial_\nu G^{-1}(q) - \frac{3}{16\pi^2 f_\pi^2} \{i\gamma_5, G^{-1}(q)\} G(q) \{i\gamma_5, G^{-1}(q)\}$$

In his last paper Gribov argued that these effects are *likely* to lead to confinement of light quarks and, thus, to confinement of any colour states.

(V. Gribov, EPJC 1999)

Dynamical nature of the pion-axial current transition constant  $f_\pi$ :

$$f_\pi^2 = \frac{1}{8} \int \frac{d^4 q}{(2\pi)^4 i} \text{Tr} \left[ \{i\gamma_5, G^{-1}\} G \{i\gamma_5, G^{-1}\} G (\partial_\nu G^{-1} G)^2 \right] + \frac{1}{64\pi^2 f_\pi^2} \int \frac{d^4 q}{(2\pi)^4 i} \text{Tr} \left[ (\{i\gamma_5, G^{-1}\} G)^4 \right]$$

Gluon Sector

In the analysis of the quark Green function, the behaviour of  $\alpha_s$  was implied.

**An open problem:** To construct and to analyse an equation for the gluon similar to that for the quark Green function.

From this analysis a consistent picture of the coupling  $g(q)$  rising above  $g_{crit}$  in the infrared momentum region should emerge.

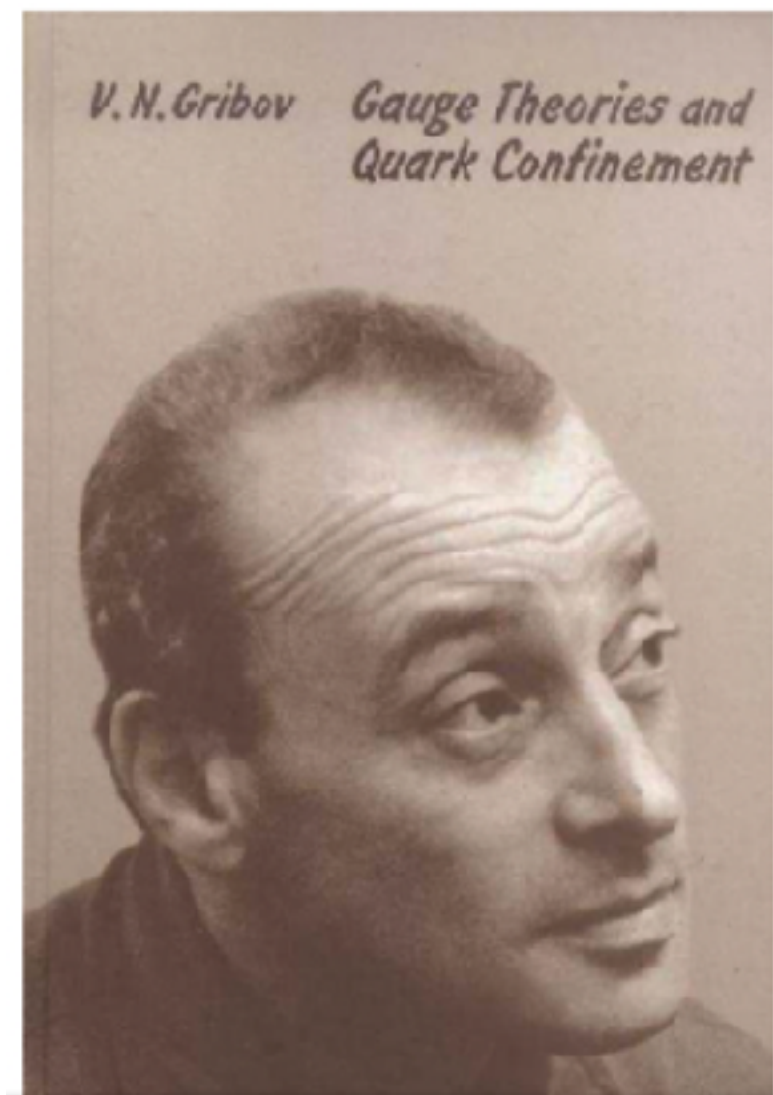
**Difficulty:** To learn to separate the *running coupling* effects from an unphysical *gauge dependent phase* that are both present in the gluon Green function.

V.G. gave the solution of the problem in the *Abelian* theory (QED) (*unpublished*)

$$\beta(g) \equiv \left( \frac{d}{d\xi} + \frac{1}{4} \frac{d^2}{d\xi^2} \right) \frac{1}{g} \simeq -\frac{n_f}{6} ([1 - \Gamma(g)]^2 - 3)^2 \simeq -\frac{2n_f}{g^2} \quad (g \gg 1)$$

- In the perturbative region we get the standard one-loop  $\beta$ -function
- In the *large coupling* regime, instead, the *charge* and *magnetic moment* contributions to the  $\beta$ -function *compensate* each other, and the *Landau pole disappears*:

$$g(\xi) \simeq 2n_f \xi = n_f \ln |k^2| \rightarrow \infty$$



An amazing success of the relativistic theory of electron and photon fields — quantum electrodynamics (QED) — has produced a long-lasting negative impact: it taught generations of physicists that came into the business in/after the 70's “not to worry”.

*Indeed, nowadays one takes a lot of things for granted :*

- One rarely questions whether the alternative roads to constructing QFT — *secondary quantization*, *functional integral* and the *Feynman diagram* approach — really lead to the *same quantum theory* of interacting *fields*
- One feels ashamed to doubt an elegant, powerful, *but potentially deceiving* technology of translating the dynamics of quantum fields into that of statistical systems
- One takes the original concept of the “*Dirac sea*” — the picture of the fermion content of the vacuum — as an anachronistic model
- One was taught to look upon the problems that arise with field-theoretical description of point-like objects and their interactions at very small distances (*ultraviolet divergences*) as purely technical : *renormalize and forget it.*

**QED** : physical objects — *electrons and photons* — are in *one-to-one correspondence* with the fundamental fields that one puts into the local QED Lagrangian.

*The role of the QED Vacuum is “trivial”: it makes *e.m. charge* (and the electron mass operator) *run*, but does not affect the *nature* of the interacting fields.*

**QCD** : the Vacuum changes the bare fields *beyond recognition* ...

*Gribov's message :*

*Functional integral* is not well defined.  
Use the *Feynman diagram* language.

*Euclidean rotation* is hardly applicable.

*Dirac sea* is alive and likely to play an active part in understanding the nature of hadrons.

*UV* and *IR* domains are interweaved.

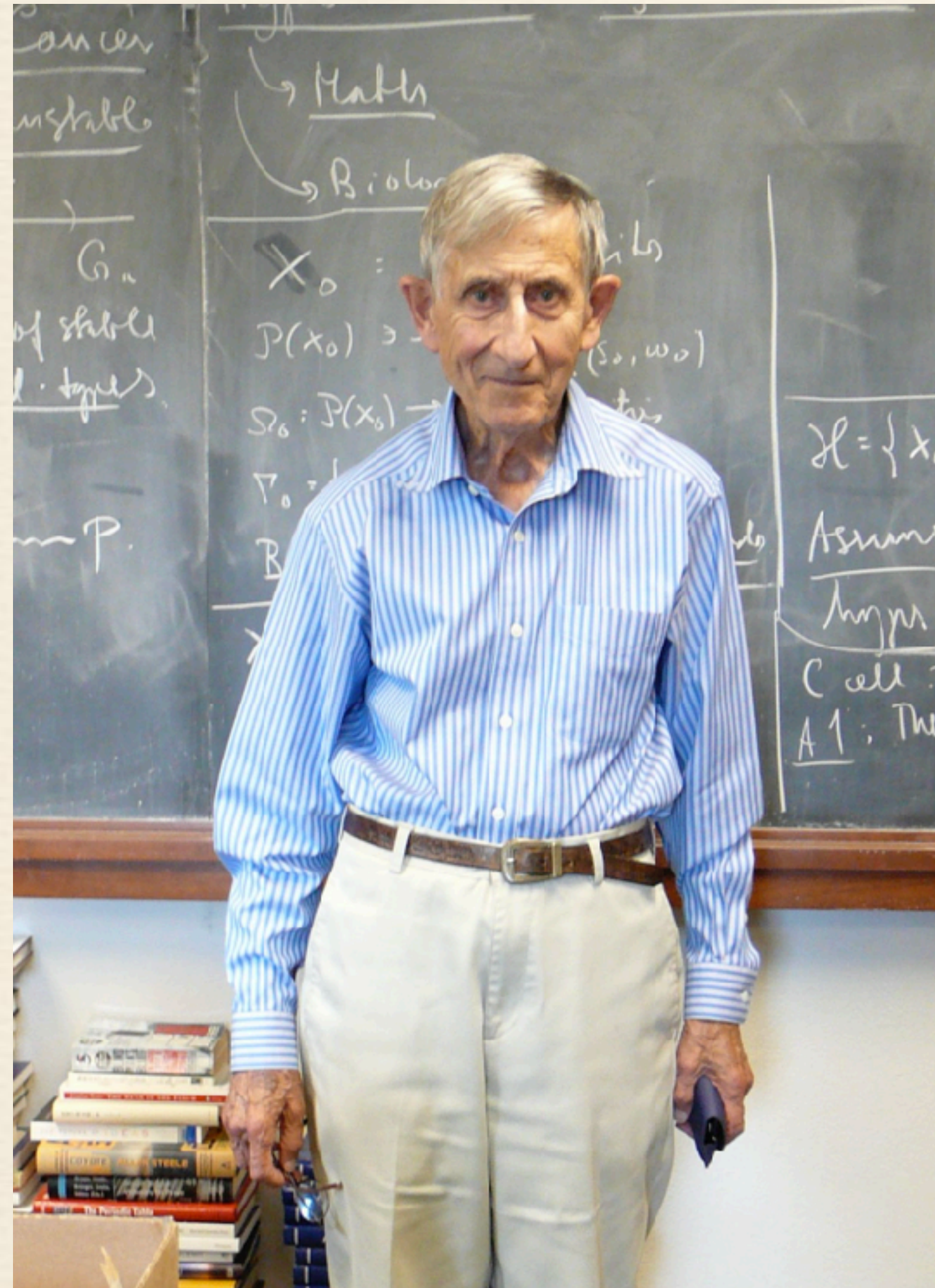
### CHALLENGES

to understand *analytic structure* of quarks and gluon propagators

to understand how *unitarity* manifests itself in Green functions and interaction amplitudes of "non-flying objects"

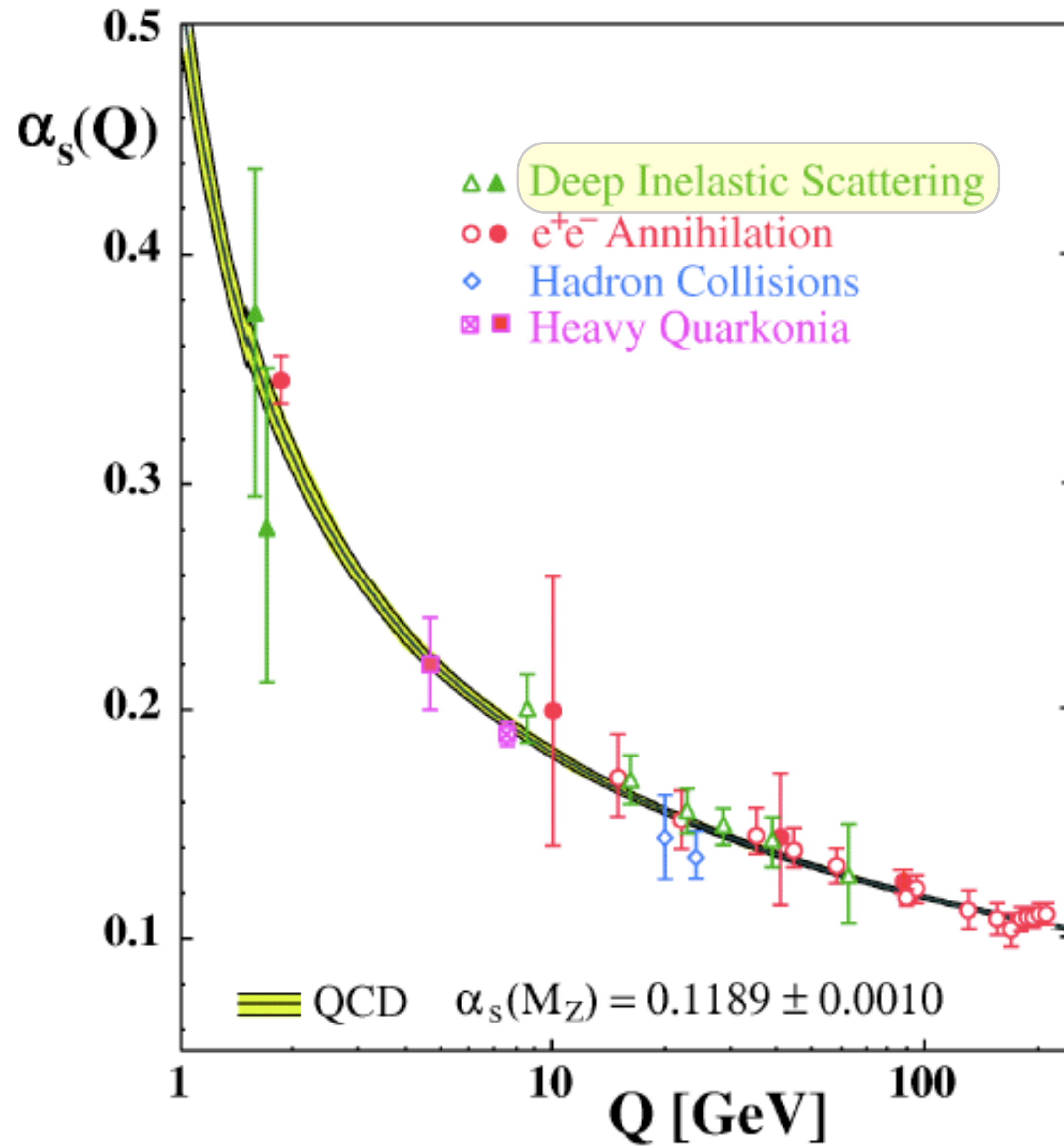
The rest will follow. *Eventually*

**1958** Dyson : *“the correct meson theory will not be found in the next hundred years”*



We have **35** years left to prove Freeman Dyson wrong ...  
or maybe not

## Summary of the QCD coupling measurements (2008)



Speaking of “***perturbative QCD***” can have two meanings :

- {1} *In a strict sense of the word, perturbative (PT) approach implies representing an answer for a (calculable) quantity in terms of series in a (small) expansion parameter.*
- {2} *In a broad sense, PT means applying ***the language of quarks and gluons*** to a problem, be it of perturbative (short-distance, small-coupling) or non-perturbative nature.*

**The quark–gluon picture works well across the board.**

Moreover, in many cases it seems to work ***too well***.

(Classical example - the story of **baryon magnetic moments** where the naive quark model counting works better than sophisticated dynamical approaches)

This is another worry: “***too good to be true***” *ain't good enough*.

Looking at multi-particle production in hard (small-distance driven) processes, one often wonders :

***... “ where is ” confinement ?..***



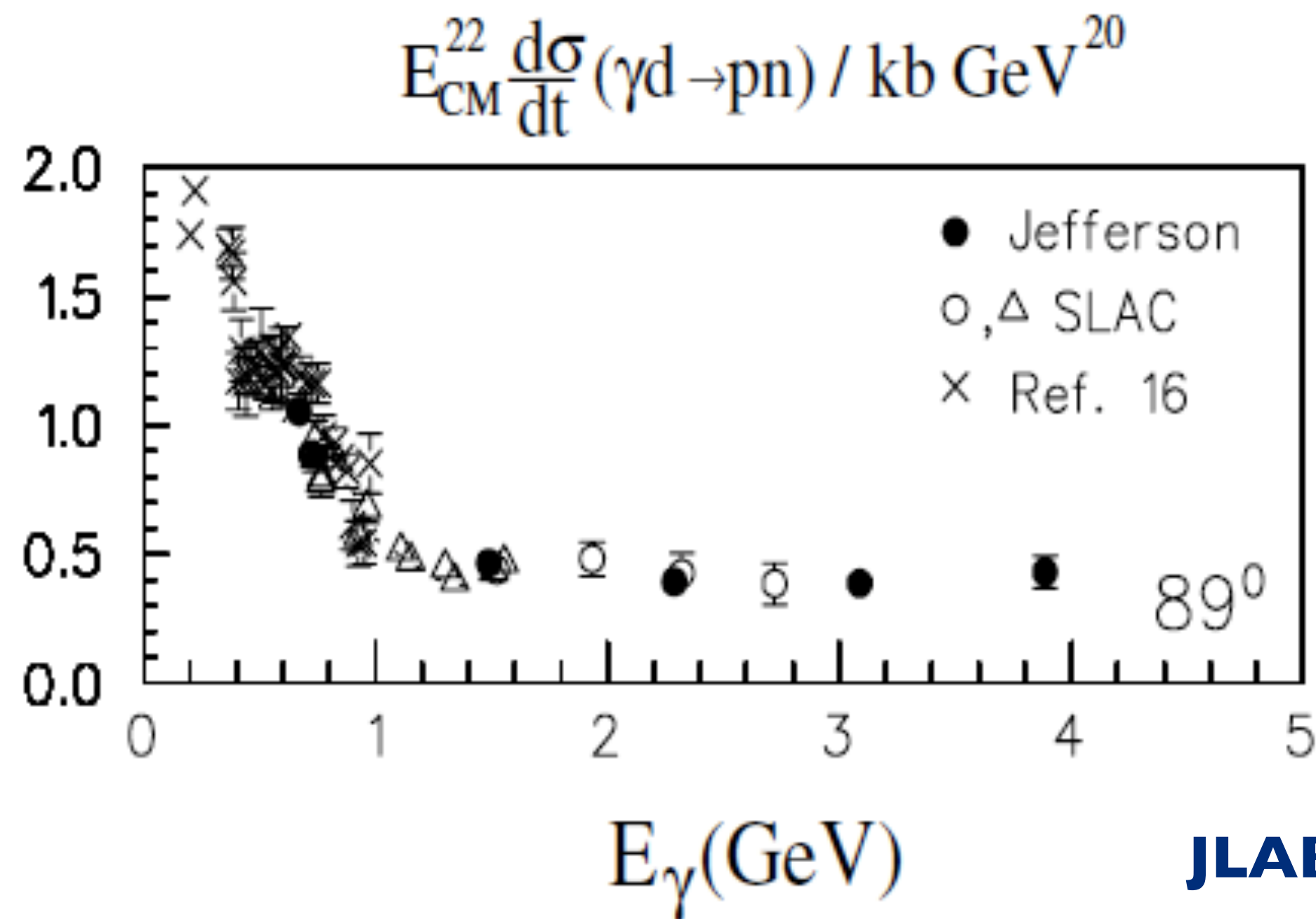
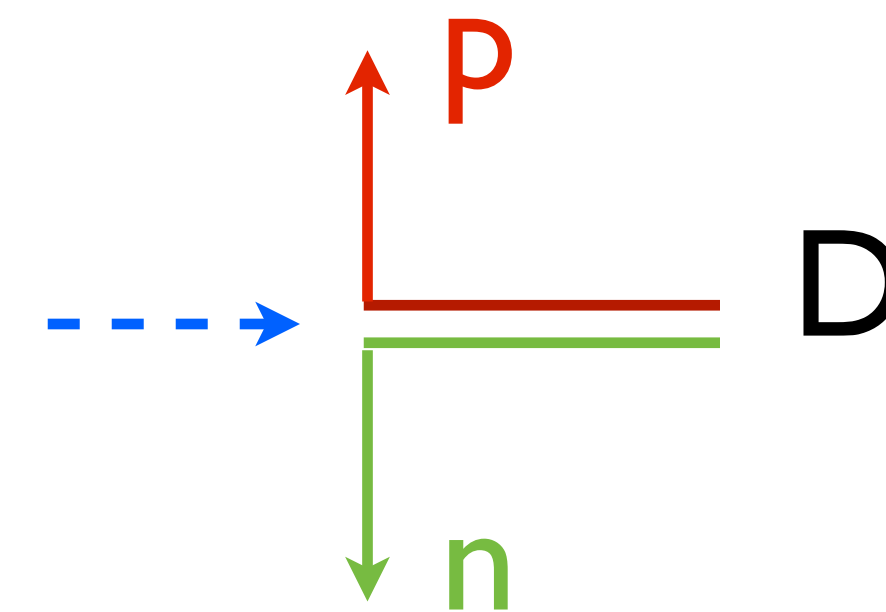
# Dimensional counting (“quark counting rules”)

large angle scattering in the high energy / momentum transfer regime

$$\frac{d\sigma}{dt} = \frac{f(\Theta)}{s^{K-2}}; \quad \frac{t}{s} = \text{const}$$

*K* the number of participating elementary fields (quarks, leptons, intermediate bosons, etc)

Example : deuteron break-up by a photon,  $\gamma + D \rightarrow p + n$

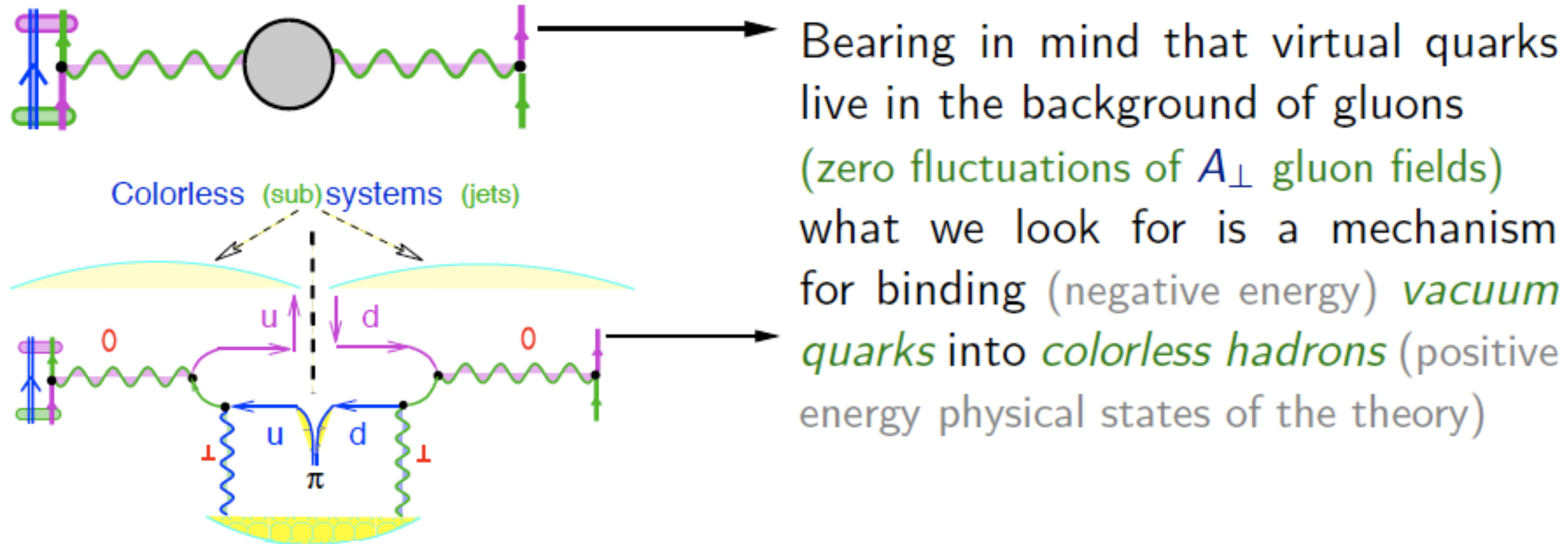


$K = 1 + 6 + 6 = 13$

it is very difficult to digest how the naive asymptotic regime settles that early !..

$d\sigma \sim \alpha_s^{10} (q^2/N)$

What happens with the Coulomb field when the sources move apart?



V.Gribov suggested such a mechanism — the supercritical binding of light fermions subject to a Coulomb-like interaction. It develops when the coupling constant hits a definite “critical value” (Gribov 1990)

$$\frac{\alpha}{\pi} > \frac{\alpha_{\text{crit}}}{\pi} = C_F^{-1} \left[ 1 - \sqrt{\frac{2}{3}} \right] \simeq 0.137$$

$$\left( C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3} \right)$$