

# Specific features of Heavy Quarks

## Dead Cone and around

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## Plan :

1. Open flavour mesons. Parton model: "leading particle effect"
2. Remark: Heavy Quark and QCD coupling
3. Radiation off massive quark
4. Heavy Quark initiated jets:  
Energy Loss, light hadron multiplicity and Dead Cone
5. Gluon radiation and hadrons. LPHD
6. Gluon formation and hadronisation  
Non-perturbative effects and Dead Cone (good news)
7. Beauty. ALICE vs DELPHI

Heavy Quark's operational definition :

$$M_Q \gg \Lambda_{\text{QCD}}$$

Finite mass is the only source of HQ specificity (*but the **top** one*)

**Consequences :** Hadron structure

A heavy quark and its light partner antiquark, inside an **open flavour** meson have the **same velocity**. Therefore, they share meson energy as

$$E_Q/E_q \simeq M_Q R \quad \text{with} \quad R \sim 1/\mu, \quad \mu = \mathcal{O}(\Lambda_{\text{QCD}})$$

- effective light quark "mass"

**An old parton model prediction.**

**Hidden flavour** mesons - quasi-Coulomb structure

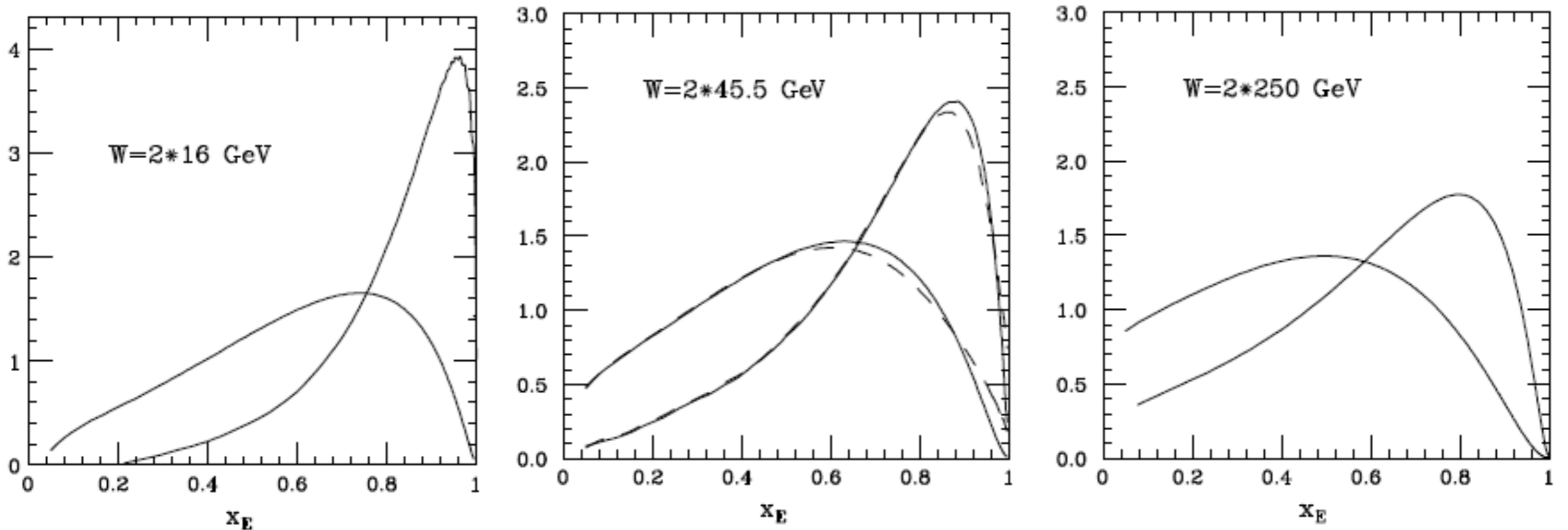
**C-family - marginally; B-family - way better.**



# Heavy quark fragmentation

The *leading particle effect* is clearly seen experimentally in the HQ fragmentation functions.

**pQCD calculable**  $Q \rightarrow Q(x_E)$  spectra (D & Khoze & Troyan, PRD 1995)



Evolution of inclusive c, b from  $W = 32 \text{ GeV}$  to  $W = 500 \text{ GeV}$  c.m.s. energy

**Mark :** In spite of the fact that the “**soft gluon corner**”  $1 - x_E \sim \Lambda/M_Q$  is, formally speaking, **a non-PT domain**, the spectrum stays under PT control

From the very start of the now almost 50-year-old HQ history, they were considered to be (p)QCD friendly beasts:

**large mass** = **large momentum scale** = **short distances** = **small coupling** = ...

And, indeed, this is true. **SOMETIMES**

QCD (**QED**) interaction strength - running coupling - **does not depend** on the nature of the (colour) charge bearer

*In particular, on the quark mass !*

Ward identity ; Conservation of (colour) current

What actually matters for quantifying the scale of the coupling argument is **characteristic gluon virtuality** which may - or may not - be driven by  $M_Q$ .

Depends on the **observable** one is looking at

$$dw_V = \frac{C_F \alpha_s}{\pi} \frac{dz \beta}{z v} \int_{-1}^1 d \cos \Theta_c \left\{ 2(1-z) \frac{\beta^2 \sin^2 \Theta_c}{(1-\beta^2 \cos^2 \Theta_c)^2} + z^2 \left[ \frac{1}{1-\beta^2 \cos^2 \Theta_c} - \frac{1}{2} \right] \zeta_V^{-1} \right\}$$

"Classical part" of gluon radiation

independent of the production process and of the nature of the emitters !

$z$  gluon energy fraction

$\beta$  quark velocity

$\Theta_c$  gluon emission angle

} in the cms of  $Q\bar{Q}$

$$\tilde{\gamma}_{q \rightarrow q(x)+g} = \frac{C_F \alpha_s}{\pi} \left[ \frac{x}{1-x} + (1-x) \cdot \frac{1}{2} \right]$$

$$\tilde{\gamma}_{g \rightarrow g(x)+g} = \frac{C_A \alpha_s}{\pi} \left[ \frac{x}{1-x} + (1-x) \cdot (x + x^{-1}) \right]$$

Multiple hadron production in hard processes is derived from the QCD parton cascade processes that are dominated by gluon bremsstrahlung.

An essential difference in the structure of **heavy-** and light-quark jets ( $q = u, d, s$ ) results from dynamical restriction on the phase space for primary gluon radiation : radiation off an energetic quark Q with mass M and energy  $E_Q \gg M$  is **suppressed** inside the forward cone with an opening half-angle  $M/E_Q$ , - the “**dead cone**” .

This is in full analogy with QED where photon radiation is suppressed at **small angles** with respect to the direction of the radiating massive charged particle.

Suppression of the **energetic** gluon emission with low transverse momenta  $k_t$  results, in turn, in the decrease of the heavy quark **energy losses**.

This provides pQCD explanation of the **leading particle effect** and reduces **medium induced radiative** energy losses (LMP effect).

Dead cone depleting light hadron multiplicity.

Multiplicity of accompanying gluon radiation (NLL) :

$$N_{q\bar{q}}(W) - N_{Q\bar{Q}}(W) = N_{q\bar{q}}(\sqrt{e}M) \cdot [1 + \mathcal{O}(\alpha_s(M))]$$

$$N_{q\bar{q}}^h(W) - N_{Q\bar{Q}}^h(W) = \text{const}(W).$$

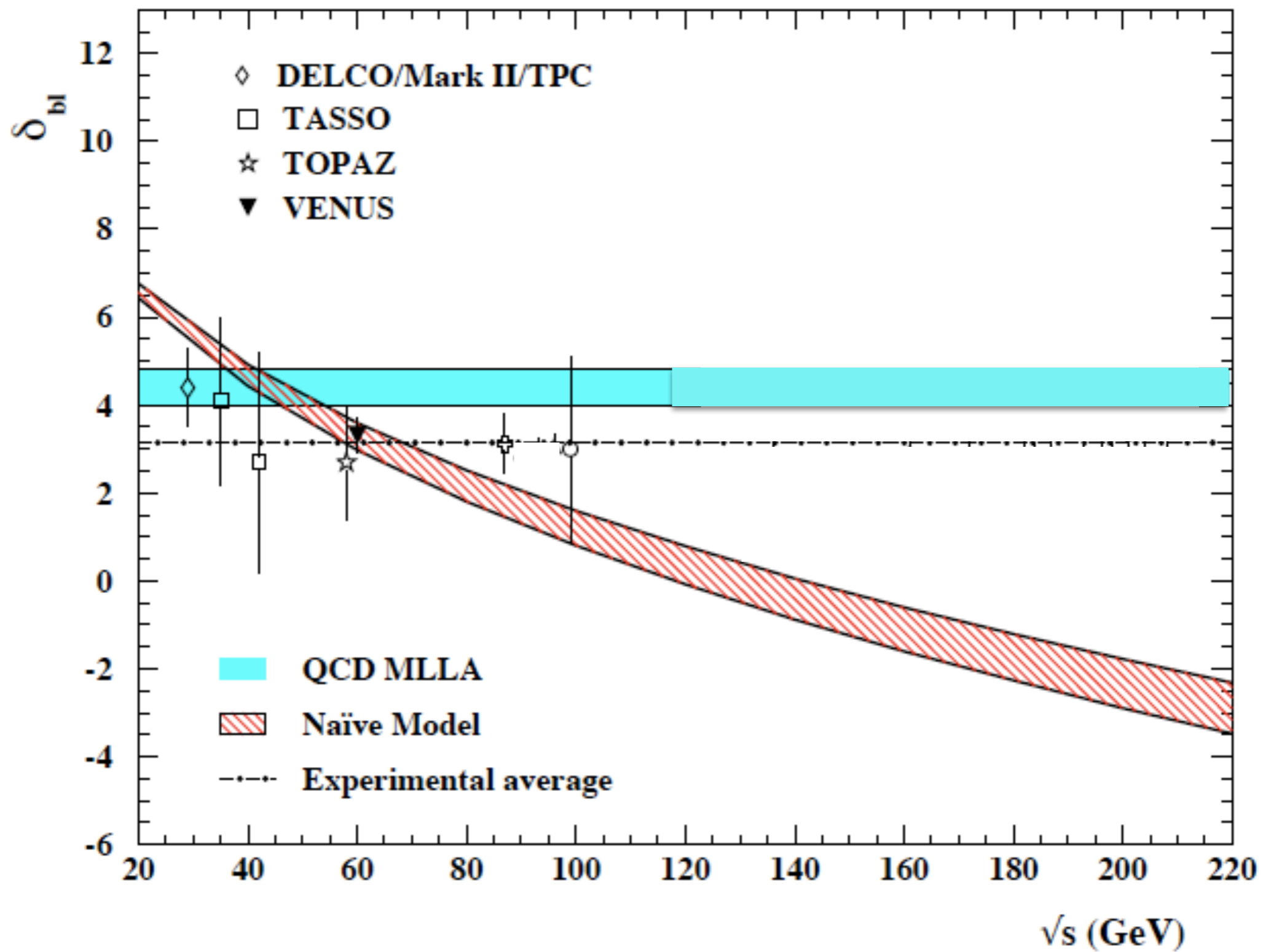
This QCD prediction is in marked contrast with the expectation of the so-called *naive model*, which related the multiplicities in light and heavy quark events based on the idea of the *reduction of the energy scale* :

$$N_{Q\bar{Q}}^h(W) = N_{q\bar{q}}^h((1 - \langle x_Q \rangle)W) \quad \langle x_Q \rangle = \frac{2 \langle E_Q \rangle}{W}, \quad 1 - \langle x_Q \rangle = \mathcal{O}(\alpha_s(W))$$

$$N_{q\bar{q}}^h(W) - N_{Q\bar{Q}}^h(W) \propto \sqrt{\alpha_s} \ln \frac{1}{\alpha_s} \cdot N_{q\bar{q}}^h(W)$$

wrong

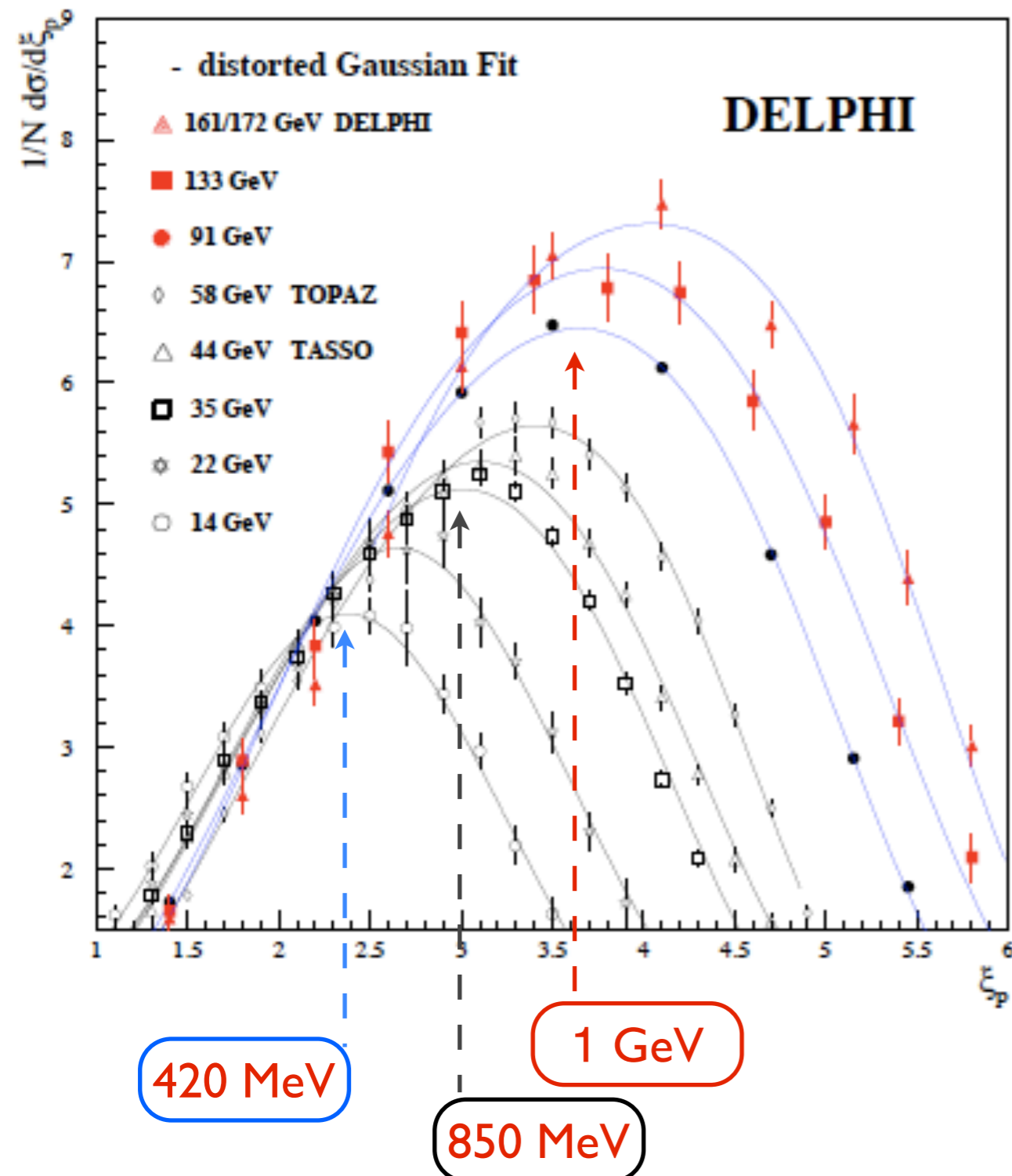




Energy dependence of the multiplicity difference btw bottom and light-q jets

pQCD predicts distribution of radiated gluons. How about hadrons?

From (pre)LEP, HERA, Tevatron, LHC experiments we learn that **energy spectra** of hadrons in jets are mathematically similar to that of gluons.



Observation of parton-hadron similarity was initially met with serious scepticism:

**disturbingly small** hadron momenta!

By looking at hadron content of a jet with **restricted opening angle** one studies smaller hardness scales but in a **cleaner environment** :

due to **Lorentz boost**, hadrons forming characteristic coherent "QCD hump" become **relativistic**

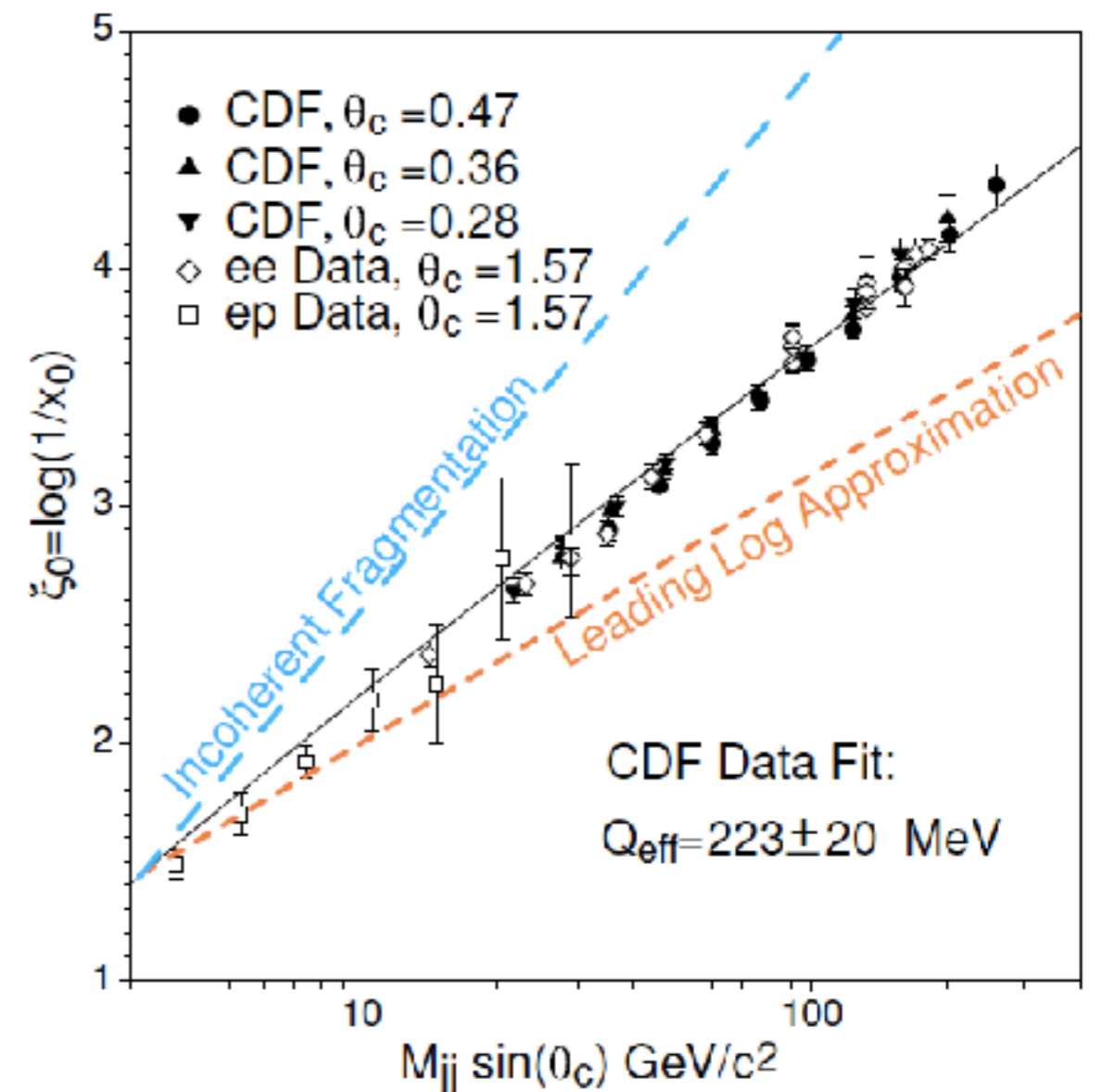
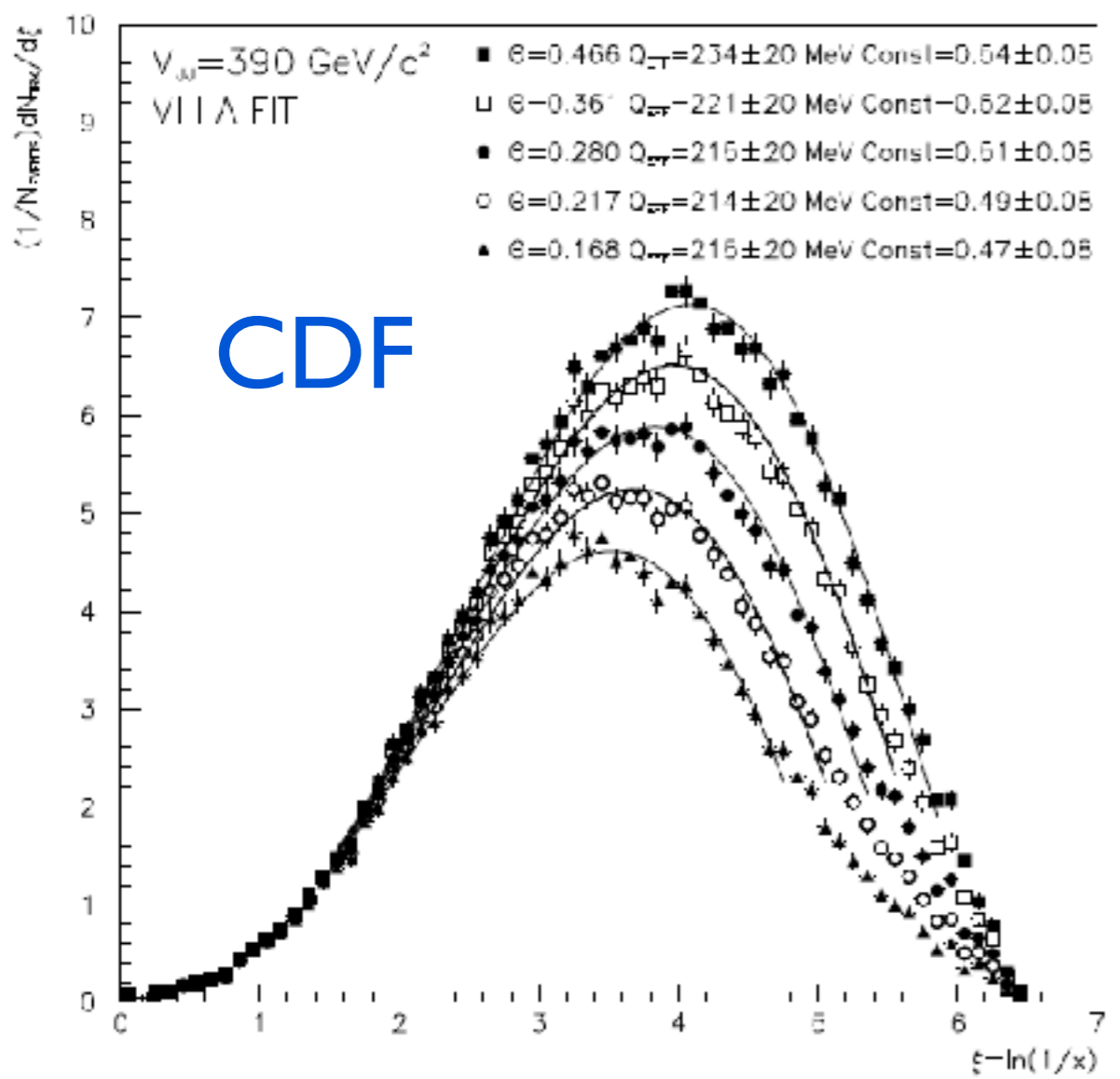
**Local Parton-Hadron Duality (LPHD)** hypothesis verified

Selecting hadrons inside a cone **0.14** around a quark jet with  $E_{jet}=100$  GeV one would see that very *dubious*  $Q=14$  GeV curve but now with the maximum *boosted* from **0.45 GeV** to **6 GeV** !

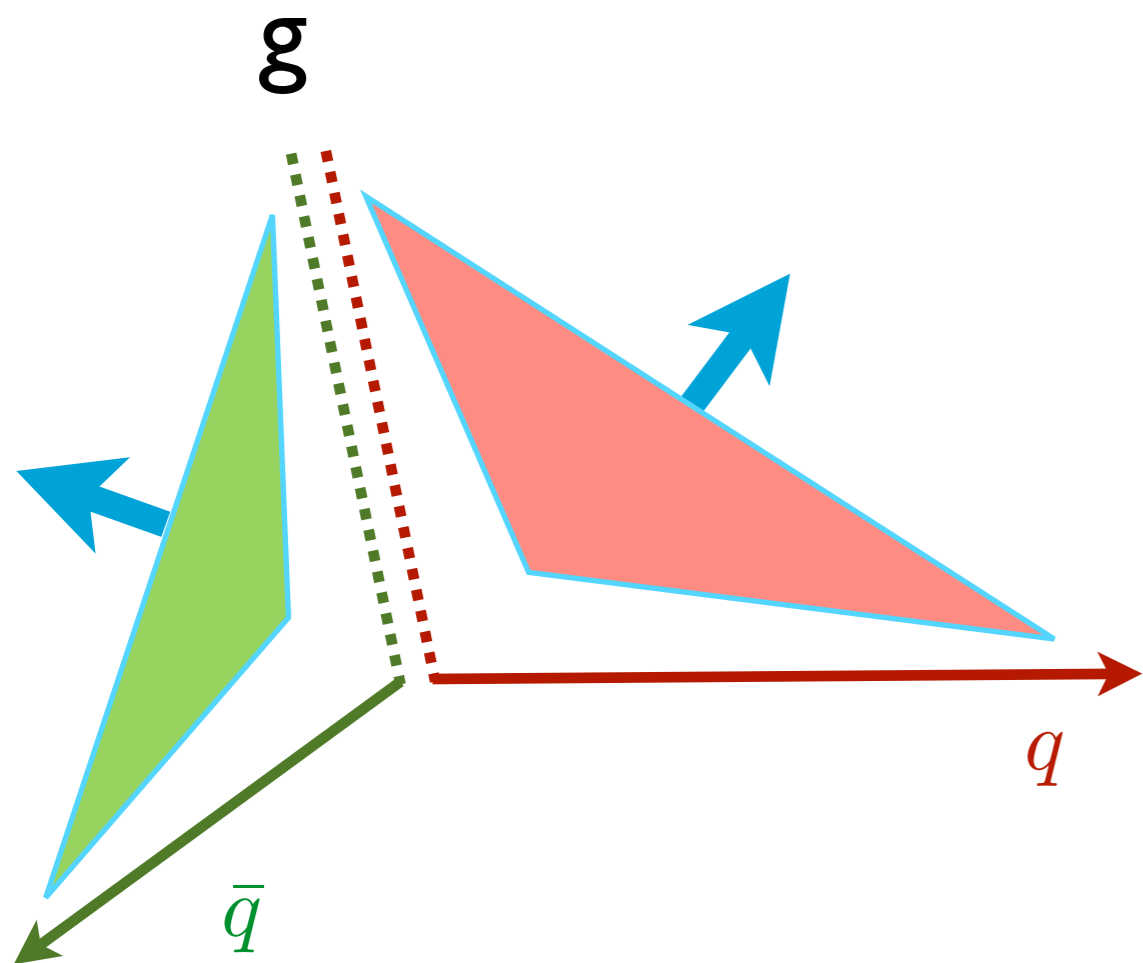
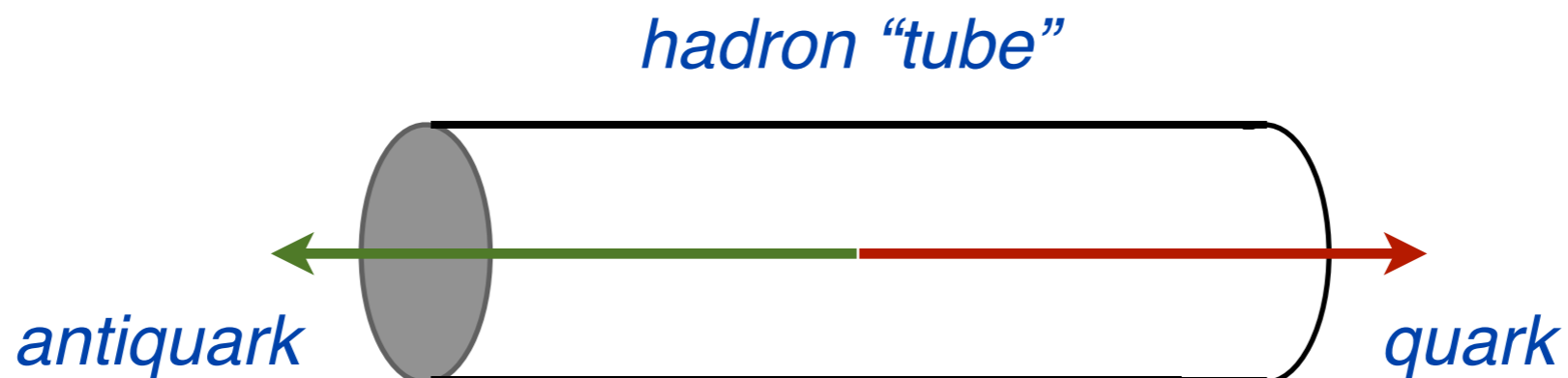
**jets with restricted “opening angle”**

Position of the Hump as a function of the *hardness* of the jet  $Q = M_{jj} \sin \Theta_c$  is a **parameter-free** pQCD prediction

The plot combines  $e^+e^-$ , DIS and  $hh$  data !



## Lund model



## Lund "String effect"

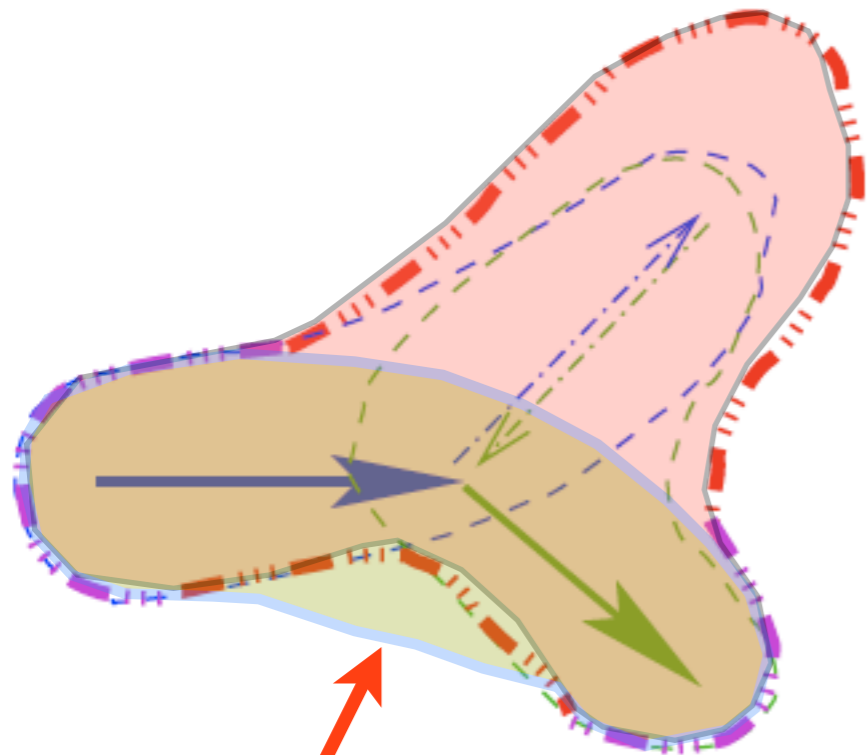
*kinematical effect of a boosted hotdog*

Lund's stress on **topology** of the dominant **colour flow** found support from pQCD

$$8 = 3 * 3$$

The story of inter-jet hadron flows that manifest subtle dynamics of **coherent soft gluon radiation** off multi-parton antennae are even more spectacular than that of inclusive energy spectra.

**String Effect** is just one (but the most famous) example of "**QCD Radiophysics**"



depletion of particle production in the  $q\bar{q}$  valley

Comparison of hadron flows between 3-jet

$$q\bar{q} + g$$

and  $q\bar{q} + \gamma$  in the same kinematics

QCD prediction :

$$\frac{dN_{q\bar{q}}^{(q\bar{q}\gamma)}}{dN_{q\bar{q}}^{(q\bar{q}g)}} \simeq \frac{2(N_c^2 - 1)}{N_c^2 - 2} = \frac{16}{7}$$

(experiment:  $2.3 \pm 0.2$ )

Importantly, information about angular distribution of glue is imprinted upon miserable **100-300 MeV momentum** pions!

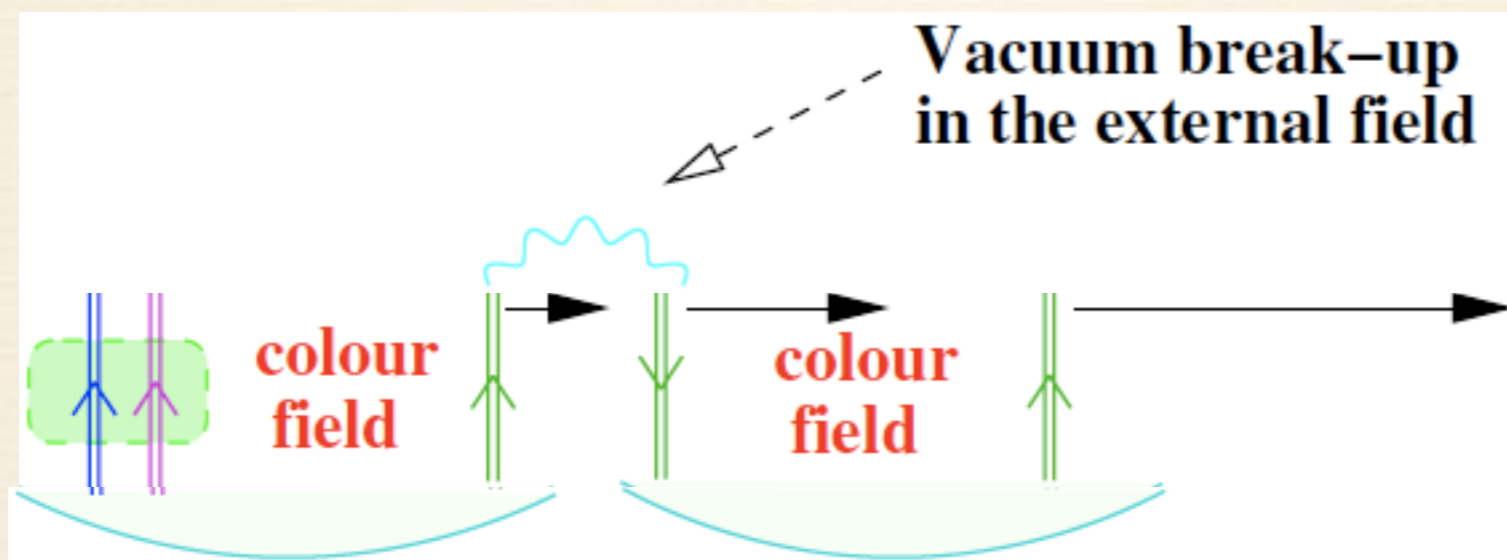
**Message:** confinement – *transformation of QCD partons into hadrons* – has **non-violent** nature: no visible energy–momentum reshuffling at the hadronisation stage!

# Kogut–Susskind picture

- In a DIS a *green* quark in the proton is hit by a virtual photon
- The quark leaves the stage and the *colour field* starts to build up



- A *green–anti-green* quark pair pops up from the vacuum, splitting the system into two *globally blanched* sub-systems.



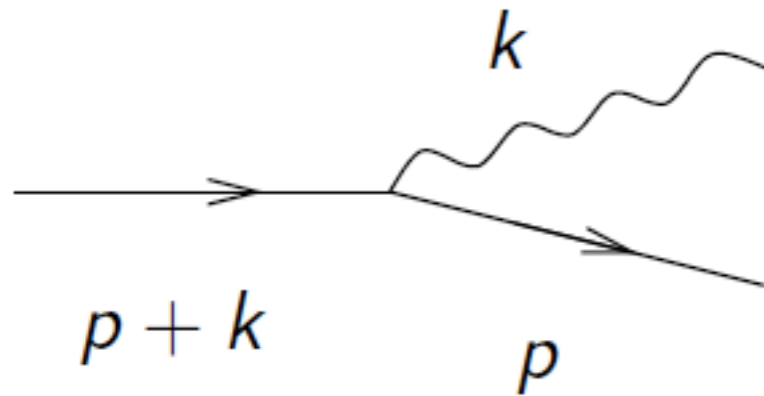
“One” hadron per  $\frac{\Delta\omega}{\omega}$ ; Hadron multiplicity  $\propto \ln Q$ .

# Gluon's life and death

**What is the condition** for a **gluon** to behave as an **independent coloured object** and thus as an **additional source of new particles?**

It takes certain time to emit a gluon.

The formation time can be simply estimated as a **lifetime** of the virtual **(p + k)** quark state



Making use of the Heisenberg uncertainty principle, with account of the Lorentz contraction,

$$t_g^{\text{form}} \sim \frac{1}{M_{\text{virt}}} \cdot \frac{E}{M_{\text{virt}}} = \frac{E}{(p+k)^2} \approx \frac{E}{kE\Theta^2} \approx \frac{k}{k_{\perp}^2}$$

Comparing with the hadronization time,  $t_g^{\text{hadr}} \approx kR^2$ ,

$$t_g^{\text{form}} \sim \frac{k}{k_{\perp}^2} < t_g^{\text{hadr}} \sim kR^2$$

the gluon's being is guaranteed *iff* its transverse momentum is large:

$$k_{\perp} > R^{-1} = \text{a few hundred MeV.}$$

The “**get-born-before-dying**” condition agrees with the **coupling** behaviour :  $\alpha_s(k_{\perp}^2)$

- $R^{-1} \ll k_{\perp} \ll k \sim \sqrt{Q^2}$

- the domain of **quasi-collinear** hard parton splittings leading to the scaling violation effects in DIS structure functions and jet fragmentation (inclusive particle distributions)

- $R^{-1} \ll k_{\perp} \sim k \ll \sqrt{Q^2}$

- **large angle soft** gluon emission responsible for drag effects in interjet multiplicity flows

- $R^{-1} \ll k_{\perp} \ll k \ll \sqrt{Q^2}$

- **double-logarithmic** (soft & collinear) gluon bremsstrahlung off quarks and gluons causing jet multiplicity grow with energy and determining QCD parton Form Factors

**All these are legitimate, PT-controllable, QCD sub-processes.**

**Parton pairs with small relative transverse momenta lie beyond PT control.**

Look at gluon radiation at the **lower edge** of the PT phase space :  $k_{\perp} \sim R^{-1}$ .

An appearance of a “**gluer**” is a signal of switching on of the **real strong interaction** :

$$\left( t^{\text{form}} \sim t^{\text{hadr}} \right)_{\text{gluer}}$$

Inclusive spectrum of **gluers** reproduces the **Feynman hadron plateau** !

$$dN = \left[ \int_{k_{\perp} \sim R^{-1}} \frac{dk_{\perp}^2}{k_{\perp}^2} 4C_F \frac{\alpha_s(k_{\perp}^2)}{4\pi} \right] \frac{dk}{k} = \text{const} \cdot \frac{dk}{k}$$



Hadronisation time: **in the rest frame**,  $t_{\text{hadr}} \sim R$

**With account of the Lorentz boost,**

$$t_{\text{hadr}} = ER^2 \longrightarrow t_Q = ER^2 \cdot \frac{1}{M_Q R} = \frac{R}{\Theta_Q}$$

**Light quark/gluon**                      **Heavy quark**

Switching on the NP strong interaction (= hadronisation) can be triggered by looking at formation time of a "**gluer**" - gluon on

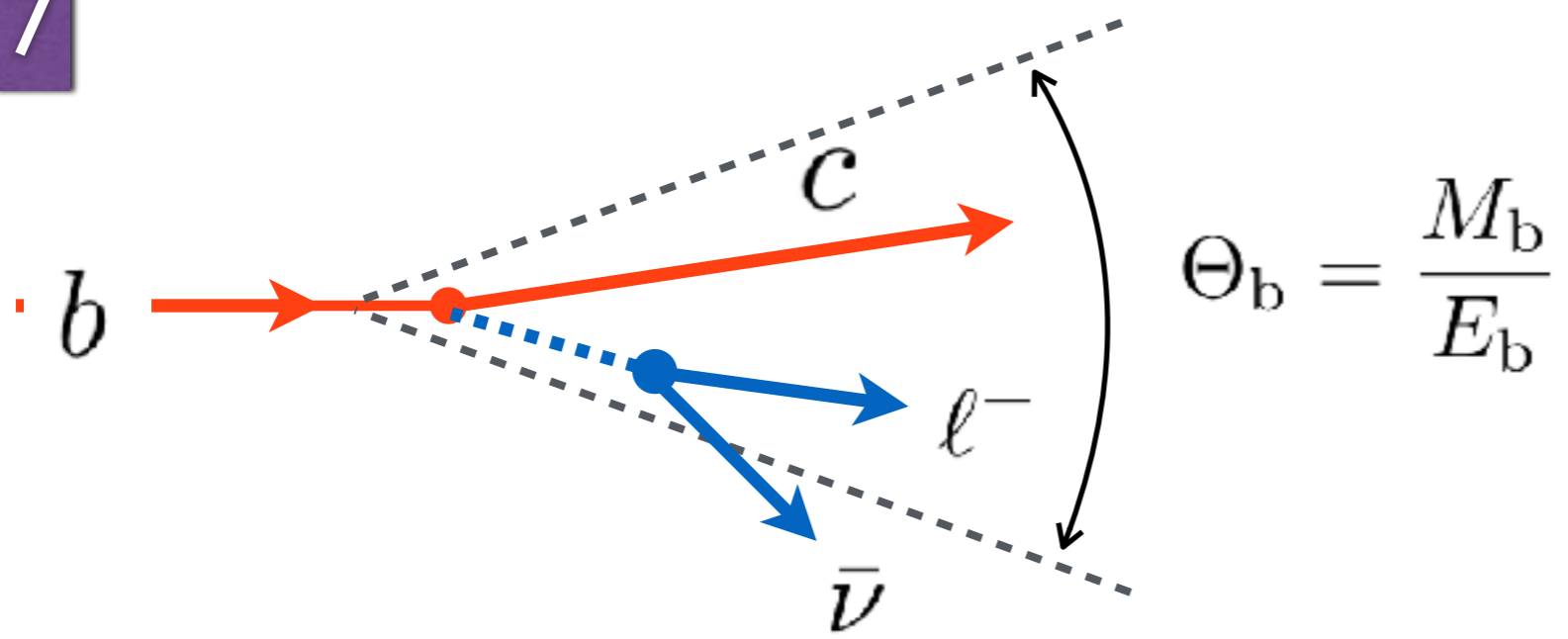
the edge of PT:  $k_{\perp(\text{gluer})} \sim R^{-1}$ ,  $\alpha_s(k_{\perp(\text{gluer})}^2) = \mathcal{O}(1)$ .

**Angles of gluers (= produced hadrons)**

$$\Theta_{\text{gluer}} = \frac{(k_{\perp})_{\text{gluer}}}{\omega} = \frac{1}{\omega R} = \frac{R}{\omega R^2} = \frac{R}{t_{\text{hadr}}} \gtrsim \frac{R}{t_Q} = \Theta_Q$$

The very last gluer responsible for  $c \longrightarrow D$ .  
All the rest have **larger** angles. **No NP filling-in!**

**Good news !**



IF **C**- and **B**- quark masses were **indeed** ("parametrically") large...

Look at light hadron multiplicity in a quark jet:

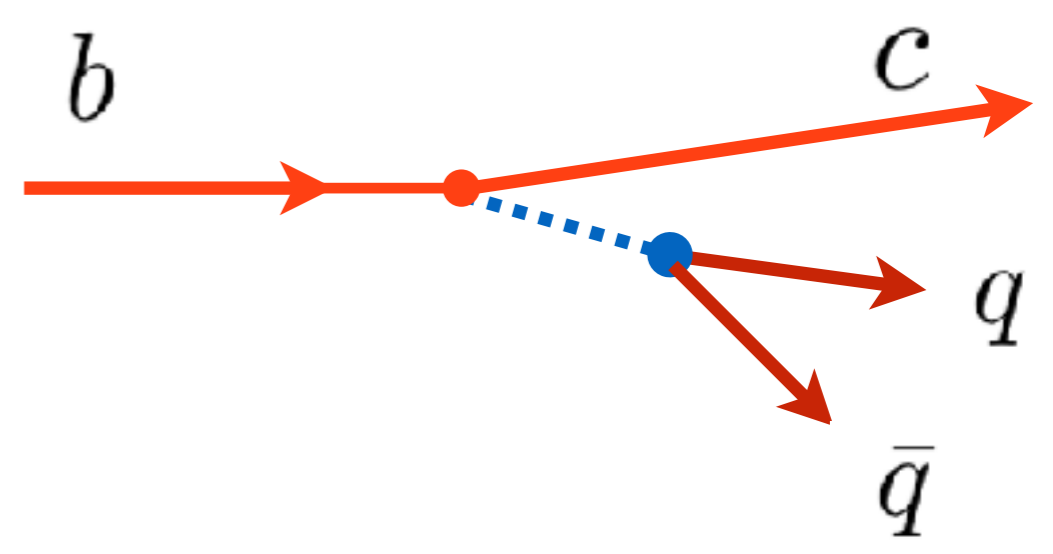
$$N_{\text{hadr}}(E) = N_q(E) - \underbrace{N_q(M_b)}_{\text{B}} + [N_q(M_b) - N_q(M_c)] + N_q(M_c)$$

.....  
C

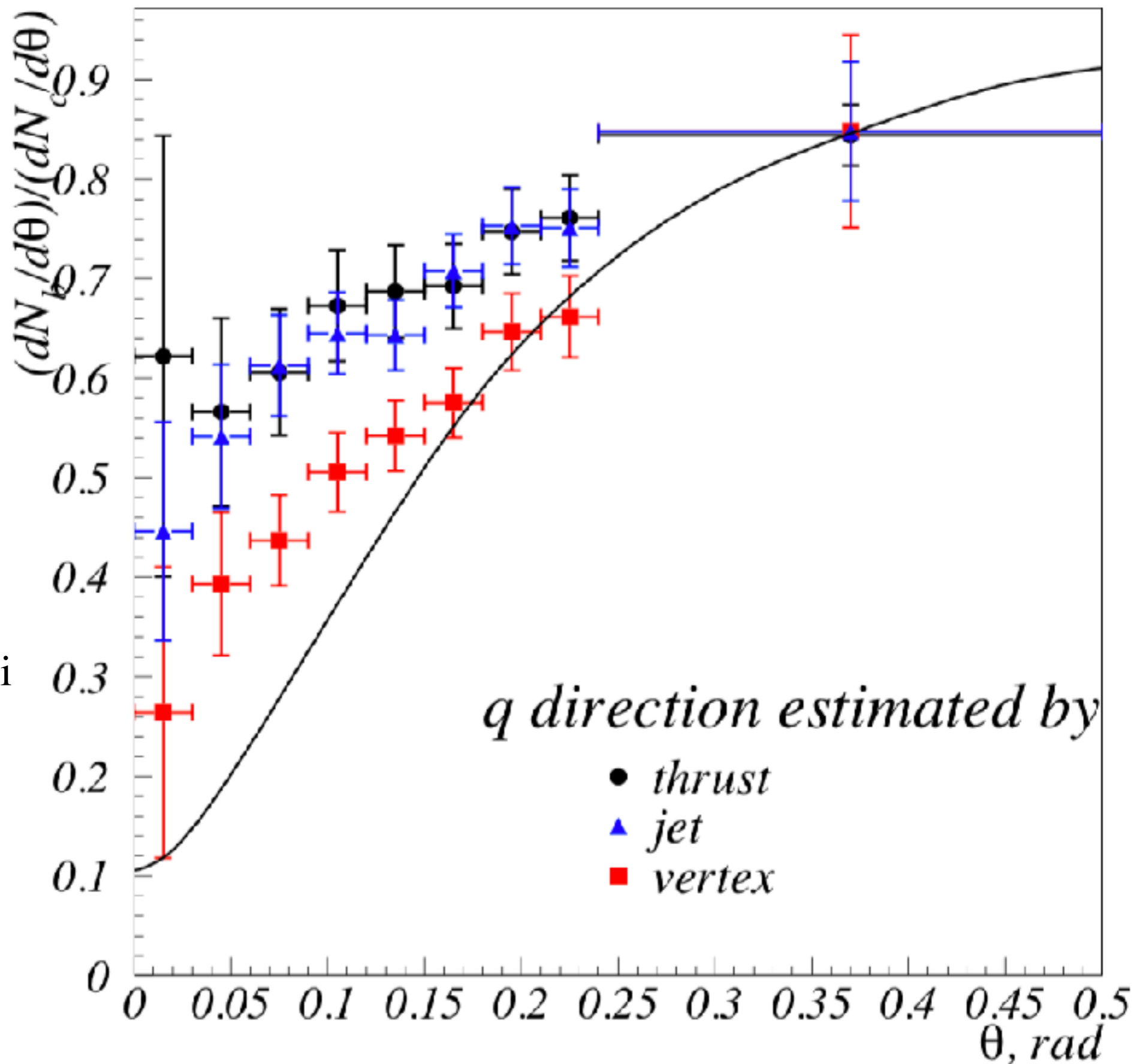
**Subtlety:**

hadron multiplicities in **semi-leptonic** and "**3-jet**" **B**-decays differ by an "extra **E+E-**"

Certain statistical contamination of the **C**-cone possible.



# DELPHI **B/C** ratio (2004). Which direction to use?..



analysis by

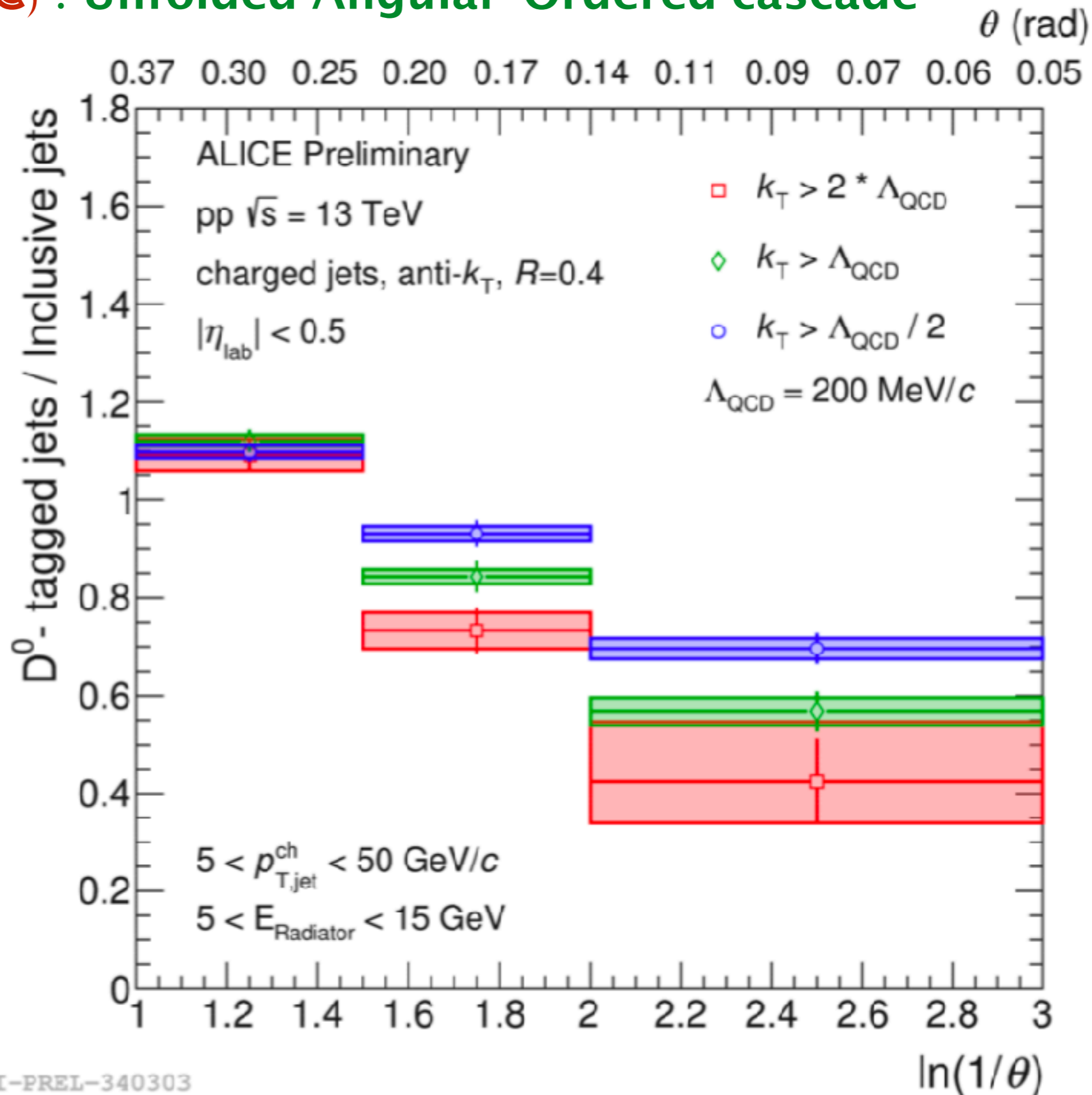
Battaglia, Orava, Salmi

**Helsinki**

initiated by

**V.A. Khoze**

# ALICE (C) . Unfolded Angular-Ordered cascade



*Studies of inter-jet particle flows ( QCD radiophysics )  
intra-jet parton multiplication ( inclusive spectra )*

*at LEP and elsewhere*

*have taught us about the key features of hadroproduction :*

The *colour field* that an ensemble of hard **partons** develops, determines, on the “one-to-one” basis, the structure of final flows of **hadrons**.

*When viewed globally, confinement is about “renaming”  
a flying-away quark into a flying-away pion  
rather than “pulling” opposite colours back together.*

*Applied to Heavy Quarks, “gentle confinement”  
manifests itself in the **Dead Cone** phenomenon*

**Bravo ALICE !**

**and B-bis !**