DGLAP and BFKL

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1. Logarithmic theory;

$\alpha << 1$ **but** $\alpha \ln(Q^2/Q_0^2) \sim 1$

2. Space-time picture;

(detector resolution; EFT)

3. Evolution $(t_i >> t_{i+1})$

A) DGLAP

i) evolution equation (splitting functions)

ii) conservation law(s)

iii) IR regularization (plus prescription)

iv) double Log's

v) anomalous dimension

vi) factorization scheme(s)

vii)unintegrated PDF(x, k_T)

B. BFKL

i) evolution equation

gluon reggeization (bootstrap)

ii) diffusion in $\ln k_T$

iii) $j = 1 + \omega_0$ **intercept**

iv) evolution 'trajectory'

v) Mueller-Navelet, Mueller-Tang

vi) correlations

vii)secondary Reggeons

If coupling α is dimensionless next loop integral reads

$$\alpha \int \frac{d^4k}{k^4} \quad ==> \quad \alpha \ln(...)$$

For $\alpha << 1$ we can sum up $\sum_n C_n(\alpha \ln(...))^n$

There is No charge without the field around it $\quad e \to e\gamma \to e(e^+e^-) \to ...$

time-life of fluctuation $\Delta t \sim 1/\Delta E$

DGLAP - $\Delta t \sim 1/k_{i,T}$ **(in Breit frame)**

BFKL - $\Delta t \sim E_i/k_{i,T}^2$ **(in target rest frame)**

DGLAP evolution

$$\frac{da(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi}\sum_b \left[\int P_{ab}(z)b(\frac{x}{z},Q^2)\frac{dz}{z} - a(x,Q^2)\int P_{ba}(z)dz\right]$$

$a = q(x), g(x)$

$$P_{qq} = C_F\tfrac{1+z^2}{1-z}$$

$$P_{gq} = C_F\tfrac{1+(1-z)^2}{z}$$

$$P_{qg} = T_R(z^2+(1-z)^2)$$

$$P_{gg} = 2N_c\left(z(1-z)+\tfrac{1-z}{z}+\tfrac{z}{1-z}\right)$$

symmetry

$z \to 1-z$

$$\sum_a \int xa(x,Q^2))dx = const \qquad \int q_f(x,Q^2)dx = const$$

'+" prescription

$$\frac{f(z)}{(1-z)_+} = \frac{f(z)-f(1)}{1-z}$$

Soft $(1-z) \to 0$ **gluon does not change**

white **PDF**

IR divergence is canceled

Double Log(s)

$P_{gg}(z << 1) = 2N_c/z \ ==> \ \alpha_s \int (dq^2/q^2)(dz/z)$

$xg(x, Q^2) \sim \exp\left(\sqrt{\frac{4N_c\alpha_s}{\pi}\ln(1/x)\ln(Q^2/Q_0^2)}\right)$

$$a(x, Q^2) \propto \left(Q^2/Q_0^2\right)^{\gamma}$$

At $x << 1$ **anom. dimension.**

$$\gamma = \sqrt{\alpha_s N_c/\pi)(\ln(1/x)/\ln(Q^2/Q_0^2)}$$

increases with x decreases

Factorization

Two blocks are separated by the LOG^c
cell $\int^{k_{i+1}} dk_i^2/k_i^2$ **with** $k_i << k_{i+1}$
Splitting and coefficient functions
are calculated with $k_i^2 = 0$.
(power k_i^2/k_{i+1}^2 **corrections are neglected)**

Strong Δt_i and/or $\theta_i = k_{i,t}/E_i$ ordering

(both in DGLAP and in BFKL)

Scheme dependence starting from NLO

$$a^{\overline{\mathrm{MS}}}(x) \; = \; a^{\mathrm{phys}}(x) - \frac{\alpha_s}{2\pi} \int \frac{dz}{z} \sum_b \delta P_{ab}(z) b^{phys}(x/z)$$

due to ϵ/ϵ **terms in** $\overline{\mathrm{MS}}$

Input PDF(x, Q_0)

$+$ **boundary cond.**^{n} **PDF**$(x = 1, Q) = 0$

$$a(x, Q^2) \; = \; \int dx_0 \sum_b b(x_0, Q_0^2) \cdot G_{ab}(x_0, Q_0; x, Q)$$

input **evolution**

(In BFKL==> boundary at $Q = Q_0$**,**

input at $x = x_0$**)**

Unintegrated PDF(x, k_T, μ)

$$a(x,\mu) = \int^{\mu} d^2 k_T f_a(x, k_T, \mu)$$

$$f_a = \frac{T(k_T,\mu)}{k_T^2}\frac{\alpha_s}{2\pi}\sum_b \int P_{ab}(z) b(\frac{x}{z}, Q^2 = \frac{k_T^2}{1-z}) dz$$

BFKL evolution (gluon only)

$$\frac{df(x,q_T)}{d\ln(1/x)} = \frac{N_c\alpha_s}{\pi}\int K(q_T,q'_T)f(x,q'_T)d^2q'_T$$

$$K(q,q')f(q') = \frac{1}{(q-q')^2}\left[f(q') - \frac{q^2 f(q)}{q'^2+(q-q')^2}\right]$$

gluon reggeizat^n

No IR divergence at $q \to q'$

Gauge

DGLAP - axial $G_{\mu\nu} = G^{\perp}_{\mu\nu}$

$$G_{\mu\nu} = \left[[g_{\mu\nu} - \frac{q_\mu n_\nu + n_\mu q_\nu}{(q.n)} \right] \qquad q^2 = n^2 = 0$$

BFKL - different n for different cells

need to sum up the diagrams

DGLAP $==> f(x.q) \propto 1/q^2 \times (q^2/Q_0^2)^\gamma$

with anom. dim. $\gamma \sim \alpha_s << 1$

BFKL at $\alpha_s \ln(1/x) >> 1 \;\; ==> \gamma \to 1/2$

$f(x,q) \propto x^{-\omega_0}/\sqrt{q^2 Q_0^2} \qquad \omega_0^{LO} = \frac{N_c \alpha_s}{\pi} 4 \ln 2$

(beam - target symmetry)

Diffusion:

i) in b_t $\Delta b_{ti} \sim 1/q_{Ti}$

ii) in $\ln q_{Ti}$ $\Delta \ln q_{Ti} \sim O(1)$

$$< \Delta y > = < \ln x_i - \ln x_{i+1} > \sim 1/\omega_0 \sim 3$$

Dipole representation

$$\frac{d}{dY}N(x,y;Y) = \frac{\alpha_s N_c}{2\pi^2}\int d^2z \frac{(x-y)^2}{(x-z)^2(y-z)^2}$$
$$\times\,[N(x,z;Y) + N(y,z;Y) - N(x,y;Y)$$
$$-N(x,z:Y)N(y,z,:Y)]$$

For a small density N the last term in square brackets can be neglected and equation reproduces the conventional BFKL equation in coordinate representation. However for a large $N \to 1$ the r.h.s. vanishes and we reach the saturation $N = 1$.

two high E_T jets ($\Delta y >$ few$/\omega_)$

a) inclusive — Mueller-Navelet

b) gap between 2 jets - Mueller-Tang

Jet-Jet correlat^s die out for $\Delta y \gg \omega_0$

$(\omega_{n\geq 1} \leq 0 \quad \omega_{colour} \leq 0)$

Secondary Reggeons

($q\bar{q}$ state in t-channel)

No explicit q_{Ti} ordering

$$\int_{Q_0^4/s}^{s} \frac{dq_T^2}{q_T^2} \sim \ln^2(s/Q_0^2)$$

DLogs $==> \sum_n C_n(\alpha_s \ln^2 s)^n$

$==> \sigma \sim s^{\omega_R - 1} \quad \omega_R = \sqrt{2C_F\alpha_s/\pi}$

($\omega_R \sim 0.5$ **for** $\alpha_s = 0.3$)