Classical point-like electron with finite electromagnetic and total masses

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- Gravitation and Particle Physics
- Reissner-Nordström Solution
- Energy-Momentum Tensors of Electromagnetic and Gravitational Fields
- Total Inert Mass of Classical Electron
- "Electromagnetic Mass" of Classical Electron
- Charge Conjugation
- Discussion and Conclusions

- Ratio of electromagnetic to gravitational forces for two electrons

  $\frac{\mathcal{F}_{el}}{\mathcal{F}_{gr}} = \frac{e^2}{R^2} \Big/ \frac{km^2}{R^2} = 4.2 \cdot 10^{42},$

  where $e = 4.8 \cdot 10^{-10}$ CGSE, $m = 9.11 \cdot 10^{-28}$ g, $k = 6.67 \cdot 10^{-8}$ cm^3 g^{-1} c^{-2}.

- The gravitation field does not play any role in the elementary particle structure!

  Nevertheless this statement could be wrong since the equation for the gravitational field is nonlinear.

- Infinite electromagnetic mass of the electron

$$m_{em} = \frac{\mathcal{E}_{em}}{c^2} = \frac{1}{c^2} \int \frac{\vec{E}^2}{8\pi} dV = \int \frac{e^2 dR}{2c^2 R^2} \to \infty,$$

  $dV = 4\pi R^2 dR$. $m_{em}c^2 = e^2/(2r_0)$, $r_0 = 1.4 \cdot 10^{-13}$ cm is the classical electron radius.

- Gravitational interaction in Newtonian physics

$$dU_{gr} = -\frac{k dm_1 dm_2}{R_{12}} = -\frac{k}{R_{12}} \frac{e^2 dV_1}{c^2 R_1^4} \frac{e^2 dV_2}{c^2 R_2^4} < 0.$$

$$U_{gr}/\mathcal{E}_{em} = -\frac{1}{12} \frac{r_e^2}{R_c^2},$$ where $r_e^2 = ke^2/c^4$, $r_e = 1.4 \cdot 10^{-34}$ cm.

  Gravitation can play important role at $r \le r_e$.

• Riemannian geometry

Minkowsky space-time

$$ds^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2$$
$$=\sum_{i,j} g_{ik} dx^i dx^j \equiv g_{ij} dx^i dx^j,$$

where $\mathbf{dx^0 = cdt,\ dx^1 = dx,\ dx^2 = dy,\ dx^3 = dz,\ g_{ij} =}$ **diag**$\mathbf{(1,\ -1,\ -1,\ -1)}$.

- **If the component of the metric tensor** $g_{ij} = g_{ji}$ **are functions of coordinates** x^l**, the geometry is Riemannian.**
- **The space is Euclidian (pseudo-Euclidian) if the Riemannian tensor** $R^j_{inl} \equiv 0$.
- **Metric tensor components play a role of the gravitational potentials and can be found from the Einstein equations:** $R_{ij} - \frac{1}{2} R g_{ij} = \frac{8\pi k}{c^4} T_{ij}$.

$\mathbf{T_{ij}}$ **is the energy-momentum tensor of matter,** $\mathbf{R_{ij} = R^n_{inj}}$ **is the Ricci tensor,** $\mathbf{R = R_{ij} g^{ji}}$.

- Electric and gravitational fields

$$F^{\mu 0} = F_{\mu 0} = -F_{0\mu} = \vec{E}^{\mu} = \frac{e}{r^2}\vec{n}^{\mu},$$

where $\vec{\mathbf{n}}^{\mu} = \mathbf{x}^{\mu}/\mathbf{r}$ **is the unit vector,** $\mu = \mathbf{1},\ \mathbf{2},\ \mathbf{3};\ \mathbf{r}^2 = (\mathbf{x}^1)^2 + (\mathbf{x}^2)^2 + (\mathbf{x}^3)^2.$

$$g_{00} = \Lambda, \quad g^{00} = 1/\Lambda,$$

$$\Lambda = 1 - \frac{2km}{c^2 r} + \frac{ke^2}{c^4 r^2} \equiv \frac{D(r)}{r^2} = \frac{r^2 - r_g r + (r_e)^2}{r^2},$$

where $\mathbf{r_g} = \frac{\mathbf{2km}}{\mathbf{c^2}} = \mathbf{1.35 \cdot 10^{-55}}\mathrm{cm}$ **is Schwarzschild radius of electron,**

$\mathbf{r_e^2} = \frac{\mathbf{ke^2}}{\mathbf{c^4}},\ \ \mathbf{r_e} = \mathbf{1.38 \cdot 10^{-34}}\mathrm{cm}.$ $\quad r_e \gg r_g \Rightarrow D(r) > 0.$

$$g^{\alpha\beta} = -[\delta_{\alpha\beta} + (\Lambda - 1)\vec{n}^{\alpha}\vec{n}^{\beta}],$$

where $\delta_{\alpha\beta}$ =**diag**$(1, 1, 1)$ **is the Kronecker symbol.** $\mathrm{det}(g^{ij}) = -1.$

- **General Definition of Pseudo-Tensor $\mathrm{t^{ij}}$ for the case $\mathbf{det}(\mathrm{g_{ij}}) = -\mathbf{1}$**

$$P^i = \frac{1}{c}\int (T^{i0} + t^{i0})dV,$$

where $\mathrm{T^{ij}}$ is the energy-momentum tensor of the electromagnetic field, while $\mathrm{t^{ij}}$ denotes the energy-momentum pseudo-tensor of the gravitational field.

$$T^{ij} + t^{ij} = \frac{\partial h^{ijl}}{\partial x^l} \equiv \sum_{l=0}^{3} \frac{\partial h^{ijl}}{\partial x^l},$$

where $\mathrm{h^{ijl}} = -\mathrm{h^{ilj}}$ is antisymmetric for j and l and is given by (L.D. Landau and E.M. Lifshitz, 1947)

$$h^{ijl} = \frac{c^4}{16\pi k}\frac{\partial}{\partial x^n}[g^{ij}g^{ln} - g^{il}g^{jn}].$$

- **Main property of $(\mathrm{T^{ij}} + \mathrm{t^{ij}})$**

$$\frac{\partial[T^{ij}+t^{ij}]}{\partial x^j} \equiv \frac{\partial[T^{i0}+t^{i0}]}{\partial x^0} + \frac{\partial[T^{i1}+t^{i1}]}{\partial x^1} + \frac{\partial[T^{i2}+t^{i2}]}{\partial x^2} + \frac{\partial[T^{i3}+t^{i3}]}{\partial x^3} = 0.$$

Total Inert Mass of Classical Electron

- **Energy-Momentum Tensors for the Reissner-Nordström solution**

The metric tensor g^{ij} depends on r only. Greek indexes $\alpha,\ \beta,\ \ldots = 1,\ 2,\ 3$; $\det(g_{ij}) = -1$.

$$T^{00} + t^{00} = \frac{\partial h^{00\alpha}}{\partial x^\alpha} = \frac{c^4}{16\pi k}\frac{\partial^2}{\partial x^\alpha \partial x^\beta}[g^{00}g^{\alpha\beta}]$$

- **Total energy of the system of electromagnetic and gravitational fields**

$$\mathcal{E} = P^0 c = \int (T^{00} + t^{00}) dV =$$

$$\frac{1}{8\pi}\int_0^\infty \left\{ \frac{r^4}{D^2}\left[\frac{e^2}{r^4} - \frac{4km^2}{r^4} + \frac{4kme^2}{c^2 r^5} - \frac{ke^4}{c^4 r^6}\right]\right\} 4\pi r^2 dr,$$

$$\mathcal{E} = mc^2.$$

where $D = r^2 - r_g r + r_e^2 = r^2 - \frac{2km}{c^2} r + \frac{ke^2}{c^4}$ and $dV = 4\pi r^2 dr$.

THERE IS NO ANY POINT-LIKE PARTICLE WITH MASS m BUT mc^2 IS THE TOTAL ENERGY OF THE SYSTEM OF ELECTROMAGNETIC AND GRAVITATIONAL FIELDS.

- Large- and small-r behaviour of the integrand for the total energy $\mathcal{E}$

$$r \gg r_e, \quad D \approx r^2, \quad r^4/D^2 \approx 1,$$

$$T^{00} + t^{00} \approx \frac{e^2}{8\pi r^4}, \ \int_{r_e}^{\infty} \left\{ \frac{e^2}{8\pi r^4} \right\} 4\pi r^2 dr = \frac{e^2}{2r_e}.$$

$$r \leq r_e, \quad D \sim r_e^2, \quad r^4/D^2 \sim (r/r_e)^4,$$

$$(T^{00} + t^{00}) 4\pi r^2 \rightarrow -e^2/(2r_e^2).$$

Contribution of large distances ($\mathrm{r} > \mathrm{r_e}$) is positive and is about $\mathrm{e^2/r_e}$, while contribution of small distances ($\mathrm{r} < \mathrm{r_e}$) is negative and is also about $-\mathrm{e^2/r_e}$.
The total integral is equal to $\mathrm{mc^2} \ll \mathrm{e^2/r_e}$
since $\mathrm{mc^2} = 0.511$ MeV while $\mathrm{e^2/r_e} \sim 10^{21}$ MeV.

"Electromagnetic Mass" of Classical Electron

- "Electromagnetic mass" (energy) of electron

$$\mathcal{E}_{em} = m_{em}c^2 = \int T^{00} dV = \int T_0^0 g^{00} dV,$$

where $\mathbf{T_0^0 = \tilde{E}^2/(8\pi) = e^2/(8\pi r^4), \;\; dV = 4\pi r^2}$,

Since $\mathbf{g^{00} = r^2/D = r^2/(r^2 - r_g r + r_e^2)}$,

then $\mathbf{g^{00} \to r^2/r_e^2}$ when $\mathbf{r \to 0}$

and $\mathbf{g^{00} \to 1}$ when $\mathbf{r \to \infty}$.

$\mathbf{D = (r - r_g/2)^2 + r_{eg}^2}$ with $\mathbf{r_{eg}^2 = r_e^2 - r_g^2/4 > 0}$, since $\mathbf{r_e \gg r_g}$. Hence $\mathbf{D > 0}$ for any $\mathbf{r}$.

Therefore the integral is convergent.

$\mathbf{\mathcal{E}_{em} = 1.3 \cdot 10^{15}}$ erg $\mathbf{= 8.2 \cdot 10^{26}}$ eV; $\mathbf{m_{em} = 1.5 \cdot 10^{-6}}$ g.

$\mathbf{\mathcal{E}_{em}/mc^2 \approx 1.6 \cdot 10^{21}}$.

- "Electromagnetic mass" of positron is the same as for electron

- **What is the electron?**

$$\vec{E} = \frac{e}{r^2}\vec{n}, \quad g_{00} = \Lambda, \quad g^{00} = 1/\Lambda,$$

$$\Lambda = 1 - \frac{2km}{c^2 r} + \frac{ke^2}{c^4 r^2}, \quad g^{\mu\nu} = g^{\mu\nu}(\Lambda)$$

- **What is the positron?**

$$\vec{E} = -\frac{e}{r^2}\vec{n} \equiv \frac{|e|}{r^2}\vec{n},$$

$$g_{00} = \Lambda, \quad g^{00} = 1/\Lambda, \quad g^{\mu\nu} = g^{\mu\nu}(\Lambda)$$

- **Charge conjugation transformation**

$$e \longrightarrow -e, \; m \longrightarrow m, \; \vec{E} \longrightarrow -\vec{E}, \; g^{ij} \longrightarrow g^{ij}$$

Positron has the same mass as the electron. It is not the particle with the negative mass as in the Dirac equation.

- **Particles with $\mathrm{m} = 0$ could be "vacuum" particles**

Discussion and Conclusions

- Alteration of the space-time metric due to gravitation field can made the electron mass finite. Metric is changed at distances $r_e \sim 10^{-34}\mathrm{cm}$.
- The electromagnetic field contribution to the total electron mass is positive and dominates at $r \gg r_e$. The "electromagnetic mass" is about 10^{21} MeV/c^2.
- The gravitational field contribution to the total electron mass is negative and dominates at $r < r_e$. This contribution is also about 10^{21} MeV/c^2.
- The parameter m of the Reissner-Nordström solution is both the inert and gravitational mass.

  There are solutions with $m = m_e$ and even with masses $m = 0$ and $m < 0$.
- The solution with $m = 0$ can probably be used for the construction of the vacuum state.
- The total electron mass is a mass of the electromagnetic and gravitation fields.

  There is no need in a point-like massive particle (usual electron).
- Charge conjugation: $e \to -e$, $m \to m$, $\tilde{E} \to -\tilde{E}$, $g_{ij} \to g_{ij}$. Positron mass is positive.
- The action corresponding to the Reissner-Nordström solution is infinite:

  $$L = -\frac{1}{16\pi c} g^{ij} g^{nl} F_{in} F_{jl} - \frac{c^3}{16\pi k} R = \frac{1}{8\pi c}\frac{e^2}{r^4}.$$