Exclusive vector-meson production at HERMES

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High Energy Physics Division Seminar

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- Physics Motivation
- Phenomenological description of reaction $e + N \rightarrow e' + V + N$
- The HERMES Experiment
- Results on Spin-Density Matrix Elements. Kinematic Dependences of Observables
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- Extraction of Helicity Amplitude Ratios
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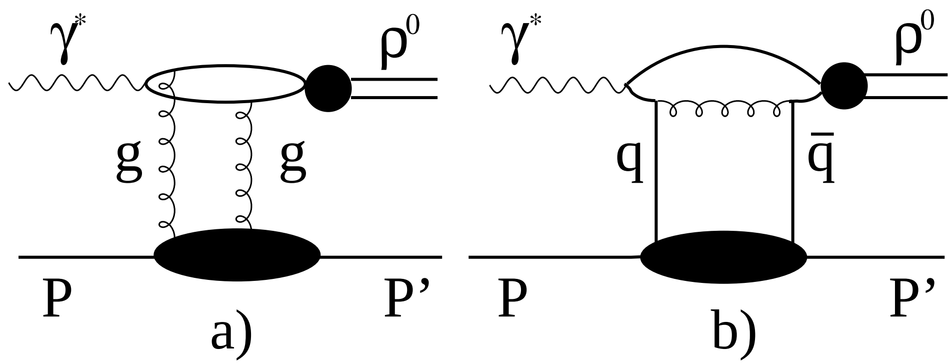
Physics Motivation



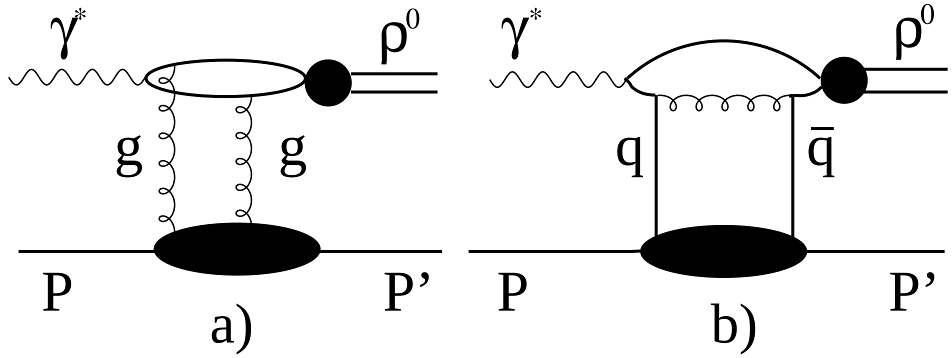
- $\gamma^* + N \rightarrow V + N$ is a perfect reaction to study both vector-meson ($V = \rho^0,\ \phi,\ \omega,\ \ldots$) production mechanism and hadron (nucleon) structure.

- Properties of Spin-Density Matrix Elements (SDMEs).
  SDMEs are coefficients in the angular distribution of final particles and therefore can be extracted from data.

- SDMEs are expressible in terms of ratios of helicity amplitudes $F_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N}$ of the $\gamma^* + N \rightarrow V + N$ reaction, hence the ratios can be obtained from angular distribution of final particles.

- Generalized Parton Distributions (GPDs) of the nucleon can be obtained from the helicity amplitude $F_{00} \equiv F_{0\frac{1}{2}0\frac{1}{2}}$ ($\gamma_L \rightarrow V_L$) for which factorization theorem is proved.

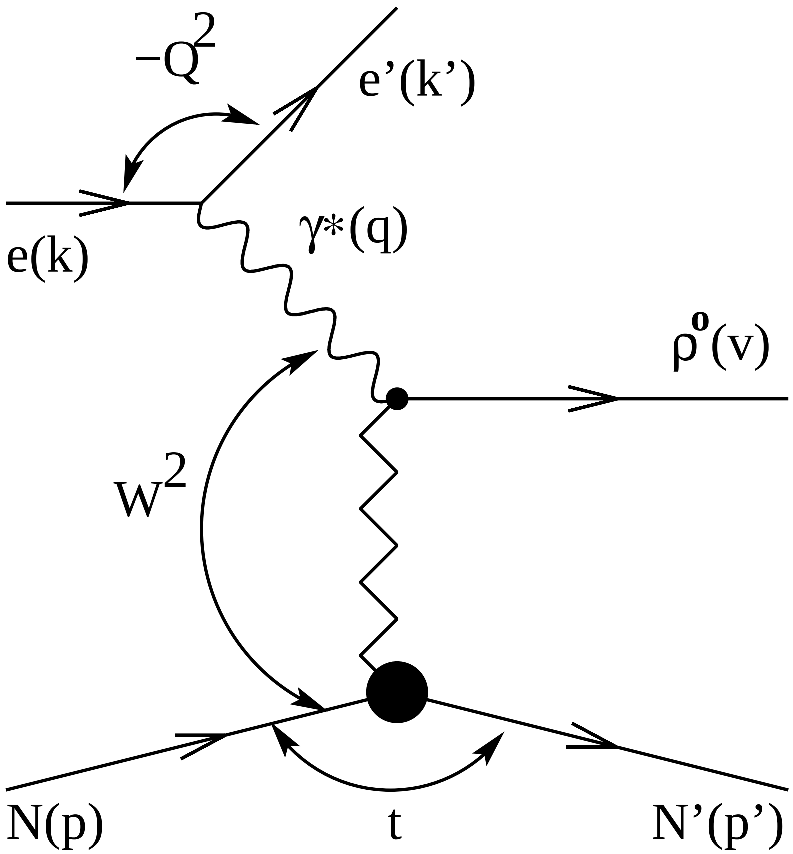
Physics Motivation



- Generalized Parton Distributions (GPDs)
  Quark GPDs: $H_q(x,\xi,t)$, $E_q(x,\xi,t)$, ...
  Gluon GPDs: $H_g(x,\xi,t)$, $E_g(x,\xi,t)$, ...
  H_q and H_g can be obtained from nucleon helicity non-flip amplitudes ($\lambda_N = \lambda'_N$).
  E_q and E_g can be extracted from nucleon helicity-flip amplitudes ($\lambda_N \neq \lambda'_N$).
- Ji's Sum Rules

$$\frac{1}{2}\int_{-1}^{1} dx x[H^q(x,\xi,t\to 0) + E^q(x,\xi,t\to 0)] =< J_q >,$$
$$\int_{-1}^{1} dx x[H^g(x,\xi,t\to 0) + E^g(x,\xi,t\to 0)] =< J_g > .$$

To use the Ji sum rules $E_q(x,\xi,t\to 0)$ and $E_g(x,\xi,t\to 0)$ are to be extracted from data on transversely polarized targets at $t \neq 0$.



$$e(\lambda) \to e'(\lambda') + \gamma^*(\lambda_\gamma),$$

$$\gamma^*(\lambda_\gamma) + N(\lambda_N) \to V(\lambda_V) + N'(\lambda'_N);$$

The helicity amplitude of the reaction
$\gamma^* + N \to V + N$

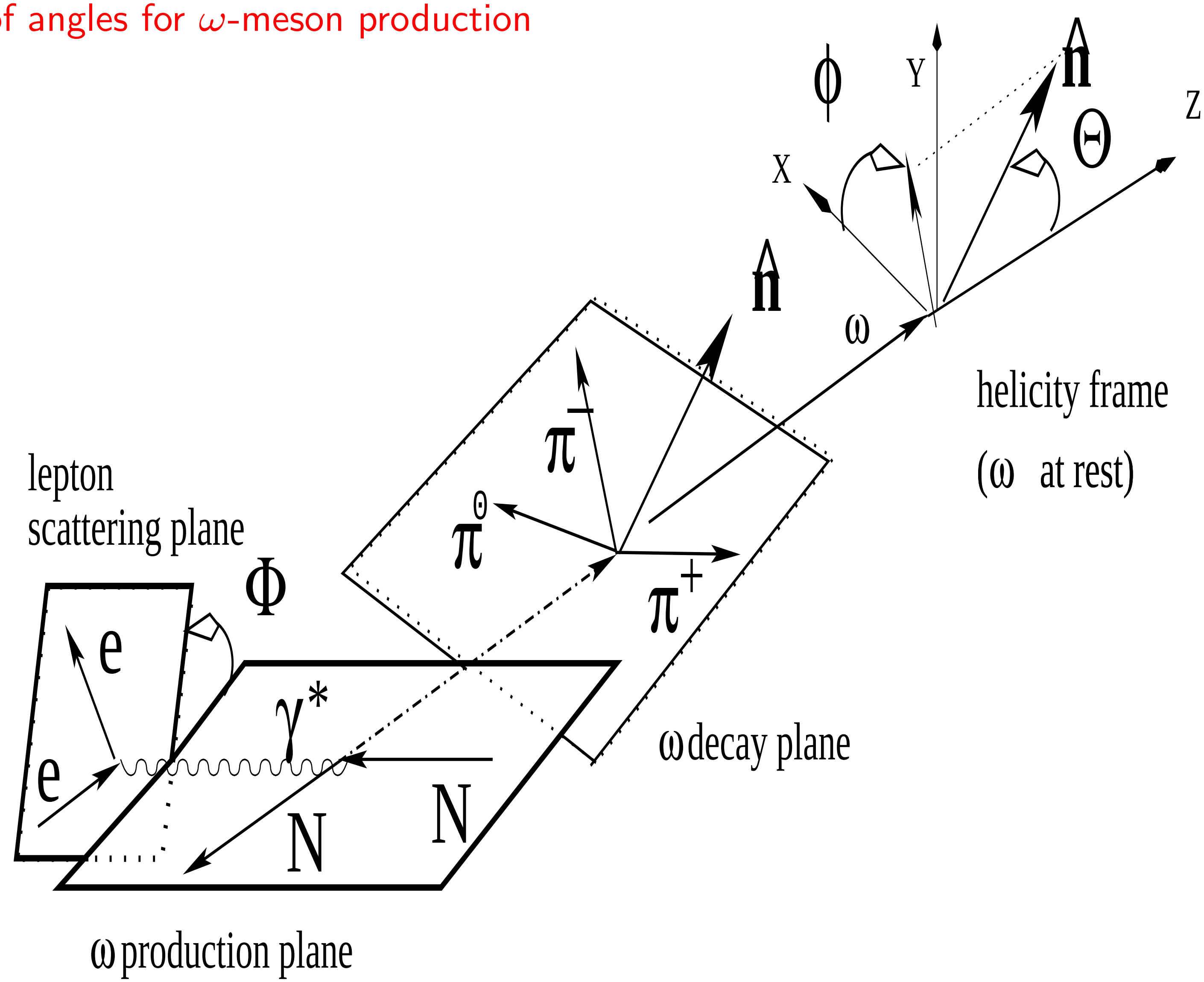
$$F_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N}$$

$$= (-1)^{\lambda_\gamma} \langle v \lambda_V p' \lambda'_N | J^\sigma_{(h)} | p \lambda_N \rangle e^{(\lambda_\gamma)}_\sigma .$$

$J^\sigma_{(h)}$ is the electromagnetic current of hadrons; $e^{(\lambda_\gamma)}_\sigma$ is the photon polarization four-vector ($\lambda_\gamma = \pm 1,\ 0$).

Phenomenological description of reaction $e + N \to e' + V + N'$

- Definition of angles for ω-meson production



- For ρ-meson production $\vec{n} = \vec{p}_{\pi^+}/|\vec{p}_{\pi^+|}$.

Phenomenological description of reaction $e + N \to e' + V + N'$

- Spin-Density Matrix Elements $r_{\lambda_V \lambda'_V}$ of the vector meson can be extracted from the angular distribution of final particles

$$\mathcal{W}(\Phi, \Theta, \phi) = \sum_{\lambda_V, \lambda'_V} Y_{1\lambda_V}(\Theta, \phi) r_{\lambda_V \lambda'_V}(\Phi) Y^*_{1\lambda'_V}(\Theta, \phi)$$

.

- Relation between spin-density matrix of virtual photon $\rho_{\lambda_\gamma \lambda'_\gamma}$ and that of vector meson: $r = \frac{F \rho F^+}{\mathcal{N}}$.

  Spin-density matrixes of γ^* and V are decomposed: $\rho = \sum \left(c_\alpha \Sigma^\alpha \right)$, $r = \sum c_\alpha r^\alpha$.

- SDMEs $r^\alpha_{\lambda_V \lambda'_V}$ in the Schilling-Wolf representation are extracted from vector-meson production data
  They can be calculated from the relation

$$r^\alpha_{\lambda_V \lambda'_V} = \frac{1}{\mathcal{N}_\alpha} \sum F_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N} \, \Sigma^\alpha_{\lambda_\gamma \lambda'_\gamma} \, F^*_{\lambda'_V \lambda'_N \lambda'_\gamma \lambda_N},$$

  where Σ^α is a set of nine standard hermitian matrixes ($\alpha = 0,\ 1, \ldots,\ 8.$).

- SDMEs in the Diehl representation: $u^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma}$, $n^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma}$, $s^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma}$, $l^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma}$

The HERMES Experiment

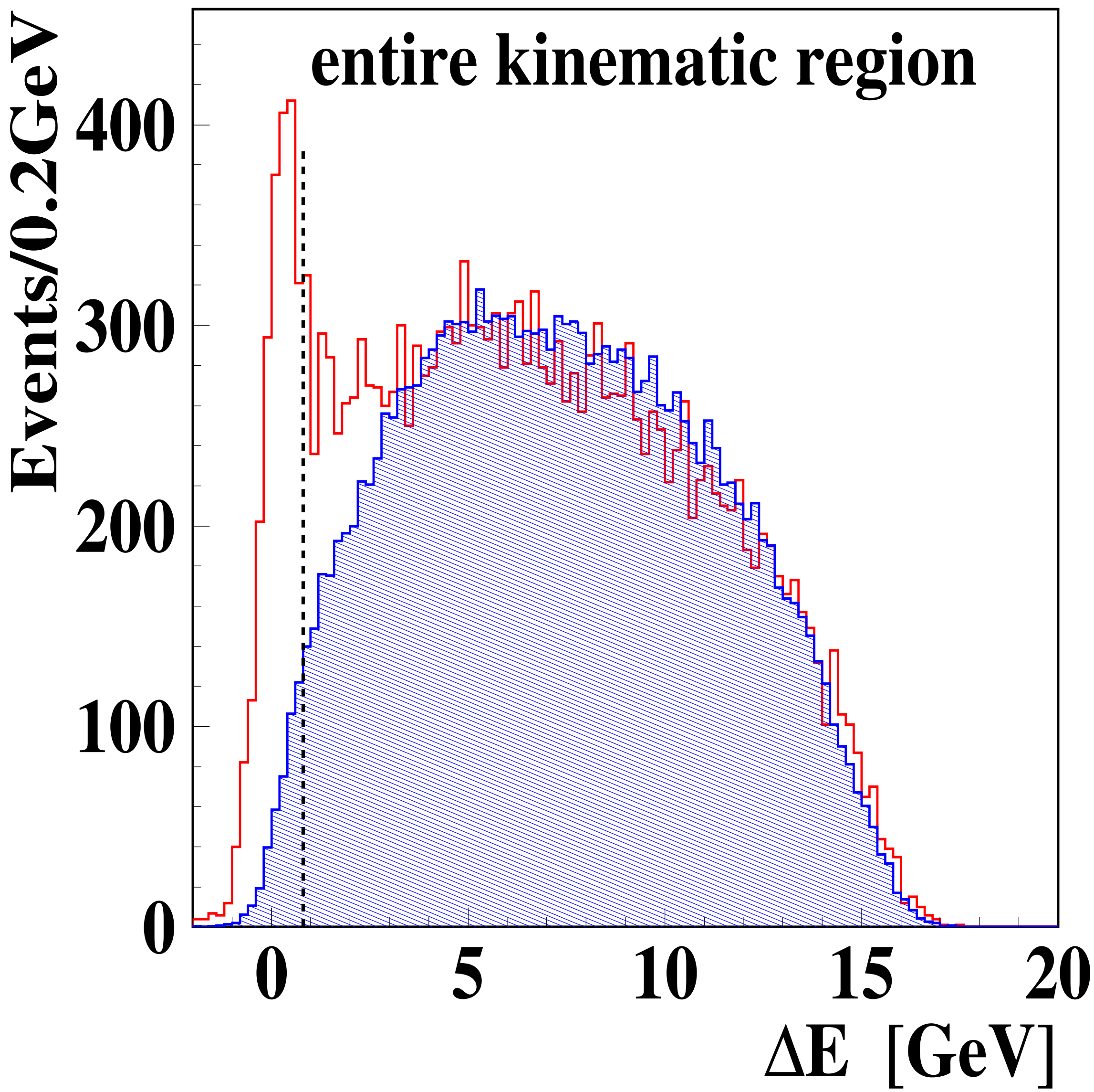
ω-meson production

- Longitudinally polarized electron/positron beam with energy of 27.6 GeV.
- 6.3 GeV $> W > 3.0$ GeV, $Q^2 > 1$ GeV^2, $-t' = -(t - t_{max}) < 0.2$ GeV^2.
- Recoil nucleon was not detected. Missing mass criterion was used.
  $\Delta E = \frac{M_X^2 - M_p^2}{2M_p}$; $-1.0\ \mathrm{GeV} < \Delta \mathrm{E} < 0.8$ GeV;
  M_X (M_p) mass of recoil system (proton).
- 2260 events with exclusive ω meson produced on unpolarized proton, 1332 on unpolarized deuteron and 279 exclusively produced ω mesons with unpolarized beam on transversely polarized proton ($|\vec{S}_\perp| \approx 0.72 \pm 0.06$) were accumulated.

ρ-meson production

- 8741 events with exclusive ρ-mesons produced with unpolarized and longitudinally polarized beam on transversely polarized proton ($|\vec{S}_\perp| \approx 0.72 \pm 0.06$) for $-t' \leq 0.4$ GeV^2 were obtained.

ΔE distribution for ω meson production



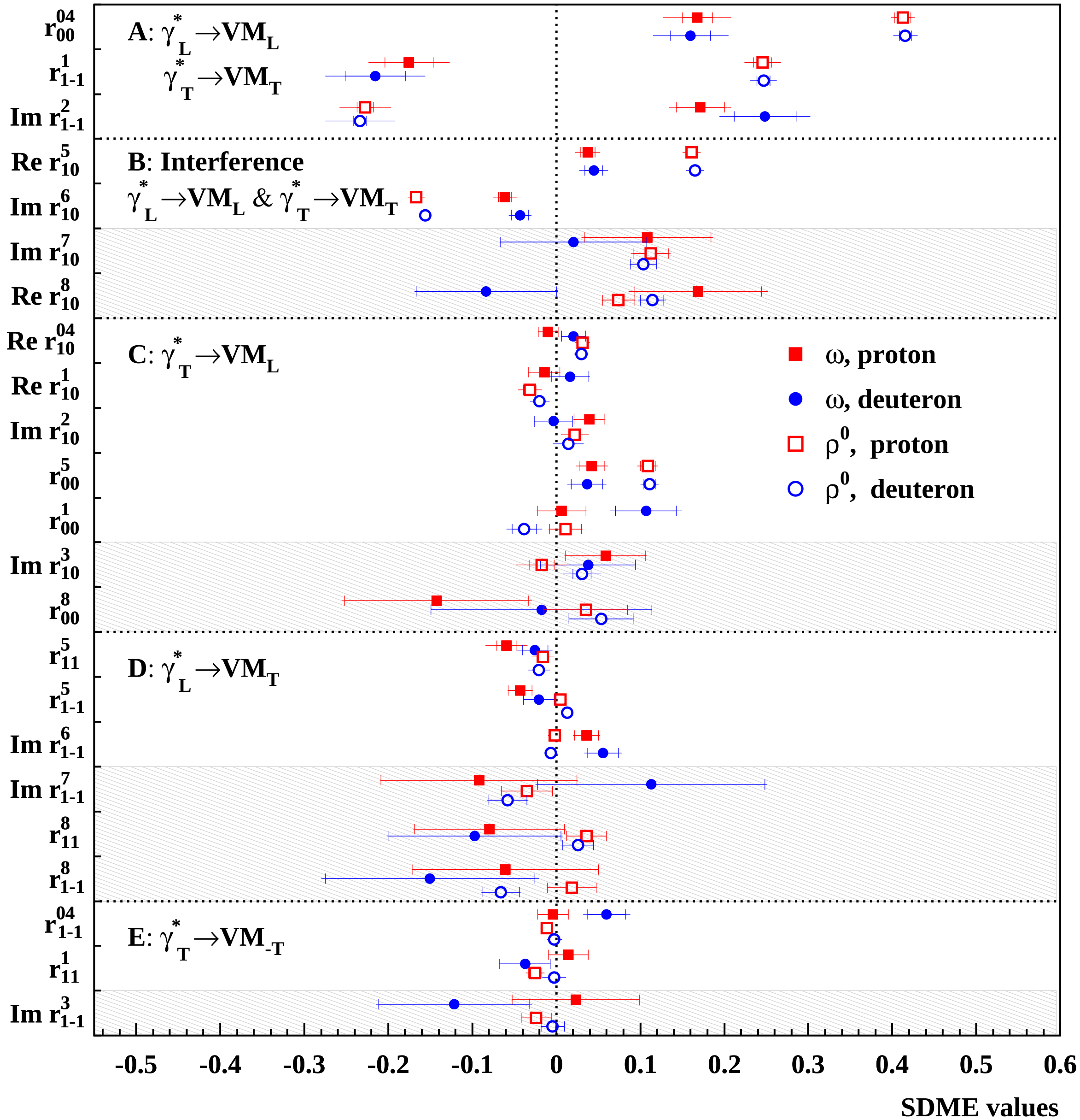
$16\% < \text{fraction of background} < 26\%$ for increasing $-t'$ is subtracted.

SDMEs from ω-meson production on unpolarized targets

A. Airapetian et al. (the HERMES Collaboration), Eur. Phys. J. C74, (2014) 3110.

Results on Spin-Density Matrix Elements

- Comparison of results for ω and ρ^0 mesons for entire kinematic region



SDMEs r^1_{1-1}, $\mathrm{Im}\{r^2_{1-1}\}$ for ω mesons have opposite signs to those for ρ mesons.

Results on Spin-Density Matrix Elements

- Enigma of the $\rho - \omega$ difference for class A SDMEs

$$\mathrm{Im}\{r^2_{1-1}\} - r^1_{1-1} = \frac{1}{\mathcal{N}}(-|T_{1\frac{1}{2}1\frac{1}{2}}|^2 - |T_{1-\frac{1}{2}1\frac{1}{2}}|^2 + |U_{1\frac{1}{2}1\frac{1}{2}}|^2 + |U_{1-\frac{1}{2}1\frac{1}{2}}|^2) > 0.$$

$T_{\lambda_V \mu_N \lambda_\gamma \lambda_N}$ is Natural Parity Exchange (NPE) amplitude.

$U_{\lambda_V \mu_N \lambda_\gamma \lambda_N}$ is Unnatural Parity Exchange (UPE) amplitude.

In Regge Phenomenology, NPE amplitudes are due to exchanges of Pomeron, ρ, ω, f_2, a_2, ...

reggeons with natural parity ($J^P = 0^+,\ 1^-,\ 2^+, \ldots$).

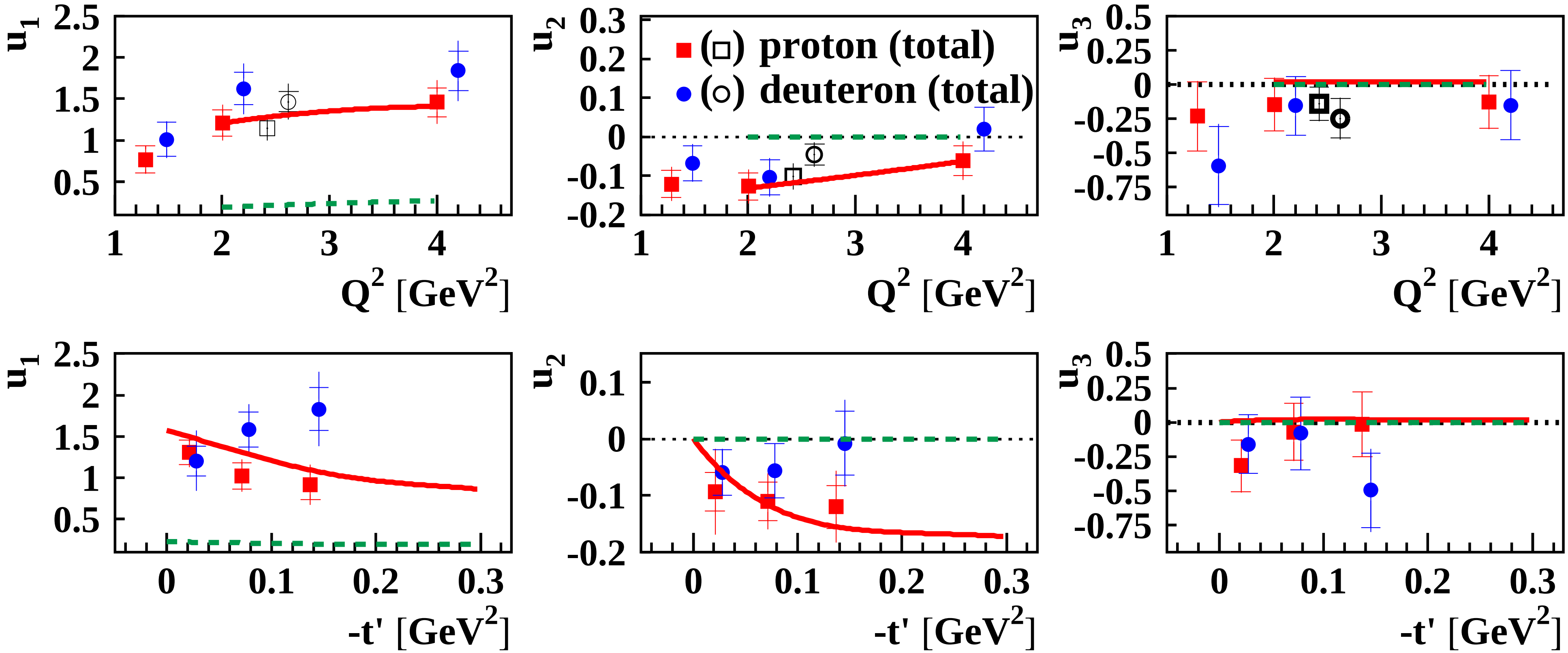
UPE amplitudes are due to exchanges of π, a_1, ... reggeons

with unnatural parity ($J^P = 0^-,\ 1^+,\ 2^-, \ldots$).

Pion exchange at intermediate W and Q^2.

Vertices $\gamma\omega\pi$ and πNN are big $\Rightarrow |U_{11}| > |T_{11}|$ Dominant UPE contribution.

Test of Unnatural-Parity Exchange



$u_1 = 1 - r^{04}_{00} + 2r^{04}_{1-1} - 2r^1_{11} - 2r^1_{1-1}$, $\quad u_1 = \widetilde{\sum} \frac{4\epsilon|U_{10}|^2 + 2|U_{11}+U_{-11}|^2}{\mathcal{N}}, \epsilon = \frac{n^\gamma_L}{n^\gamma_T} \approx 0.8,$

$u_2 = r^5_{11} + r^5_{1-1}$, $u_3 = r^8_{11} + r^8_{1-1}$, $\quad u_2 + iu_3 = \sqrt{2}\widetilde{\sum}\frac{(U_{11}+U_{-11})U^*_{10}}{\mathcal{N}}.$

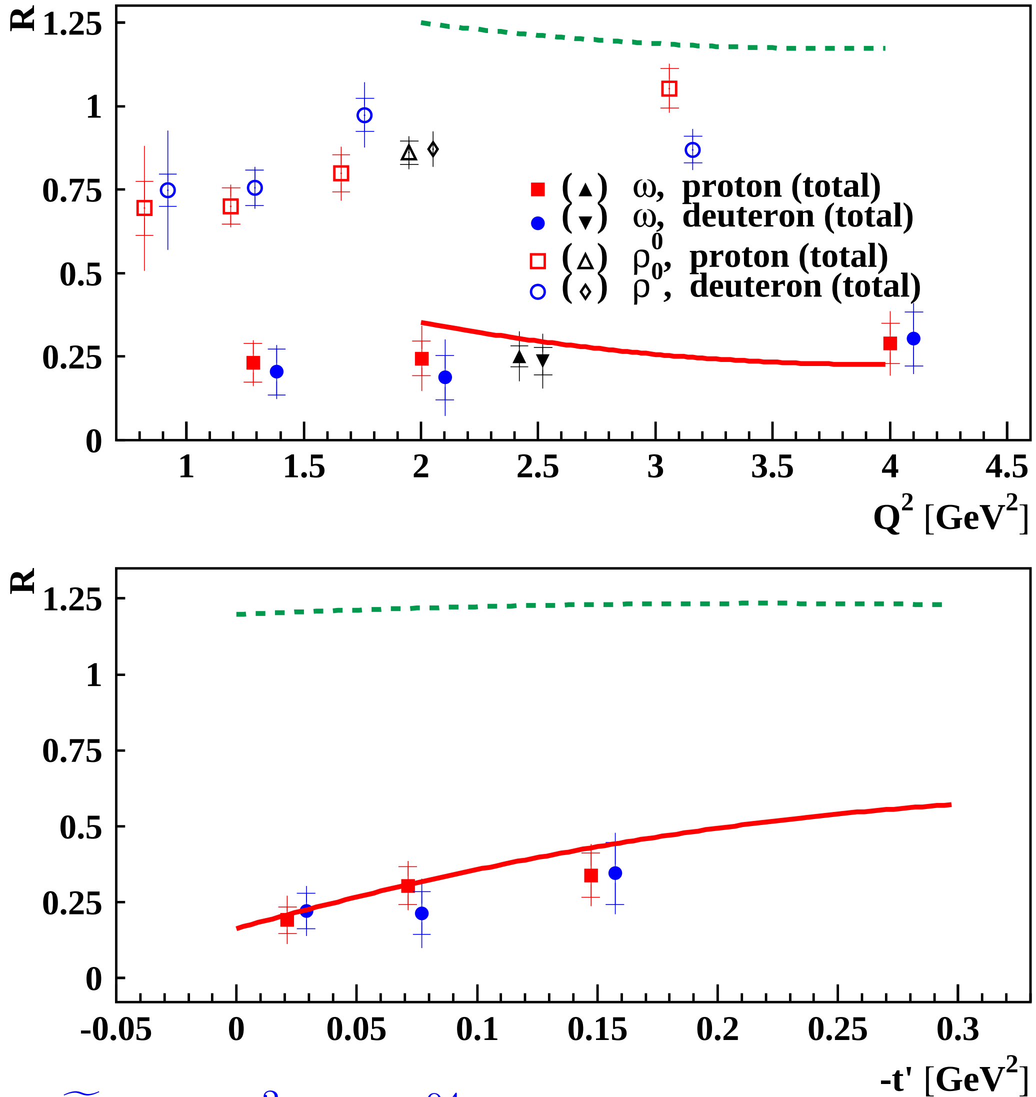
$|u_1| \gg |u_2| \sim |u_3|.$

Empty black squares and circles show results integrated over all kinematic region.

Curves show result of Goloskokov-Kroll calculations.

Longitudinal-to-Transverse Cross-Section Ratio

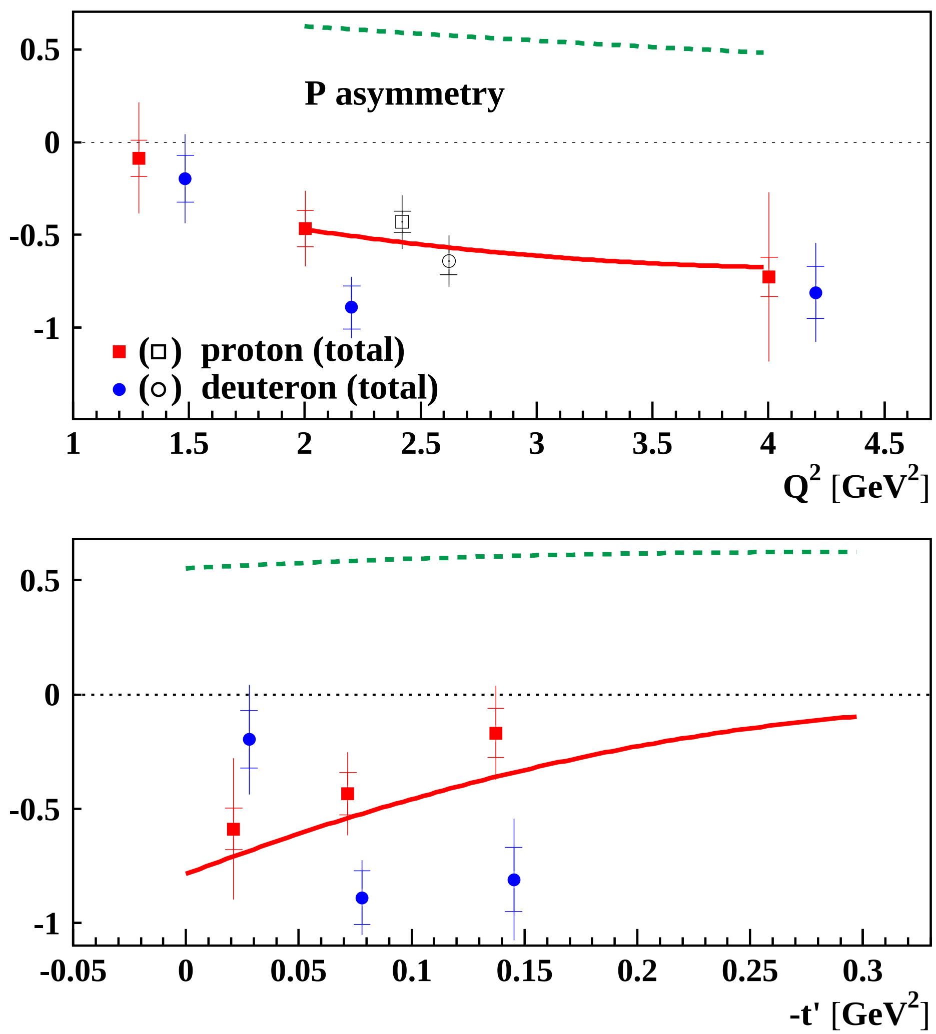
Kinematic dependence of $R = d\sigma_L/d\sigma_T$



$$R = \frac{\widetilde{\sum}_{\lambda_\omega}|F_{\lambda_\omega 0}|^2}{\widetilde{\sum}_{\lambda_\omega}|F_{\lambda_\omega 1}|^2} \approx \frac{1}{\epsilon}\frac{r^{04}_{00}}{1-r^{04}_{00}} \approx |T_{00}|^2/[|T_{11}|^2 + |U_{11}|^2].$$

Curves show result of Goloskokov-Kroll calculations.

UPE-to-NPE Asymmetry



$$P = \frac{d\sigma_T^N - d\sigma_T^U}{d\sigma_T^N + d\sigma_T^U} \approx \frac{2r^1_{1-1} - r^1_{00}}{1 - r^{04}_{00}}$$

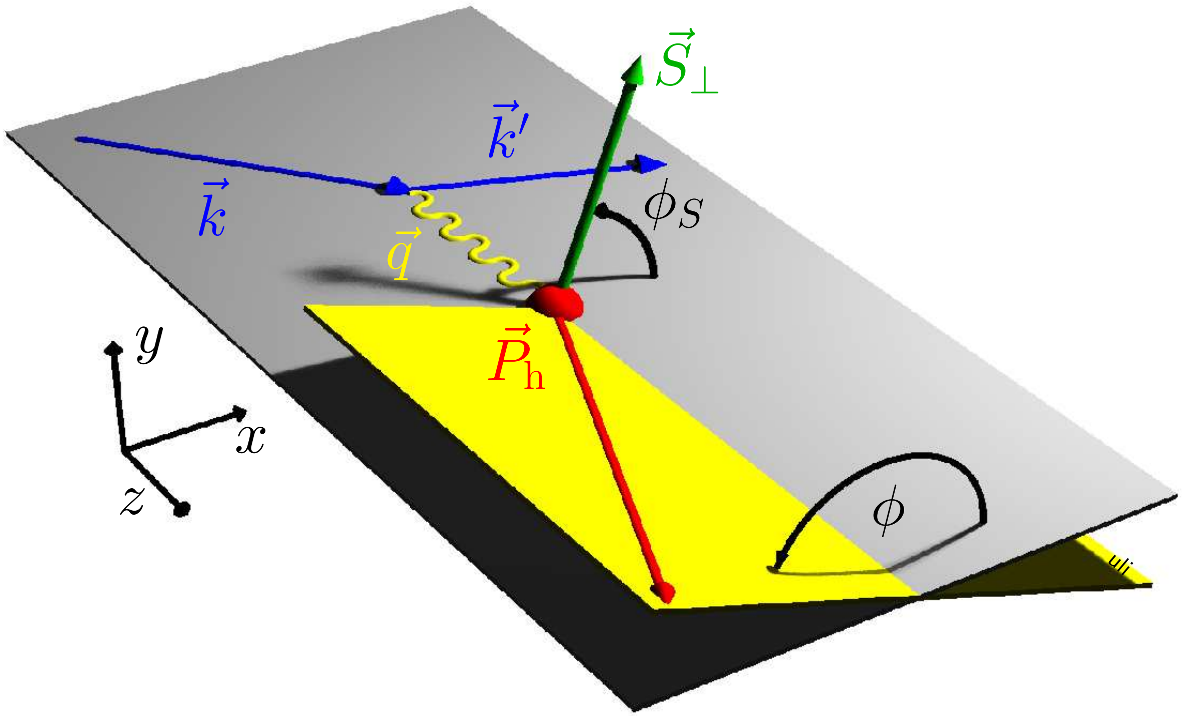
Curves show result of Goloskokov-Kroll calculations.

Empty square and circle show results integrated over all kinematic region.

Transverse-target-spin asymmetry from ω-meson production on transversely polarized proton

A. Airapetian et al. (the HERMES Collaboration), Eur. Phys. J. C75 (2015) 600.

Transverse-Target-Spin Asymmetry



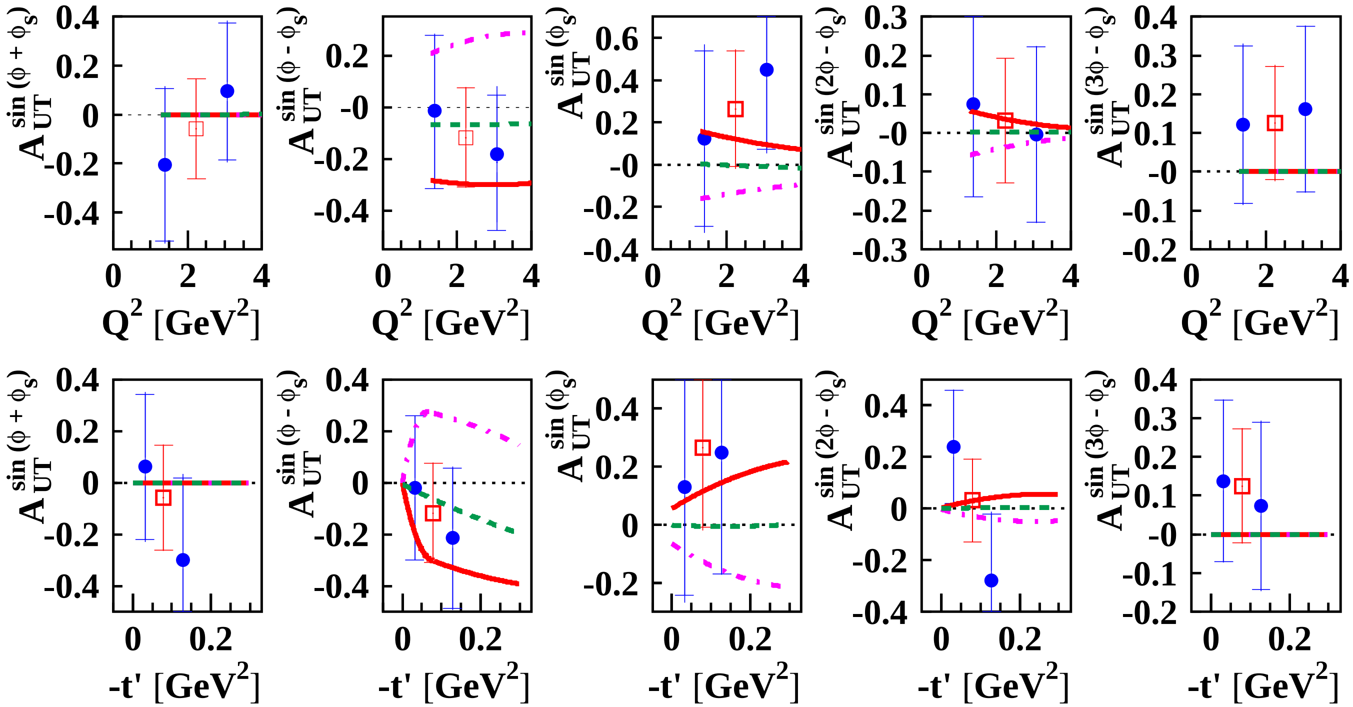
$$\mathcal{W}(\phi,\phi_S) = 1 + A_{UU}^{\cos(\phi)}\cos(\phi) + A_{UU}^{\cos(2\phi)}\cos(2\phi)$$

$$+S_\perp[A_{UT}^{\sin(\phi+\phi_S)}\sin(\phi+\phi_S) \quad + A_{UT}^{\sin(\phi-\phi_S)}\sin(\phi-\phi_S)$$

$$+A_{UT}^{\sin(\phi_S)}\sin(\phi_S) \quad + A_{UT}^{\sin(2\phi-\phi_S)}\sin(2\phi-\phi_S) \quad +A_{UT}^{\sin(3\phi-\phi_S)}\sin(3\phi-\phi_S)].$$

Transverse-Target-Spin Asymmetry

Asymmetry for total sctatistics

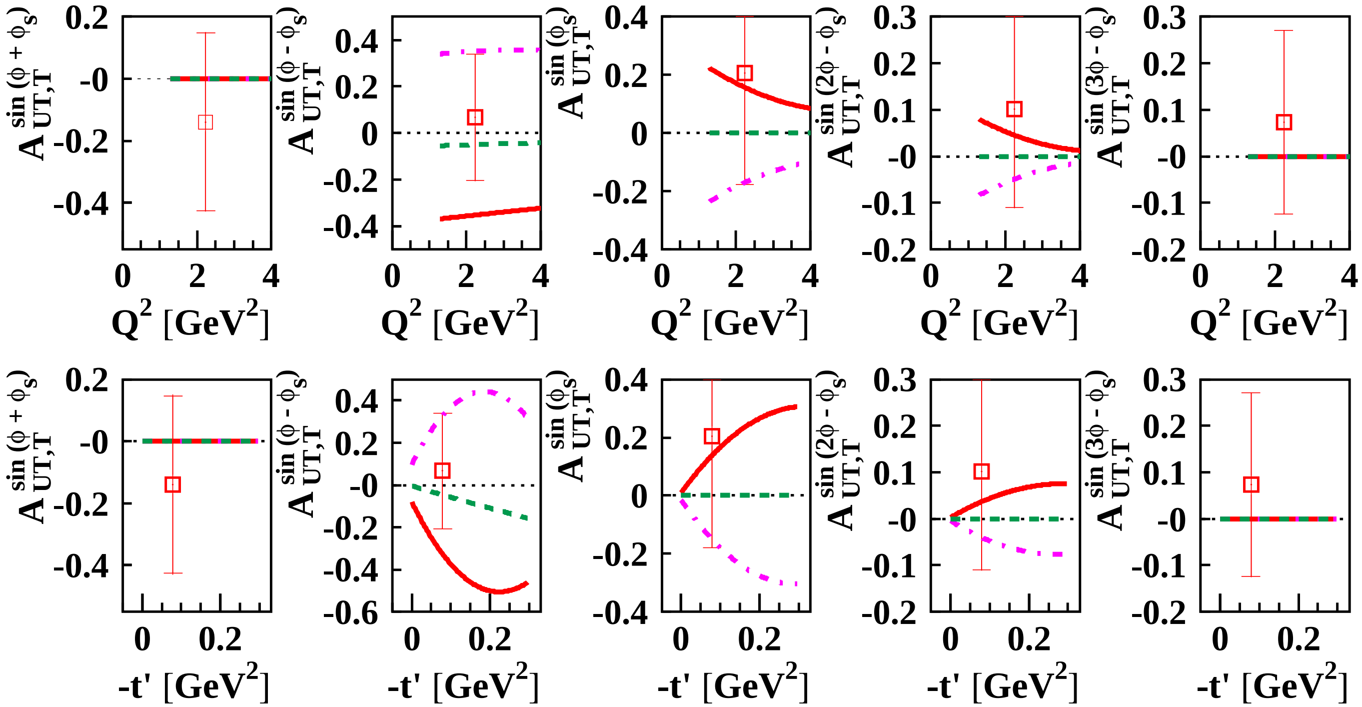


Curves show results of Goloskokov-Kroll calculations: solid (dash-dotted) lines for positive (negative) $\pi\omega$ transition form factor, dashed lines are obtained without pion exchange contribution (S. Goloskokov, P. Kroll, private communication).

Open squares show mean values of asymmetry amplitudes. HERMES: Eur. Phys. J. C 75 (2015) 600.

Transverse-Target-Spin Asymmetry

Asymmetry for transversely polarized ω mesons



Curves show results of Goloskokov-Kroll calculations: solid (dash-dotted) lines for positive (negative) $\pi\omega$ transition form factor, dashed lines are obtained without pion exchange contribution.
Open squares show mean values of asymmetry amplitudes.

Extraction of helicity amplitude ratios from ρ-meson production on transversely polarized proton

Extraction of Helicity Amplitude Ratios

More detailed definition of helicity amplitudes

Amplitudes without nucleon helicity flip:

$$T^{(1)}_{\lambda_V\lambda_\gamma} \equiv T_{\lambda_V\frac{1}{2}\lambda_\gamma\frac{1}{2}} = T_{\lambda_V-\frac{1}{2}\lambda_\gamma-\frac{1}{2}}, \;\; U^{(1)}_{\lambda_V\lambda_\gamma} \equiv U_{\lambda_V\frac{1}{2}\lambda_\gamma\frac{1}{2}} = -U_{\lambda_V-\frac{1}{2}\lambda_\gamma-\frac{1}{2}},$$

Amplitudes with nucleon helicity flip:

$$T^{(2)}_{\lambda_V\lambda_\gamma} \equiv T_{\lambda_V\frac{1}{2}\lambda_\gamma-\frac{1}{2}} = -T_{\lambda_V-\frac{1}{2}\lambda_\gamma\frac{1}{2}}, \;\; U^{(2)}_{\lambda_V\lambda_\gamma} \equiv U_{\lambda_V\frac{1}{2}\lambda_\gamma-\frac{1}{2}} = U_{\lambda_V-\frac{1}{2}\lambda_\gamma\frac{1}{2}},$$

Angular distribution is dimensionless quantity, hence it may depend on the helicity amplitude ratios only.

Amplitude ratios:

$$t^{(1)}_{\lambda_V\lambda_\gamma} = T^{(1)}_{\lambda_V\lambda_\gamma}/T^{(1)}_{00}, \; t^{(2)}_{\lambda_V\lambda_\gamma} = T^{(2)}_{\lambda_V\lambda_\gamma}/T^{(1)}_{00}, \; u^{(1)}_{\lambda_V\lambda_\gamma} = U^{(1)}_{\lambda_V\lambda_\gamma}/T^{(1)}_{00}, \; u^{(2)}_{\lambda_V\lambda_\gamma} = U^{(2)}_{\lambda_V\lambda_\gamma}/T^{(1)}_{00}.$$

Total number of independent amplitude ratios is 17 (34 real functions).

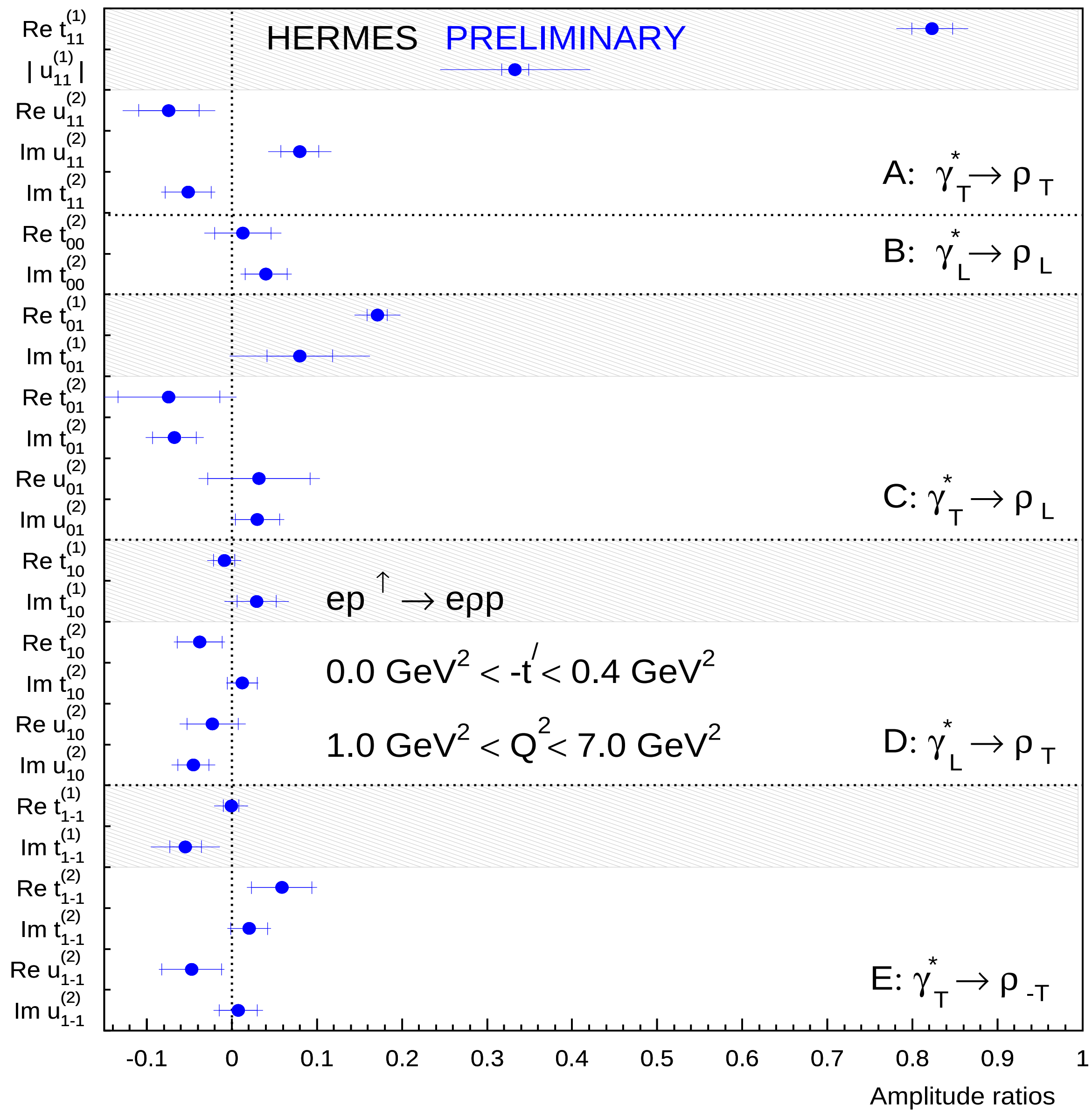
Small amplitudes can be reliably extracted if there is product of those by the amplitude $T^{(1)}_{00}$ or $T^{(1)}_{11}$ being dominant at large Q^2 and small $|t|$.

For longitudinally polarized beam and transversely polarized target only 25 parameters can be reliably extracted.

For the present data, the phase shifts of $T^{(1)}_{11}$ and $U^{(1)}_{11}$ are fixed from previous HERMES data.

Ratios $u^{(1)}_{10}$, $u^{(1)}_{10}$, $u^{(1)}_{1-1}$ are not obtained from present data since they are multiplied by small factor $\sqrt{1-\epsilon}S_L$ with the longitudinal (with respect of virtual photon) target polarization $S_L < 0.04$.

Extraction of Helicity Amplitude Ratios

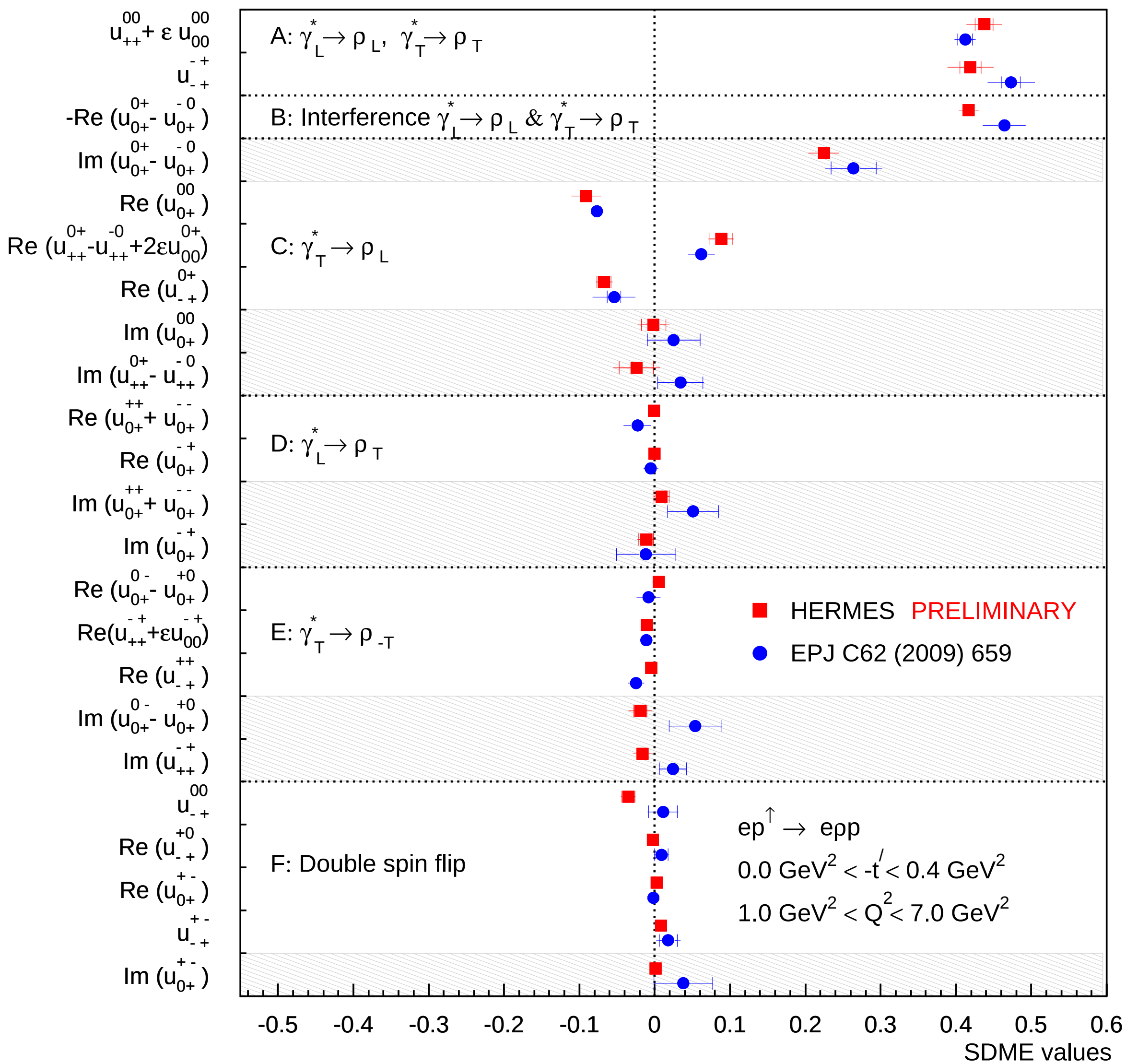


Ratios of amplitudes without nucleon spin flip are shown in shaded areas.

They were also obtained in previous HERMES analysis (Eur. Phys. J. C71 (2011) 1609).

Extraction of Helicity Amplitude Ratios

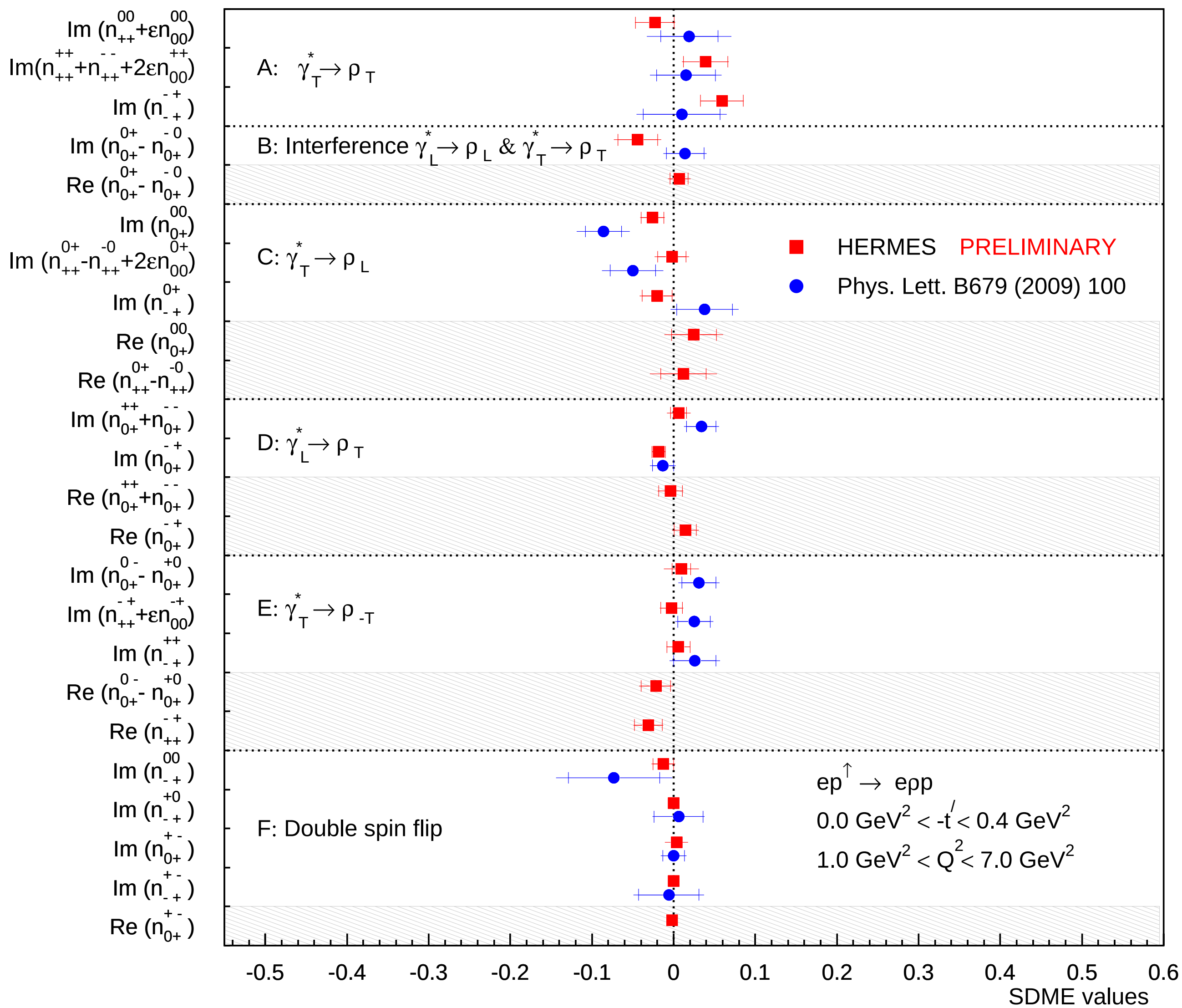
- Comparison of calculated (red points) and "direct" (blue) SDMEs $u^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma}$ in the Diehl representation



"Polarized" SDMEs (which can be obtained only with longitudinally polarized beam) are shown in shaded areas.

Extraction of Helicity Amplitude Ratios

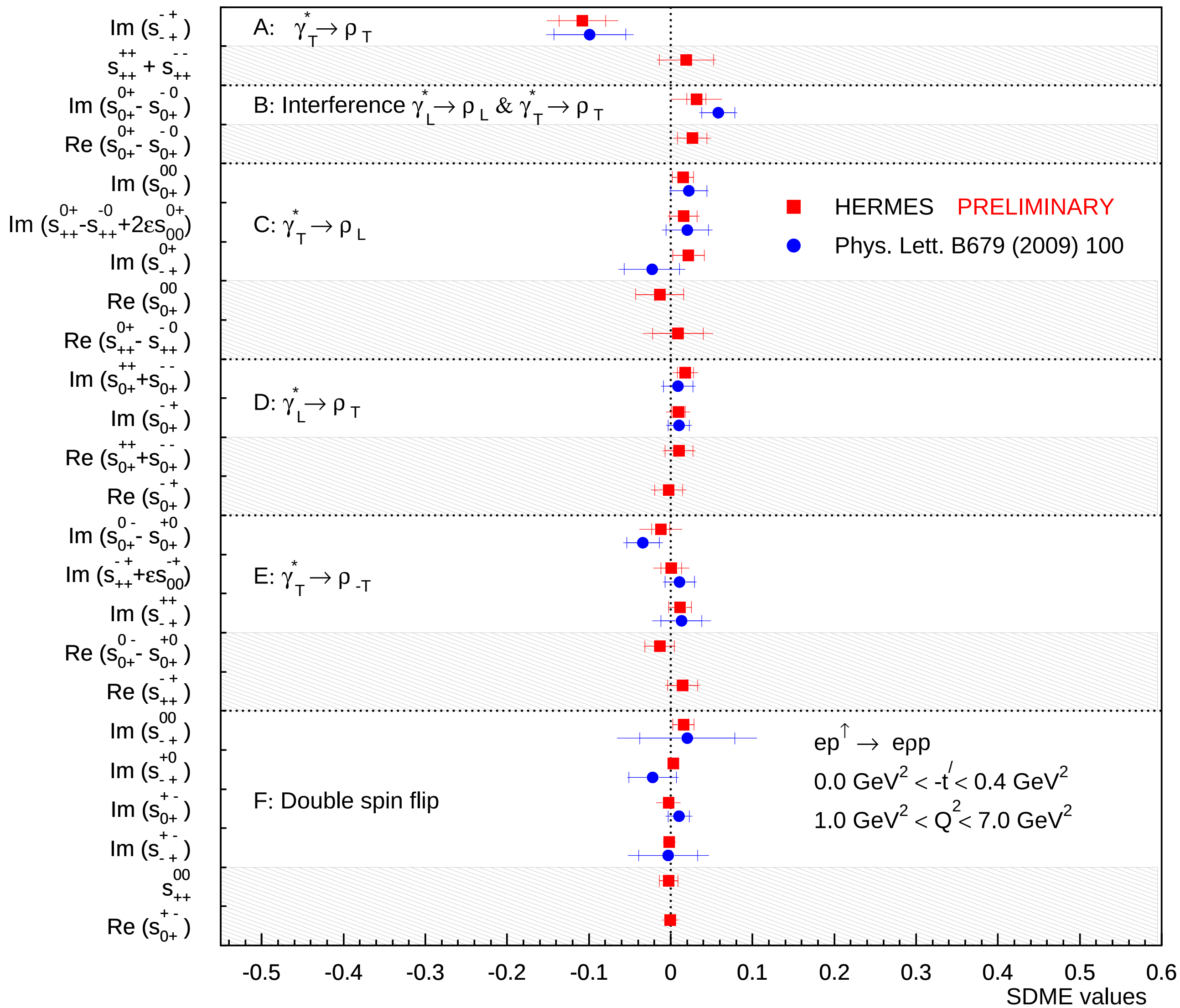
- Comparison of calculated (red points) and "direct" (blue) SDMEs $n^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma}$ in the Diehl representation



"Polarized" SDMEs (which can be obtained only with longitudinally polarized beam) are shown in shaded areas.

Extraction of Helicity Amplitude Ratios

- Comparison of calculated (red points) and "direct" (blue) SDMEs $s^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma}$ in the Diehl representation



"Polarized" SDMEs (which can be obtained only with longitudinally polarized beam) are shown in shaded areas.

Summary

- Exclusive vector-meson electroproduction in DIS is studied at HERMES using a longitudinally polarized electron/positron beam and unpolarized or transversely polarized hydrogen and deuterium targets in the kinematic region $Q^2 > 1.0$ GeV^2, 3.0 GeV $< W < 6.3$ GeV, and $-t' < 0.2$ GeV^2.
- Using an unbinned maximum likelihood method, 15 unpolarized, and, for the first time, 8 polarized spin-density matrix elements are extracted from data on exclusive ω-meson production.
- No significant differences between proton and deuteron results are seen.
- While the values of class-A and B SDMEs agree with the hypothesis of s-channel helicity conservation, the class-C SDME r^5_{00} indicates its violation.
- Using the SDMEs r^1_{1-1} and $\mathrm{Im}\{r^2_{1-1}\}$ it is shown that $|U_{11}|^2 > |T_{11}|^2$.
- The importance of UPE transitions is also shown by considering u_1, u_2, u_3 and the UPE-to-NPE asymmetry. This suggests that at HERMES energies π^0, a_1... exchanges play a significant role.
- The longitudinal-to-transverse cross-section ratio $R = d\sigma_L/d\sigma_T$ for the ω meson is $R =$ 0.25 $\pm$ 0.03 $\pm$ 0.07 at $< Q^2 >= 2.4$ GeV^2, $< -t' >= 0.08$ GeV^2, $< W >= 4.8$ GeV.
- Two cosine and five sine azimuthal modulations of the cross section on transversely polarized proton are obtained for the entire kinematic bin. Extracted asymmetry amplitudes favors probably a positive sign of $\pi\omega$ form factor.
- For the first time, the amplitude analysis of the ρ^0-meson electroproduction on the transversely polarized proton is performed by HERMES and information on 25 real functions is obtained.

- SDMEs in the Diehl Representation

$$u^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma} = (\mathcal{N}_T + \epsilon \mathcal{N}_L)^{-1} \sum_{\sigma=\pm\frac{1}{2}} \Big[T_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (T_{\lambda'_V \sigma \lambda'_\gamma \frac{1}{2}})^* + U_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (U_{\lambda'_V \sigma \lambda'_\gamma \frac{1}{2}})^* \Big],$$

$$l^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma} = (\mathcal{N}_T + \epsilon \mathcal{N}_L)^{-1} \sum_{\sigma=\pm\frac{1}{2}} \Big[T_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (U_{\lambda'_V \sigma \lambda'_\gamma \frac{1}{2}})^* + U_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (T_{\lambda'_V \sigma \lambda'_\gamma \frac{1}{2}})^* \Big],$$

$$s^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma} = (\mathcal{N}_T + \epsilon \mathcal{N}_L)^{-1} \sum_{\sigma=\pm\frac{1}{2}} \Big[T_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (U_{\lambda'_V \sigma \lambda'_\gamma -\frac{1}{2}})^* + U_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (T_{\lambda'_V \sigma \lambda'_\gamma -\frac{1}{2}})^* \Big],$$

$$n^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma} = (\mathcal{N}_T + \epsilon \mathcal{N}_L)^{-1} \sum_{\sigma=\pm\frac{1}{2}} \Big[T_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (T_{\lambda'_V \sigma \lambda'_\gamma -\frac{1}{2}})^* + U_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (U_{\lambda'_V \sigma \lambda'_\gamma -\frac{1}{2}})^* \Big],$$

where the normalization factors are

$$\mathcal{N}_T = \frac{1}{2} \sum_{\lambda'_V, \lambda'_N, \lambda_N} \Big[|T_{\lambda'_V \lambda'_N 1 \lambda_N}|^2 + |U_{\lambda'_V \lambda'_N 1 \lambda_N}|^2 \Big], \; \mathcal{N}_L = \frac{1}{2} \sum_{\lambda'_V, \lambda'_N, \lambda_N} \Big[|T_{\lambda'_V \lambda'_N 0 \lambda_N}|^2 + |U_{\lambda'_V \lambda'_N 0 \lambda_N}|$$

The NPE amplitudes $T_{\lambda'_V \lambda'_N \lambda_\gamma \lambda_N}$ and UPE amplitudes $U_{\lambda'_V \lambda'_N \lambda_\gamma \lambda_N}$ are defined by

$$T_{\lambda'_V \lambda'_N \lambda_\gamma \lambda_N} = \frac{1}{2} \Big[F_{\lambda'_V \lambda'_N \lambda_\gamma \lambda_N} + (-1)^{\lambda_N - \lambda'_N} F_{\lambda'_V - \lambda'_N \lambda_\gamma - \lambda_N} \Big],$$

$$U_{\lambda'_V \lambda'_N \lambda_\gamma \lambda_N} = \frac{1}{2} \Big[F_{\lambda'_V \lambda'_N \lambda_\gamma \lambda_N} - (-1)^{\lambda_N - \lambda'_N} F_{\lambda'_V - \lambda'_N \lambda_\gamma - \lambda_N} \Big].$$

- Dependence of angular distribution on SDMEs

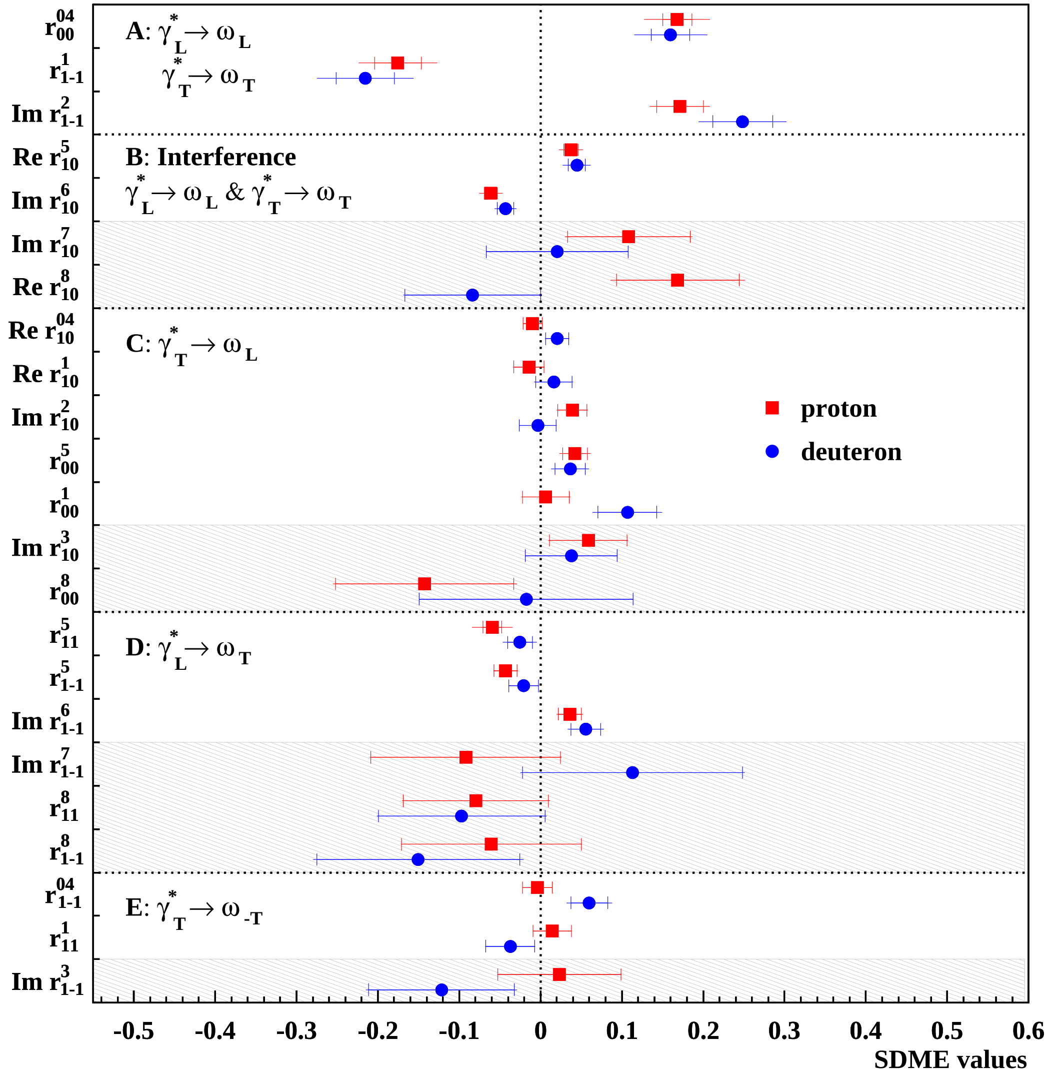
$$\mathcal{W}^{U+L}(\Phi,\phi,\cos\Theta) = \mathcal{W}^{U}(\Phi,\phi,\cos\Theta) + P_b\mathcal{W}^{L}(\Phi,\phi,\cos\Theta)$$

$$\mathcal{W}^{U}(\Phi,\phi,\cos\Theta) =$$
$$\frac{3}{8\pi^2}\Big[\frac{1}{2}(1-r^{04}_{00}) + \frac{1}{2}(3r^{04}_{00}-1)\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^{04}_{10}\}\sin 2\Theta\cos\phi - r^{04}_{1-1}\sin^2\Theta\cos 2\phi$$
$$-\ \epsilon\cos 2\Phi\Big(r^{1}_{11}\sin^2\Theta + r^{1}_{00}\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^{1}_{10}\}\sin 2\Theta\cos\phi - r^{1}_{1-1}\sin^2\Theta\cos 2\phi\Big)$$
$$-\ \epsilon\sin 2\Phi\Big(\sqrt{2}\mathrm{Im}\{r^{2}_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^{2}_{1-1}\}\sin^2\Theta\sin 2\phi\Big)$$
$$+\ \sqrt{2\epsilon(1+\epsilon)}\cos\Phi\Big(r^{5}_{11}\sin^2\Theta + r^{5}_{00}\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^{5}_{10}\}\sin 2\Theta\cos\phi - r^{5}_{1-1}\sin^2\Theta\cos 2\phi\Big)$$
$$+\ \sqrt{2\epsilon(1+\epsilon)}\sin\Phi\Big(\sqrt{2}\mathrm{Im}\{r^{6}_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^{6}_{1-1}\}\sin^2\Theta\sin 2\phi\Big)\Big],$$

$$\mathcal{W}^{L}(\Phi,\phi,\cos\Theta) = \frac{3}{8\pi^2}\Big[\sqrt{1-\epsilon^2}\Big(\sqrt{2}\mathrm{Im}\{r^{3}_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^{3}_{1-1}\}\sin^2\Theta\sin 2\phi\Big)$$
$$+\ \sqrt{2\epsilon(1-\epsilon)}\cos\Phi\Big(\sqrt{2}\mathrm{Im}\{r^{7}_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^{7}_{1-1}\}\sin^2\Theta\sin 2\phi\Big)$$
$$+$$
$$\sqrt{2\epsilon(1-\epsilon)}\sin\Phi\Big(r^{8}_{11}\sin^2\Theta + r^{8}_{00}\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^{8}_{10}\}\sin 2\Theta\cos\phi - r^{8}_{1-1}\sin^2\Theta\cos 2\phi\Big)\Big].$$

Results on Spin-Density Matrix Elements

- **SDMEs for ω mesons for entire kinematic region**



"Polarized" SDMEs (shaded areas) are measured by HERMES for the first time.

A. Airapetian et al. (the HERMES Collaboration), Eur. Phys. J. C74, (2014) 3110.

Results on Spin-Density Matrix Elements

- S-Channel Helicity Conservation (SCHC): helicity amplitudes with $\lambda_V = \lambda_\gamma$ are dominant

  If SCHC is valid all elements of classes C, D, E are zero.

- Check of SCHC relations for Class-A and B SDMEs for the proton

  $r^1_{1-1} + \mathrm{Im}[\mathrm{r}^2_{1-1}] = -0.004 \pm 0.038 \pm 0.015,$

  $\mathrm{Re}[\mathrm{r}^5_{10}] + \mathrm{Im}[\mathrm{r}^6_{10}] = -0.024 \pm 0.013 \pm 0.004,$

  $\mathrm{Im}[\mathrm{r}^7_{10}] - \mathrm{Re}[\mathrm{r}^8_{10}] = -0.060 \pm 0.100 \pm 0.018,$

  and deuteron

  $r^1_{1-1} + \mathrm{Im}[\mathrm{r}^2_{1-1}] = 0.033 \pm 0.049 \pm 0.016,$

  $\mathrm{Re}[\mathrm{r}^5_{10}] + \mathrm{Im}[\mathrm{r}^6_{10}] = 0.001 \pm 0.016 \pm 0.005,$

  $\mathrm{Im}[\mathrm{r}^7_{10}] - \mathrm{Re}[\mathrm{r}^8_{10}] = 0.104 \pm 0.110 \pm 0.023.$

Results on Spin-Density Matrix Elements

- How to make the U_{11} amplitude large (dominant) at large Q^2?

  First step: hard ρ^0 production (big Q^2).

  Modulus of amplitude $|T_{11}(\gamma + N \to \rho^0 + N)| > |T_{11}(\gamma + N \to \omega + N)|$.

  Second step: pion exchange in final state $\rho^0 + N \to \omega + N$.

  $\rho^0 \to \omega + \pi^0$, $\pi^0 + N \to N$. Vertices $\rho^0\omega\pi$ and πNN are big.

  Peripherical pion exchange (soft) is combined with hard ρ^0 production at high Q^2.

- Factorization theorem

  Factorization theorem is proved for T_{00} $(\gamma_L \to V_L)$ only. $U_{00} \equiv 0$.
  Ivanov-Kirshner: $|F_{11}/F_{00}| \propto 1/Q$ at $Q \to \infty$. $F_{00} = T_{00}$, But $F_{11} = T_{11} + U_{11}$.

  Ivanov-Kirshner statement is true for the "direct" amplitudes T_{00} and T_{11} without pion-pole contribution and final state interaction. Contrubution of U_{11} change significantly F_{11} at HERMES kinematic.

  Fractional contribution of pion exchange goes to zero at $W \to \infty$.

Unbinned Maximum Likelihood Method

- Calculation of "background" SDMEs

  Two Monte Carlo (MC) sets.

  First (normalization) MC set: uniform angular distribution ($\cos\Theta$, Φ, ϕ). Number of events is N_{MC}.

  Second (background pseudo-data) MC set for calculation of a set S_{bg} of 15 background SDMEs. Number of events N_{PD}.

  Log-likelihood function for background pseudo-data events for unpolarized (U) beam

  $-\ln L(S_{bg}) = -\sum_{i=1}^{N_{PD}} \ln \frac{\mathcal{W}^U(S_{bg},\Phi_i,\phi_i,\cos\Theta_i)}{\widetilde{\mathcal{N}}_{bg}(S_{bg})}$,

  $\widetilde{\mathcal{N}}_{bg}(S_{bg}) = \sum_{j=1}^{N_{MC}} \mathcal{W}^U(S_{bg}, \Phi_j, \phi_j, \cos\Theta_j)$.

- Calculation of physical SDMEs

  N total number of experimental events in exclusive region.

  S set of 23 SDMEs for unpolarized target and longitudinally (L) polarized beam.

  $-\ln L(S) = -\sum_{i=1}^{N} \ln \left[\frac{(1-f_{bg})*\mathcal{W}^{U+L}(S,\Phi_i,\phi_i,\cos\Theta_i)}{\widetilde{\mathcal{N}}(S,S_{bg})} + \frac{f_{bg}*\mathcal{W}^U(S_{bg},\Phi_i,\phi_i,\cos\Theta_i)}{\widetilde{\mathcal{N}}(S,S_{bg})}\right]$

  f_{bg} fraction of background events in experimental events in exclusive region.

  The total normalization factor

  $\widetilde{\mathcal{N}}(S, S_{bg}) =$
  $\sum_{j=1}^{N_{MC}} [(1-f_{bg}) * \mathcal{W}^{U+L}(S, \Phi_j, \phi_j, \cos\Theta_j) + f_{bg} * \mathcal{W}^U(S_{bg}, \Phi_j, \phi_j, \cos\Theta_j)]$

Unbinned Maximum Likelihood Method

- No background corrections

  $\ln \mathcal{L} = \sum_i^I \ln[\mathcal{W}(\mathcal{R}, (P_b)_i, (P_T)_i, \Phi_i, \Psi_i, \theta_i, \varphi_i))/N_i]$,

  $N_i = K_1 + K_2(P_b)_i + K_3(P_T)_i + K_4(P_b)_i(P_T)_i$

  $(P_b)_i$ beam polarization, $(P_T)_i$ target polarization for i-th event,
  $\mathcal{R}$ set of amplitude ratios.

  $N_{++} = \frac{1}{L}\sum_{m=1}^{L} \mathcal{W}(\mathcal{R}, (P_b = 1), (P_T = 1), \Phi_m, \Psi_m, \theta_m, \varphi_m)$

  N_{+-} corresponds to $P_b = 1$, $P_T = -1$, N_{-+} to $P_b = -1$, $P_T = 1$ etc.
  K_1, K_2, K_3, and K_4 are linear combinations of N_{++}, N_{+-}, N_{-+}, and N_{--}.

- Likelihood function with background corrections

  $\ln \mathcal{L}_{tot} = \sum_i^I \ln\Big[(1 - f_{bg})\frac{\mathcal{W}(\mathcal{R},(P_b)_i,(P_T)_i,\Phi_i,\Psi_i,\theta_i,\varphi_i)}{N_i} + f_{bg}\frac{\mathcal{W}_{bg}((P_b)_i,(P_T)_i,\Phi_i,\Psi_i,\theta_i,\varphi_i)}{N_i^{bg}}\Big]$

  Angular distribution, $\mathcal{W}_{bg}$ of background events is assumed to be independent of polarizations P_b and P_T, f_{bg} fraction of reconstructed background events.

  $N_i^{bg} = \frac{1}{L}\sum_{m=1}^{L} \mathcal{W}_{bg}((P_b = 0), (P_T = 0), \Phi_m, \Psi_m, \theta_m, \varphi_m)$

  $\ln \mathcal{L}_{tot} = \sum_i^I \ln\Big[\frac{(1-g_{bg})\mathcal{W}(\mathcal{R},(P_b)_i,(P_T)_i,\Phi_i,\Psi_i,\theta_i,\varphi_i)N_i + g_{bg}\mathcal{W}_{bg}((P_b)_i,(P_T)_i,\Phi_i,\Psi_i,\theta_i,\varphi_i)}{(1-g_{bg})N_i + g_{bg}N_i^{bg}}\Big]$

  g_{bg} is the fraction of background in 4π (before interaction of particles with detector)

Phenomenological description of reaction $e + N \to e' + V + N'$

- First process $e \to e + \gamma^*$ (QED). Spin-density matrix of virtual photon $\rho_{\lambda_\gamma \lambda'_\gamma}$.
- Second process $\gamma^* + N \to V + N$ (QCD). Helicity amplitudes $F_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N}$.

  Spin-Density Matrix of V: $r = \frac{F \rho F^+}{\mathcal{N}}$. $\rho = \sum \Big(c_\alpha \Sigma^\alpha \Big)$, $r^\alpha = \frac{F \Sigma^\alpha F^+}{\mathcal{N}}$, $r = \sum c_\alpha r^\alpha$.
- Third process. Examples: i) $V = \omega$; $\omega \to \pi^+ + \pi^- + \pi^0$ $(\to \gamma + \gamma)$.
  ω: $J^P = 1^-$, $P(3\pi) = -1$, orbital motion 1^+: $Y_{1\lambda_\omega}(\vec{n})$, $\vec{n}$ is a unit normal to ω-decay plane;
  ii) $V = \rho$; $J^P = 1^-$, $P(2\pi) = +1$, $Y_{1\lambda_\rho}(\vec{n})$, $\vec{n} = \vec{p}_{\pi^+}/|\vec{p}_{\pi^+}|$.
- Spin-Density Matrix Elements (SDMEs) $r_{\lambda_V \lambda'_V}$ of the vector meson are extracted from the angular distribution of final particles
  $\mathcal{W}(\Phi, \Theta, \phi) = \sum_{\lambda_V, \lambda'_V} Y_{1\lambda_V}(\vec{n}) r_{\lambda_V \lambda'_V} Y^*_{1\lambda'_V}(\vec{n}) =$
  $\sum_{\lambda_V, \lambda'_V} Y_{1\lambda_V}(\Theta, \phi) r_{\lambda_V \lambda'_V}(\Phi) Y^*_{1\lambda'_V}(\Theta, \phi)$.
- Quantities (SDMEs) $r^\alpha_{\lambda_V \lambda'_V}$ can be calculated from the relation
  $r^\alpha_{\lambda_V \lambda'_V} = \frac{1}{2\mathcal{N}_\alpha} \sum F_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N} \, \Sigma^\alpha_{\lambda_\gamma \lambda'_\gamma} \, F^*_{\lambda'_V \lambda'_N \lambda'_\gamma \lambda_N}$,
  where Σ^α is a set of nine hermitian matrixes:

$$\Sigma^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \Sigma^1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \Sigma^5 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \; \dots .$$