EVOLUTION OF NUCLEAR PROPERTIES IN THE LONG CHAINS OF ISOTOPES

V. I. Isakov

Petersburg Nuclear Physics Institute, NRC Kurchatov Institute, 188300 Gatchina, Russia

Study of evolution of nuclear properties in the long chains of isotopes, or isotones, is of special interest as here one can check the adequacy of theoretical models in the broad interval of $(N-Z)/A$. Previously, we studied in details the chain of $N=82$ isotones. By now there also exists considerable experimental information on different properties of Sn isotopes, from nuclei close to the doubly-magic and ultimate neutron-deficient ^{100}Sn, and right up to the neutron-excess Sn isotopes which are close to the also doubly-magic ^{132}Sn. There also exists some experimental information on the remote nuclei close to ^{48}Ni (proton excess) and ^{78}Ni (ultimately neutron-excess), which are also the presumably doubly-magical ones. These successions of isotopes are considered by us here. We offer results of calculations relating both to ground and excited states of Sn and Ni isotopes using more-or-less standard methods of calculations and assuming spherical form of the isotopes considered by us below.

HF+BCS with Skyrme 3 + constant pairing

$$E = -B = \int H(\mathbf{r})\, d\mathbf{r} - \frac{\Delta_\nu^2}{G_\nu} - \frac{\Delta_\pi^2}{G_\pi}.$$

$$E(\mathrm{HF}+\mathrm{BCS}) \Rightarrow E(\mathrm{HF}) + \sum_{i\in\pi,\nu} \varepsilon_{j_i}(\mathrm{HF}) \left[v_{j_i}^2(\mathrm{BCS})(2j_i+1) - n_{j_i} \right] - \frac{\Delta_\pi^2}{G_\pi} - \frac{\Delta_\nu^2}{G_\nu}.$$

$$\Delta E(\mathrm{BCS}) = \sum_{i\in\pi,\nu;\, j_i \neq j_{0\pi},\, j_{0\nu}} \varepsilon_{j_i}(\mathrm{HF}) \left[v_{j_i}^2(j_0)(2j_i+1) - n_{j_i} \right] +$$

$$+ \varepsilon_{j_{0\pi}}(\mathrm{HF}) \cdot \left[v_{j_{0\pi}}^2(j_{0\pi})(2j_{0\pi}-1) + 1 - n_{j_{0\pi}} \right] + \varepsilon_{j_{0\nu}}(\mathrm{HF}) \left[v_{j_{0\nu}}^2(j_{0\nu})(2j_{0\nu}-1) + 1 - n_{j_{0\nu}} \right] -$$

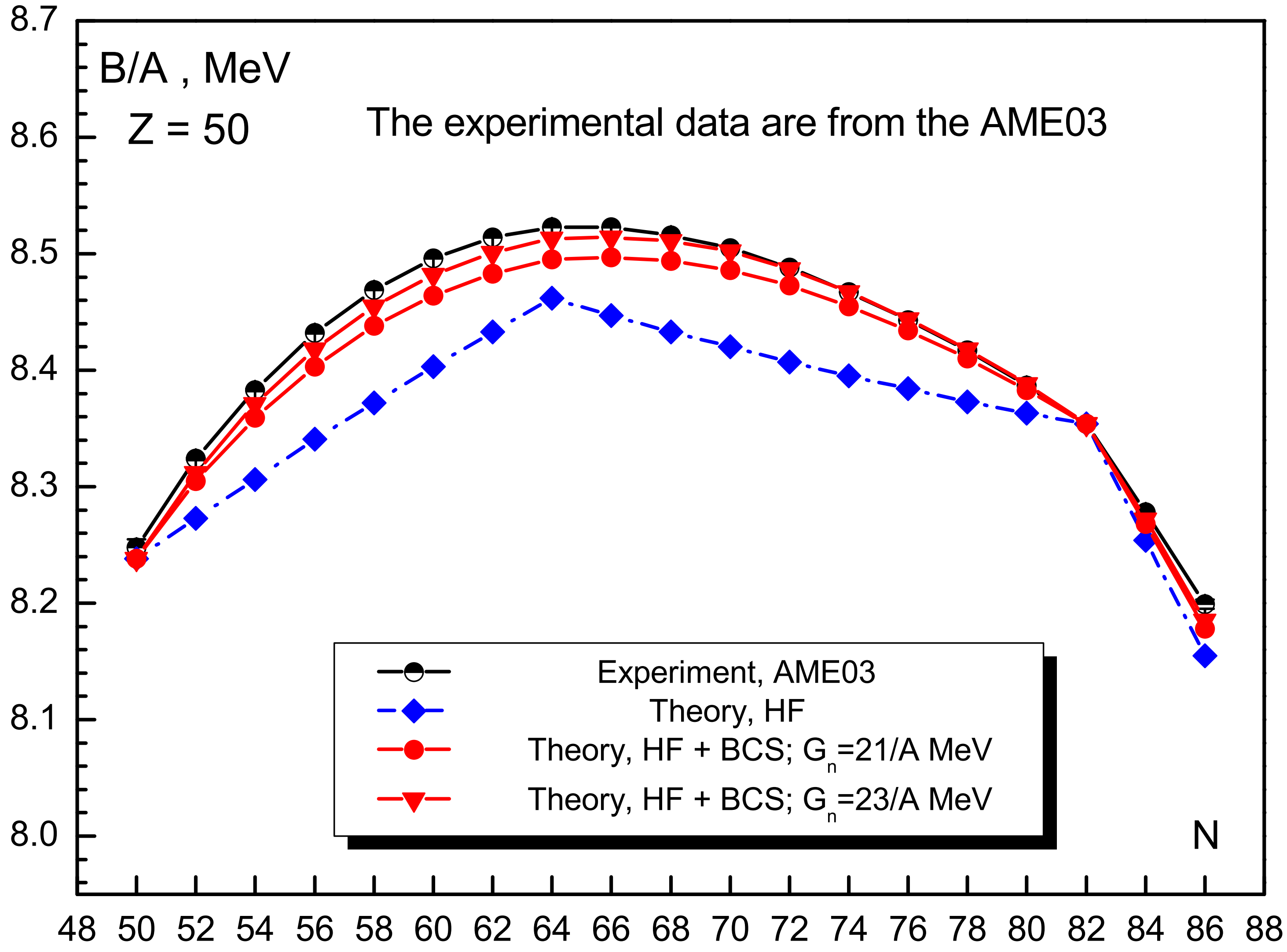
$$- \frac{\Delta_\pi(j_{0\pi})^2}{G_\pi} - \frac{\Delta_\nu(j_{0\nu})^2}{G_\nu}.$$

Phenomenological mean field potential

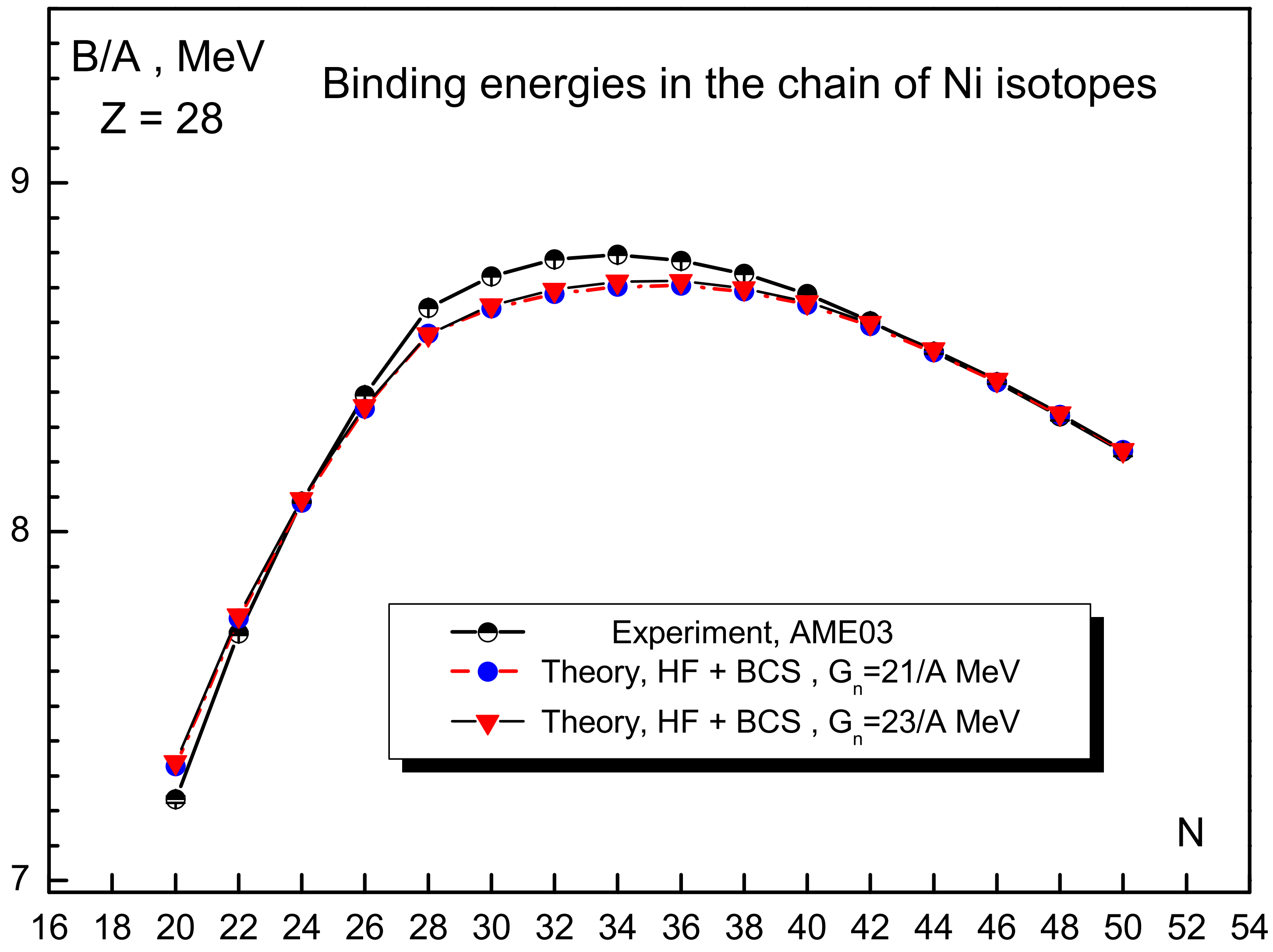
$$V(\mathbf{r},\sigma) = U \cdot f(r) + U_{ls} \cdot \frac{1}{r}\frac{df}{dr} \mathbf{l}\cdot\mathbf{s}, \quad f(r) = \frac{1}{1+\exp((r-R)/a)}.$$

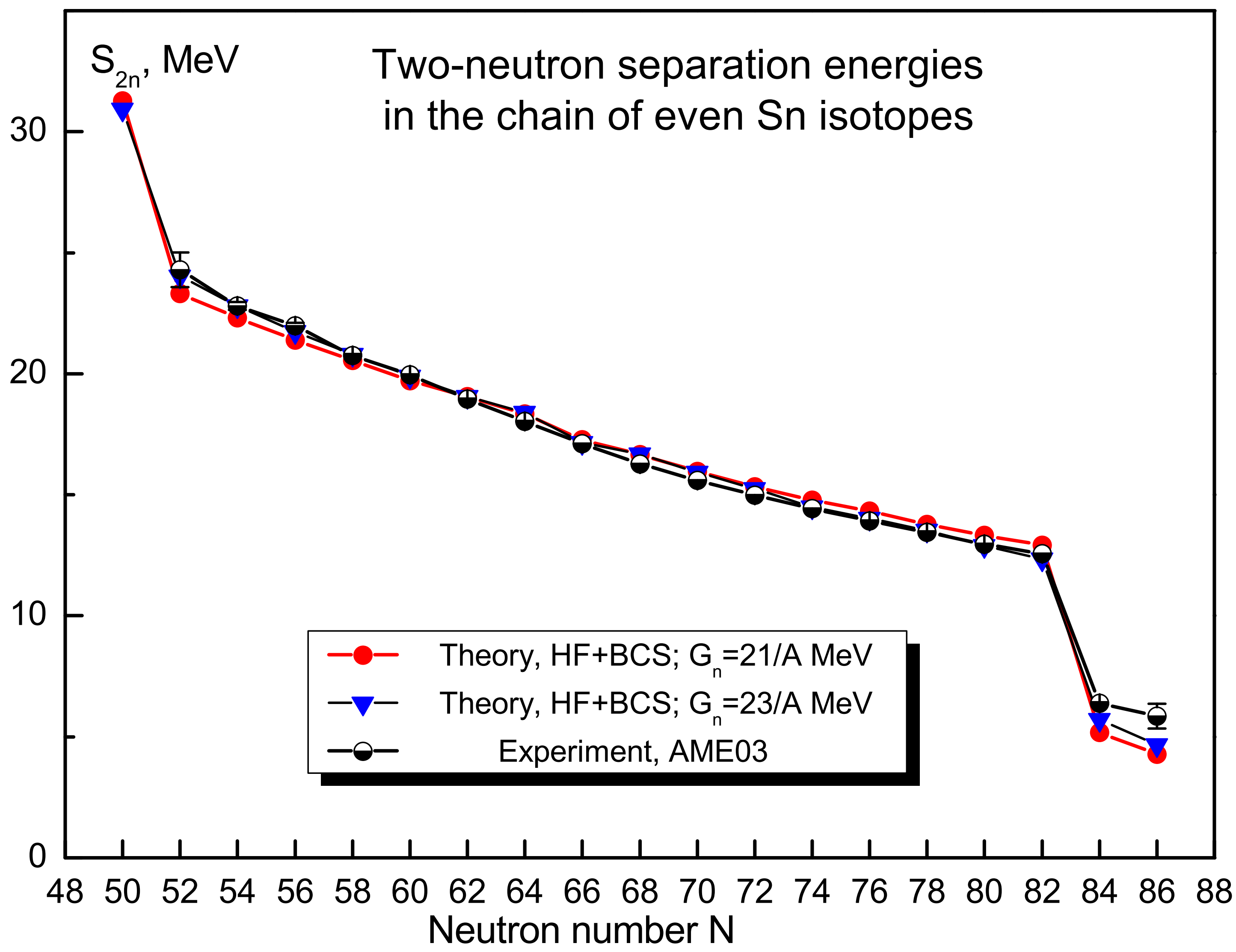
$$U = V_0 \left(1 - \beta \frac{N-Z}{A} \cdot t_z \right), U_{ls} = V_{ls} \left(1 - \beta_{ls} \frac{N-Z}{A} \cdot t_z \right)$$

$R = r_0 A^{1/3}$, t_z=1/2 for neutrons and $t_z = -1/2$ for protons. In the case of protons we added the potential of a uniformly charged sphere with $R_c = r_c A^{1/3}$. Parameters of the phenomenological potential were defined by us in the paper by V.I. Isakov, *et al.*, EPJ A**14** (2002) and they are $V_0 = -51.0, V_{ls} = 32.0$ MeV, $a_p = a_n = 0.6, r_0 = 1.27, r_{0c} = 1.25$ Fm, $\beta = 1.31, \beta_{ls} = -0.6$.

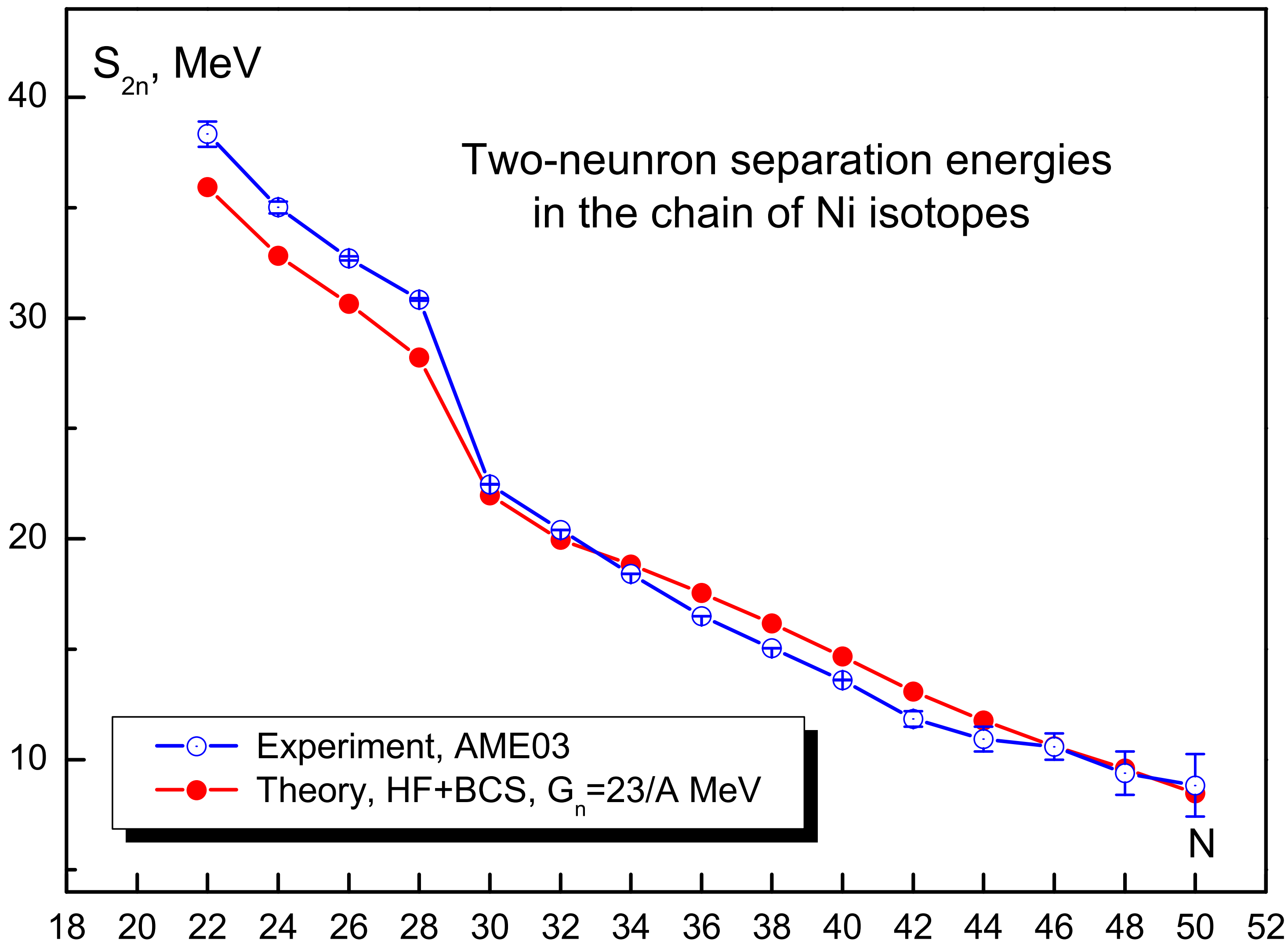


B/A , MeV
Z = 50
The experimental data are from the AME03
Experiment, AME03
Theory, HF
Theory, HF + BCS; G_n=21/A MeV
Theory, HF + BCS; G_n=23/A MeV
N
8.7
8.6
8.5
8.4
8.3
8.2
8.1
8.0
48 50 52 54 56 58 60 62 64 66 68 70 72 74 76 78 80 82 84 86 88

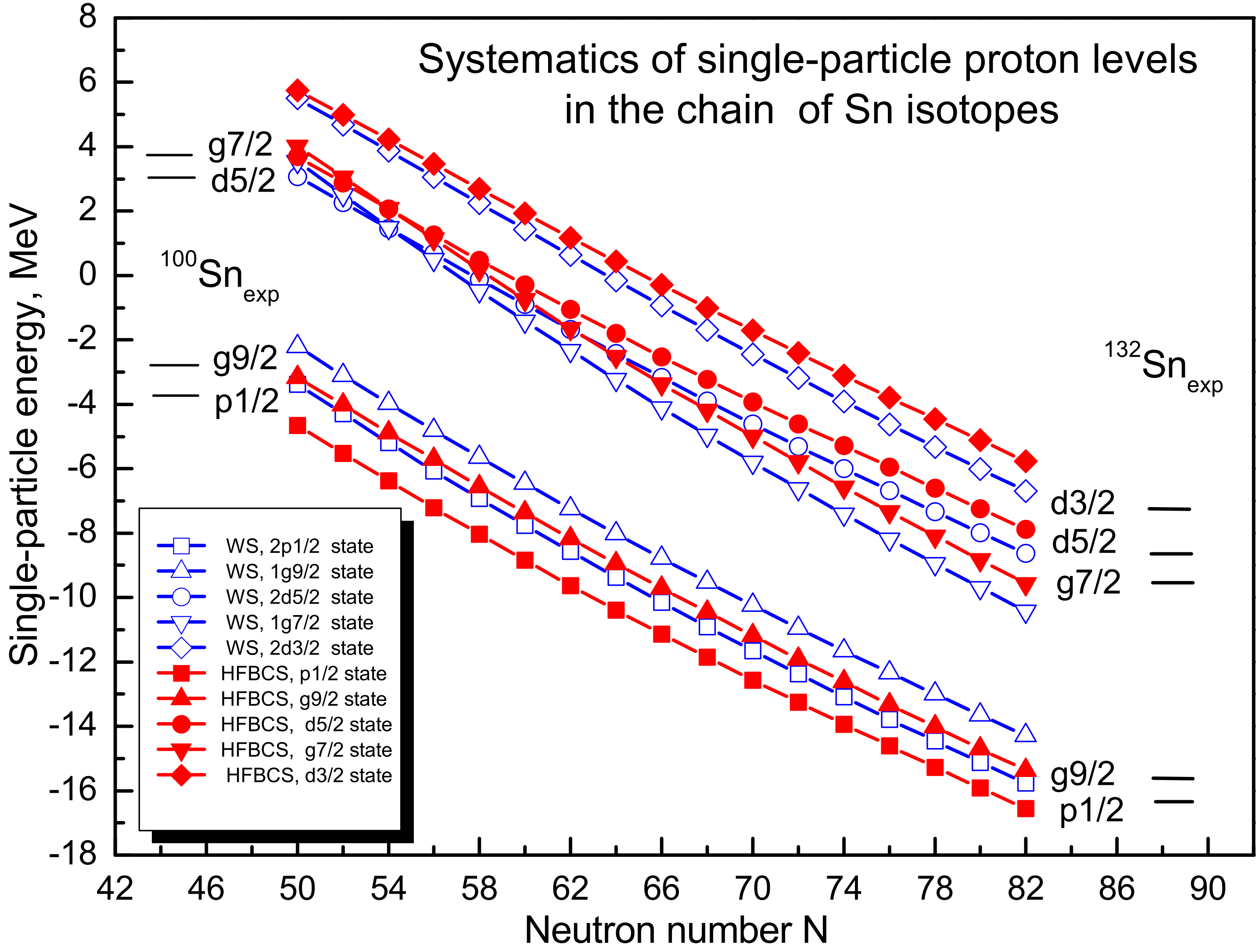

B/A , MeV
Z = 28
Binding energies in the chain of Ni isotopes
Experiment, AME03
Theory, HF + BCS , G$_n$=21/A MeV
Theory, HF + BCS , G$_n$=23/A MeV
N
9
8
7
16
18
20
22
24
26
28
30
32
34
36
38
40
42
44
46
48
50
52
54

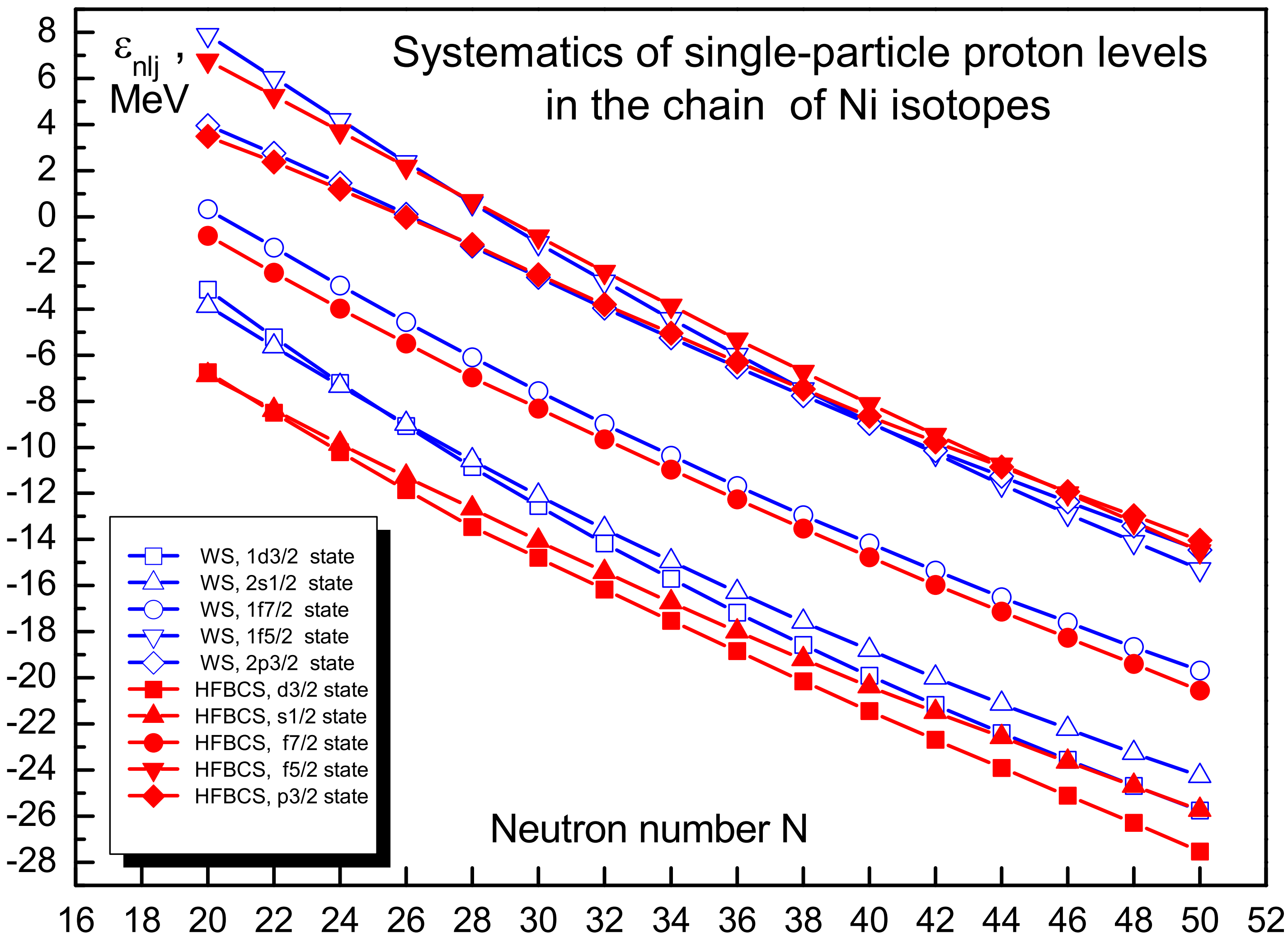


S_{2n}, MeV
Two-neutron separation energies
in the chain of even Sn isotopes
Theory, HF+BCS; G_n=21/A MeV
Theory, HF+BCS; G_n=23/A MeV
Experiment, AME03
0
10
20
30
48
50
52
54
56
58
60
62
64
66
68
70
72
74
76
78
80
82
84
86
88
Neutron number N

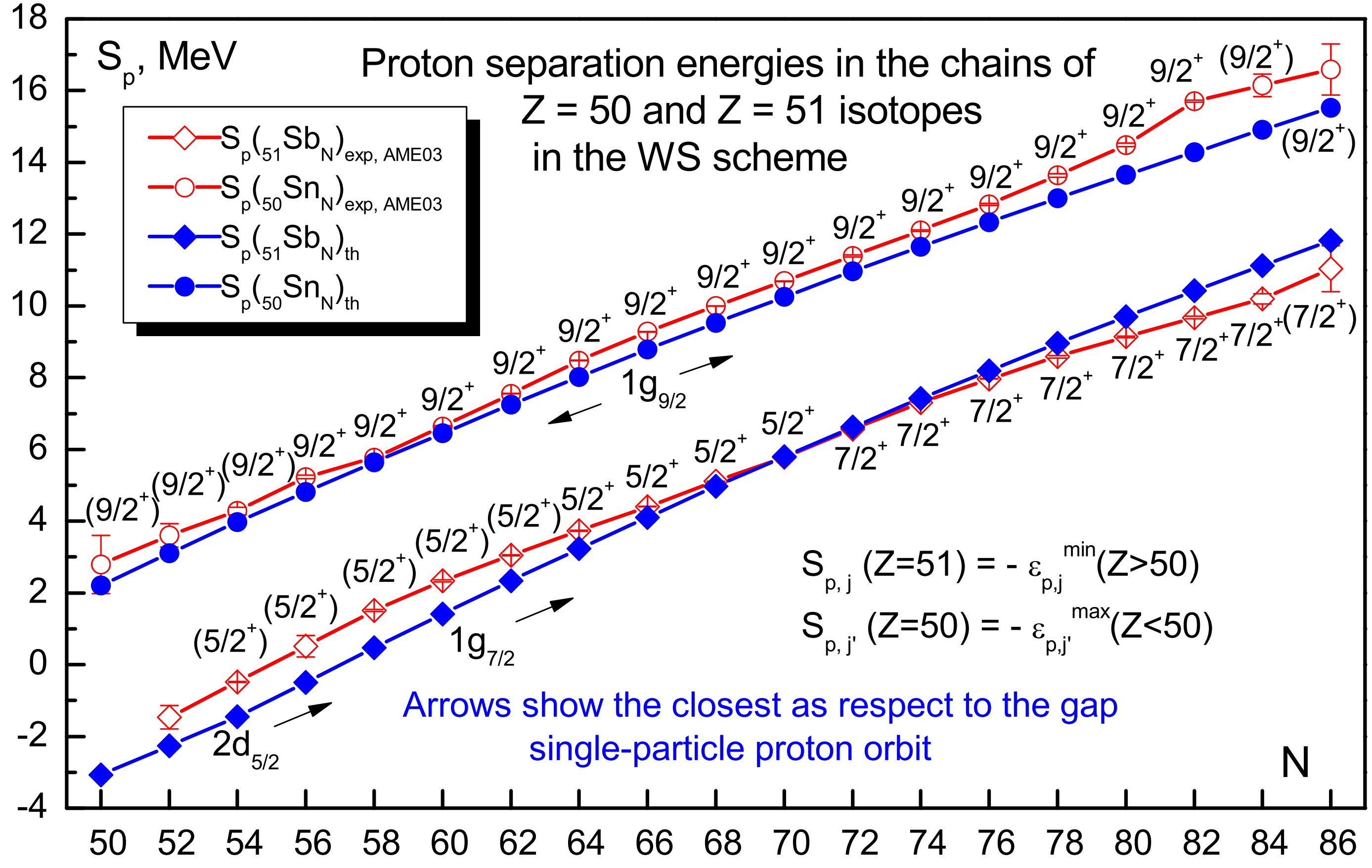


S2n, MeV
Two-neunron separation energies
in the chain of Ni isotopes
Experiment, AME03
Theory, HF+BCS, Gn=23/A MeV
N
40
30
20
10
18
20
22
24
26
28
30
32
34
36
38
40
42
44
46
48
50
52

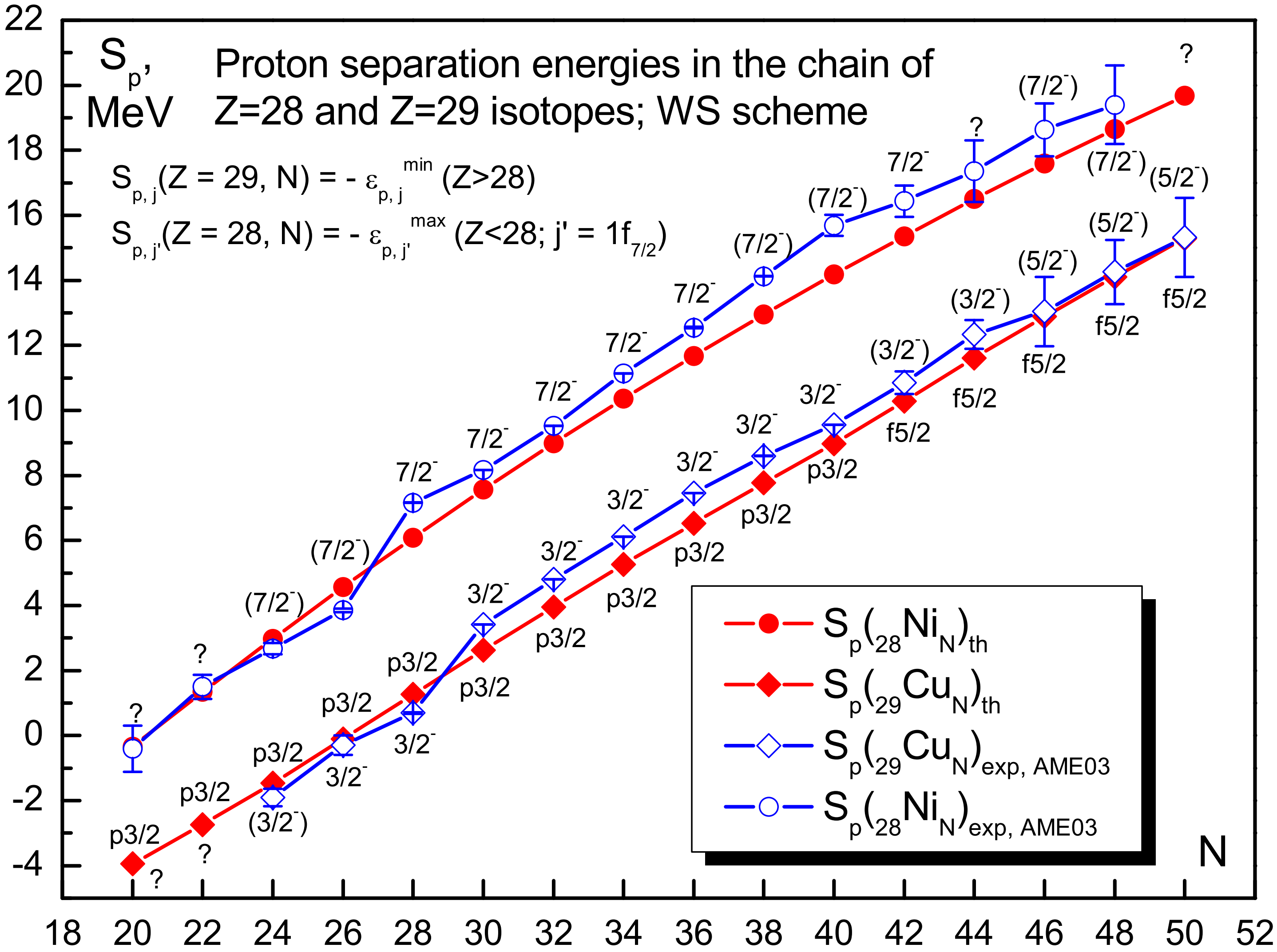

Systematics of single-particle proton levels
in the chain of Sn isotopes
Single-particle energy, MeV
Neutron number N
$^{100}Sn_{exp}$
g7/2
d5/2
g9/2
p1/2
$^{132}Sn_{exp}$
d3/2
d5/2
g7/2
g9/2
p1/2
WS, 2p1/2 state
WS, 1g9/2 state
WS, 2d5/2 state
WS, 1g7/2 state
WS, 2d3/2 state
HFBCS, p1/2 state
HFBCS, g9/2 state
HFBCS, d5/2 state
HFBCS, g7/2 state
HFBCS, d3/2 state



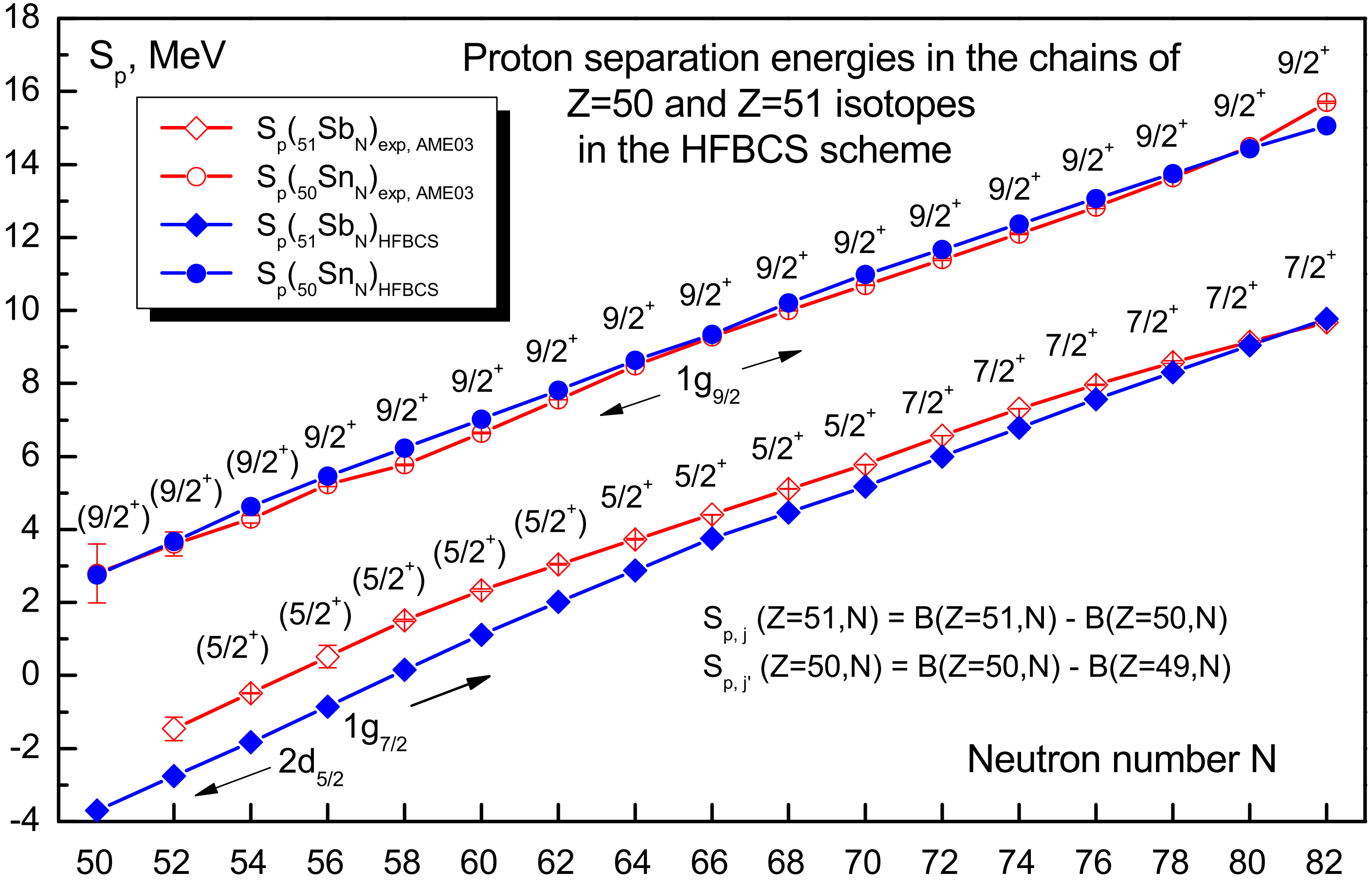
Systematics of single-particle proton levels
in the chain of Ni isotopes
ε_{nlj} ,
MeV
Neutron number N
WS, 1d3/2 state
WS, 2s1/2 state
WS, 1f7/2 state
WS, 1f5/2 state
WS, 2p3/2 state
HFBCS, d3/2 state
HFBCS, s1/2 state
HFBCS, f7/2 state
HFBCS, f5/2 state
HFBCS, p3/2 state
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
-24
-26
-28
16
18
20
22
24
26
28
30
32
34
36
38
40
42
44
46
48
50
52



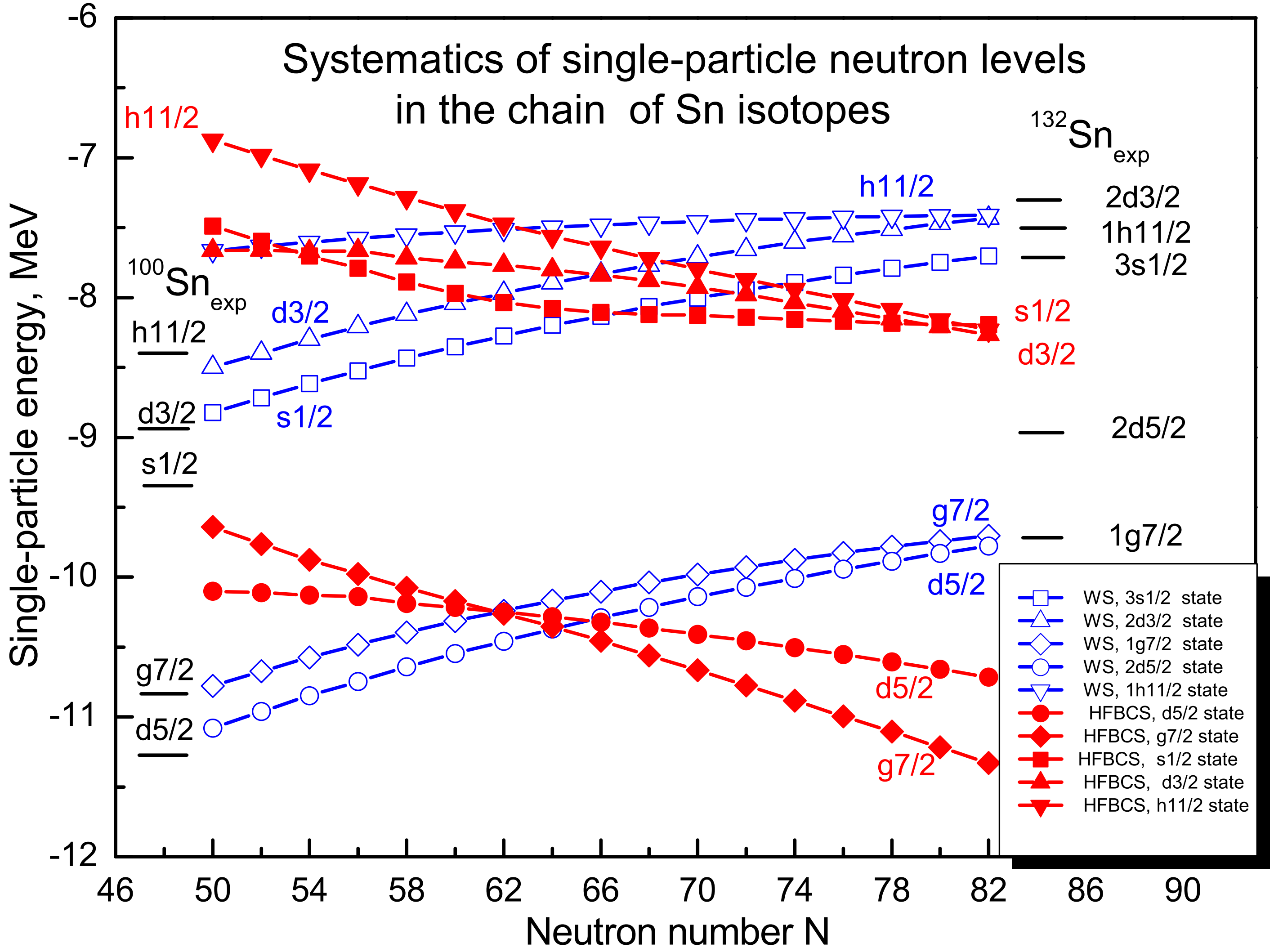
Proton separation energies in the chains of
Z = 50 and Z = 51 isotopes
in the WS scheme
S_p, MeV
S_p(51Sb_N)exp, AME03
S_p(50Sn_N)exp, AME03
S_p(51Sb_N)th
S_p(50Sn_N)th
1g9/2
1g7/2
2d5/2
S_p, j (Z=51) = - ε_p,j^min(Z>50)
S_p, j' (Z=50) = - ε_p,j'^max(Z<50)
Arrows show the closest as respect to the gap
single-particle proton orbit
N



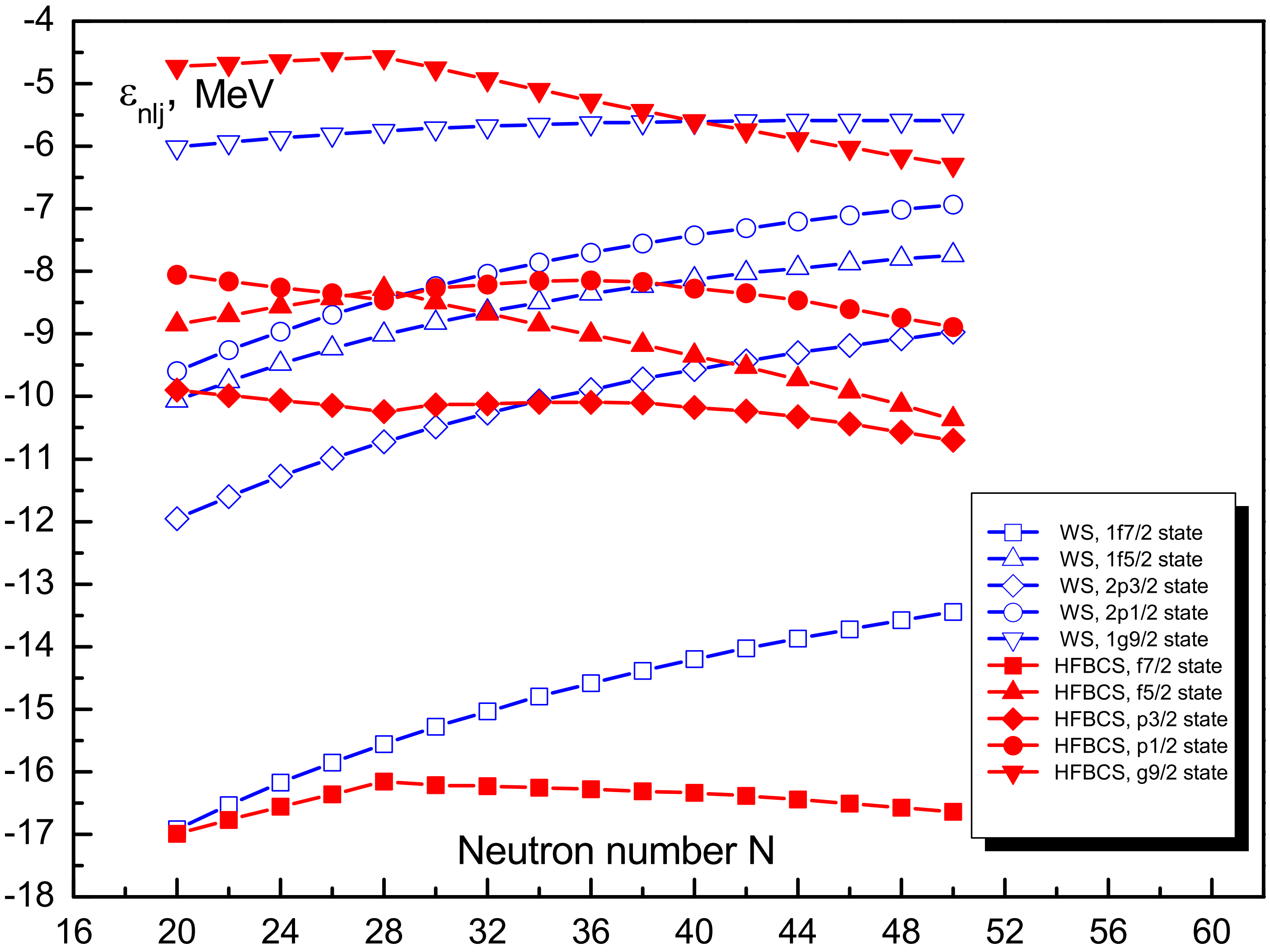
Proton separation energies in the chain of Z=28 and Z=29 isotopes; WS scheme
S_p, MeV
$S_{p,j}(Z = 29, N) = -\varepsilon_{p,j}^{min}$ (Z>28)
$S_{p,j'}(Z = 28, N) = -\varepsilon_{p,j'}^{max}$ (Z<28; j' = $1f_{7/2}$)
$S_p({}_{28}Ni_N)_{th}$
$S_p({}_{29}Cu_N)_{th}$
$S_p({}_{29}Cu_N)_{exp, AME03}$
$S_p({}_{28}Ni_N)_{exp, AME03}$
N



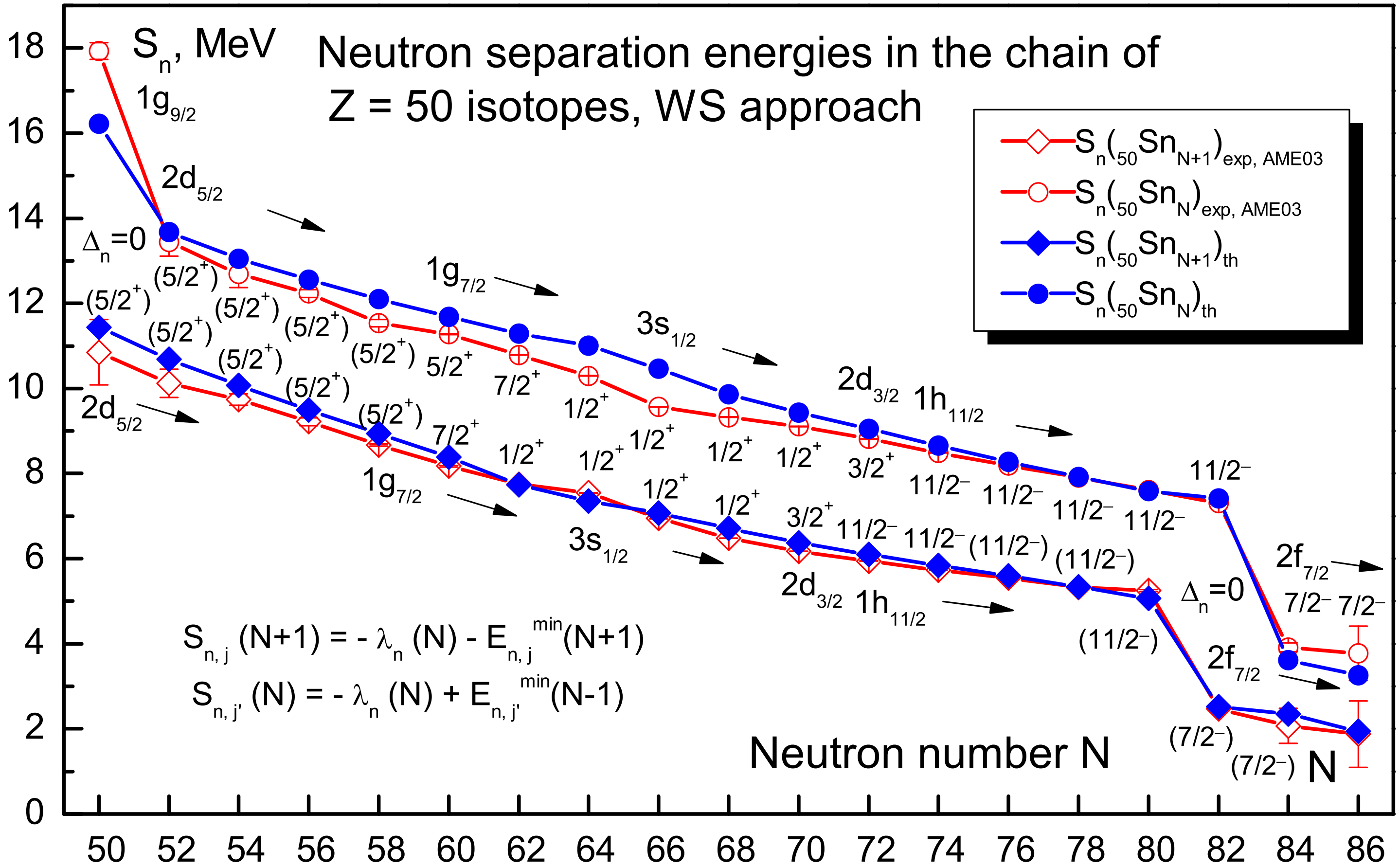
Proton separation energies in the chains of
Z=50 and Z=51 isotopes
in the HFBCS scheme
S_p, MeV
S_p(51Sb_N)exp, AME03
S_p(50Sn_N)exp, AME03
S_p(51Sb_N)HFBCS
S_p(50Sn_N)HFBCS
1g9/2
1g7/2
2d5/2
S_p, j (Z=51,N) = B(Z=51,N) - B(Z=50,N)
S_p, j' (Z=50,N) = B(Z=50,N) - B(Z=49,N)
Neutron number N



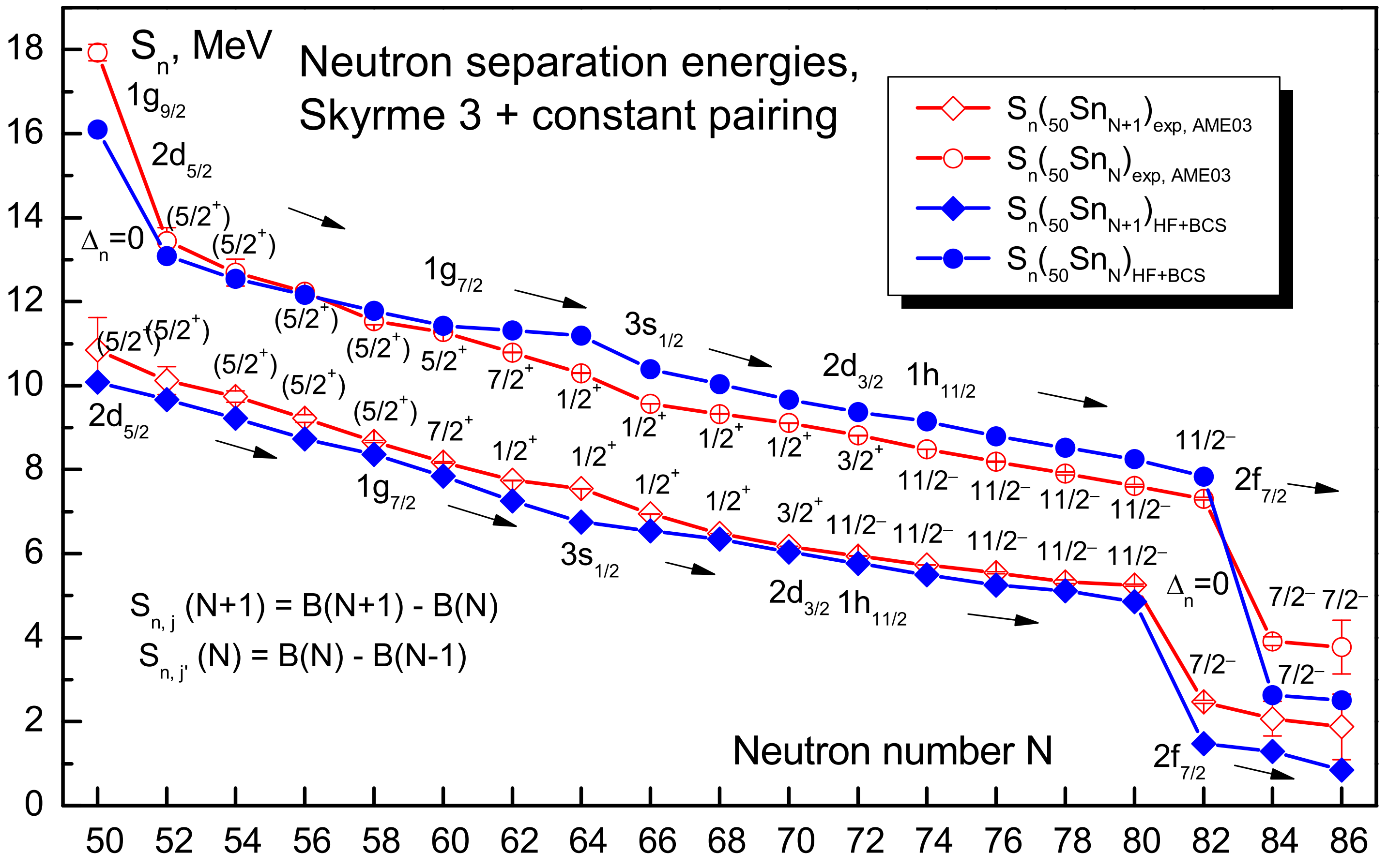
Systematics of single-particle neutron levels
in the chain of Sn isotopes
Single-particle energy, MeV
Neutron number N
-6
-7
-8
-9
-10
-11
-12
46
50
54
58
62
66
70
74
78
82
86
90
h11/2
^{100}Sn$_{exp}$
h11/2
d3/2
s1/2
g7/2
d5/2
d3/2
s1/2
^{132}Sn$_{exp}$
2d3/2
1h11/2
3s1/2
2d5/2
1g7/2
h11/2
s1/2
d3/2
g7/2
d5/2
d5/2
g7/2
WS, 3s1/2 state
WS, 2d3/2 state
WS, 1g7/2 state
WS, 2d5/2 state
WS, 1h11/2 state
HFBCS, d5/2 state
HFBCS, g7/2 state
HFBCS, s1/2 state
HFBCS, d3/2 state
HFBCS, h11/2 state



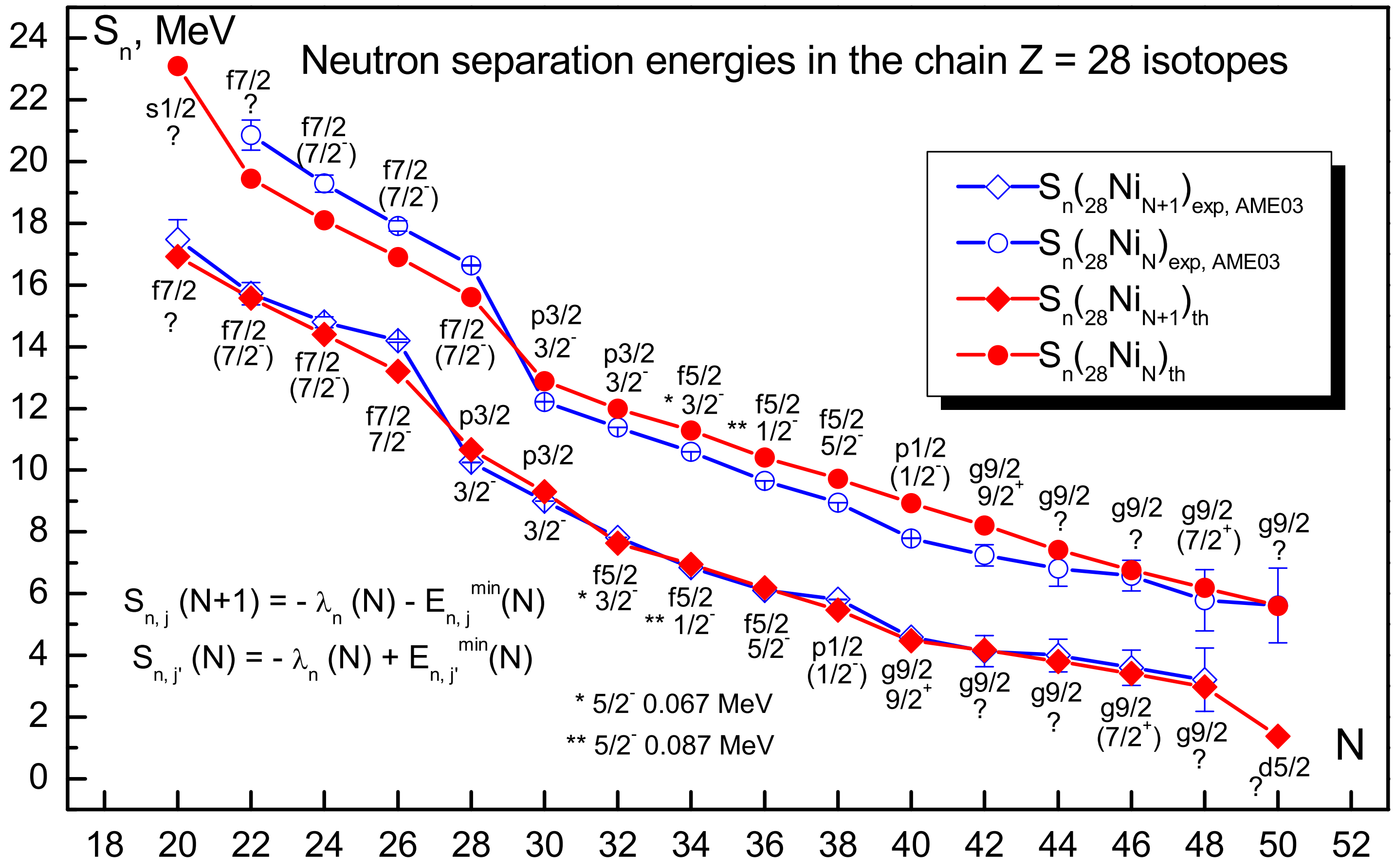
Systematics of single-particle neutron levels in the chain of Ni isotopes



Neutron separation energies in the chain of
Z = 50 isotopes, WS approach
S_n, MeV
Neutron number N
$S_n({}_{50}Sn_{N+1})_{exp, AME03}$
$S_n({}_{50}Sn_{N})_{exp, AME03}$
$S_n({}_{50}Sn_{N+1})_{th}$
$S_n({}_{50}Sn_{N})_{th}$
$S_{n,j}(N+1) = -\lambda_n(N) - E_{n,j}^{min}(N+1)$
$S_{n,j'}(N) = -\lambda_n(N) + E_{n,j'}^{min}(N-1)$



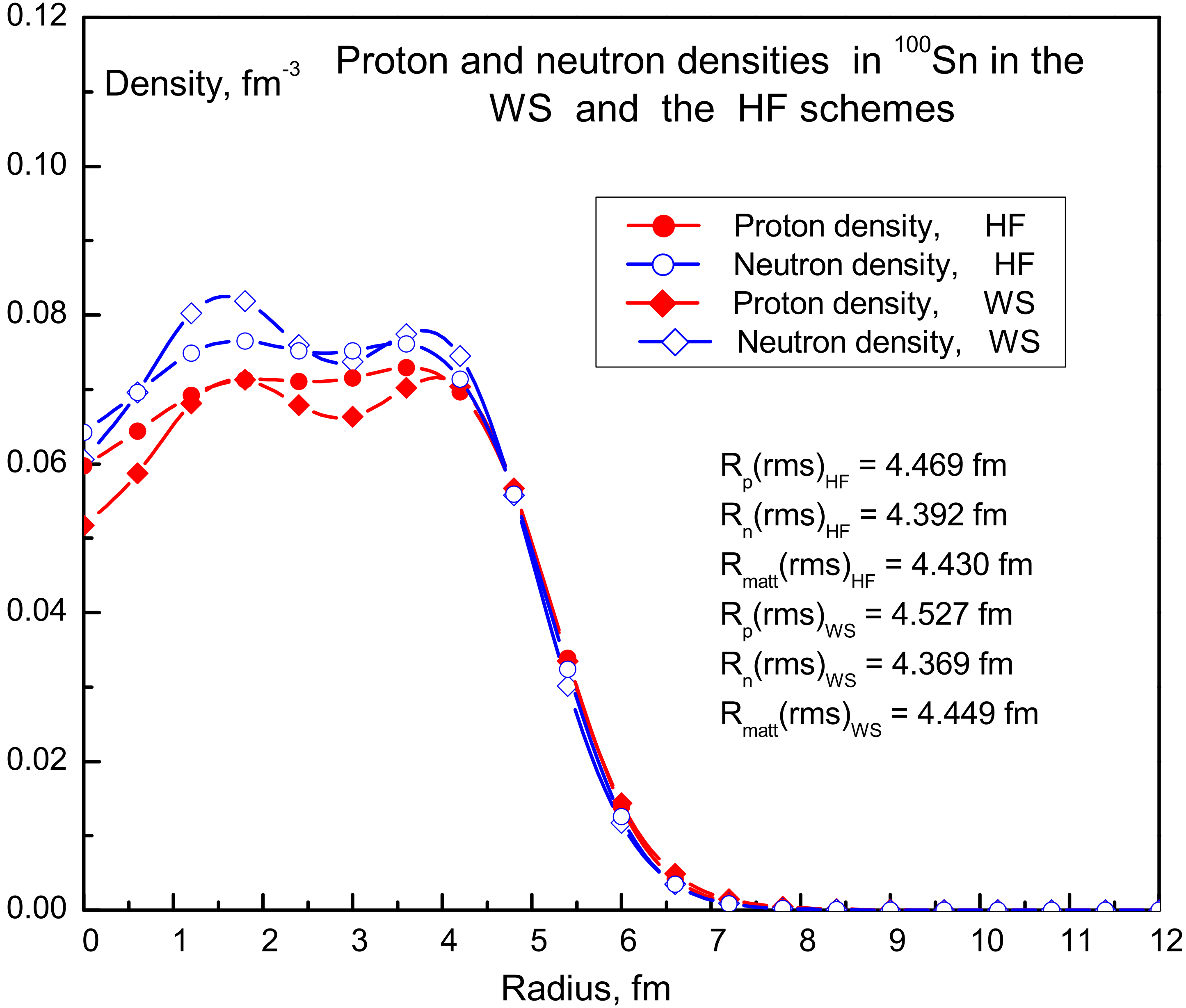
Neutron separation energies,
Skyrme 3 + constant pairing
S_n, MeV
Neutron number N
S_n(50Sn_N+1)exp, AME03
S_n(50Sn_N)exp, AME03
S_n(50Sn_N+1)HF+BCS
S_n(50Sn_N)HF+BCS
S_n, j (N+1) = B(N+1) - B(N)
S_n, j' (N) = B(N) - B(N-1)
Δ_n=0
1g9/2
2d5/2
1g7/2
3s1/2
2d3/2
1h11/2
2f7/2



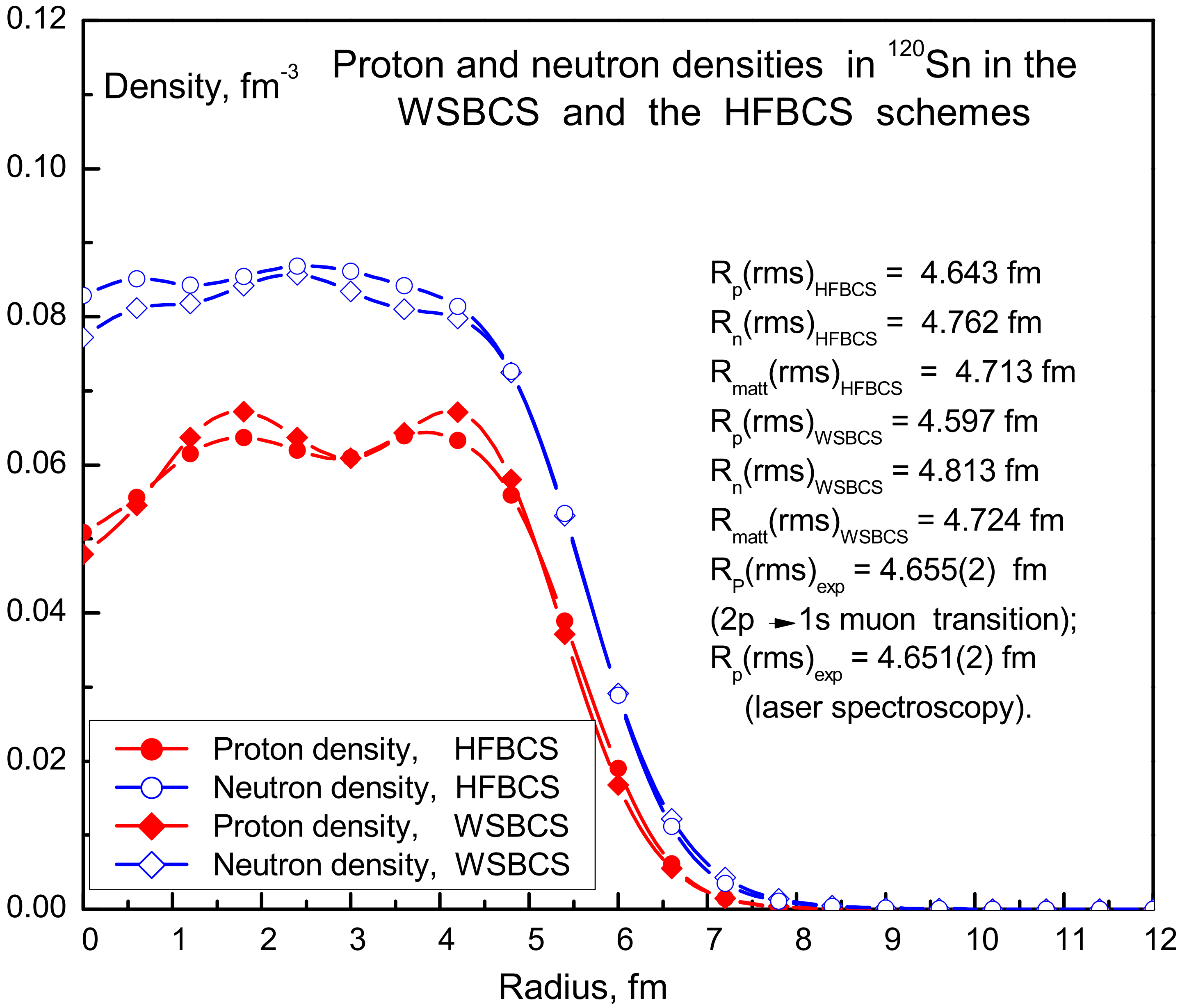
Neutron separation energies in the chain Z = 28 isotopes
S_n, MeV
N
S_n(28Ni_N+1)exp, AME03
S_n(28Ni_N)exp, AME03
S_n(28Ni_N+1)th
S_n(28Ni_N)th
S_n,j (N+1) = - λ_n (N) - E_n,j^min(N)
S_n,j' (N) = - λ_n (N) + E_n,j'^min(N)
* 5/2⁻ 0.067 MeV
** 5/2⁻ 0.087 MeV

Table 1. Root-mean-square radii R_p and R_n of the proton and the neutron distributions in the even isotopes of Sn, in the units of fm. Calculations are performed by using the HFBCS method with the Skyrme 3 interaction and constant pairing having the standard value of the pairing constant $G_n = 21/A$ MeV. Results of calculations performed by using the modified mean-field spin-orbital term are shown in square brackets.

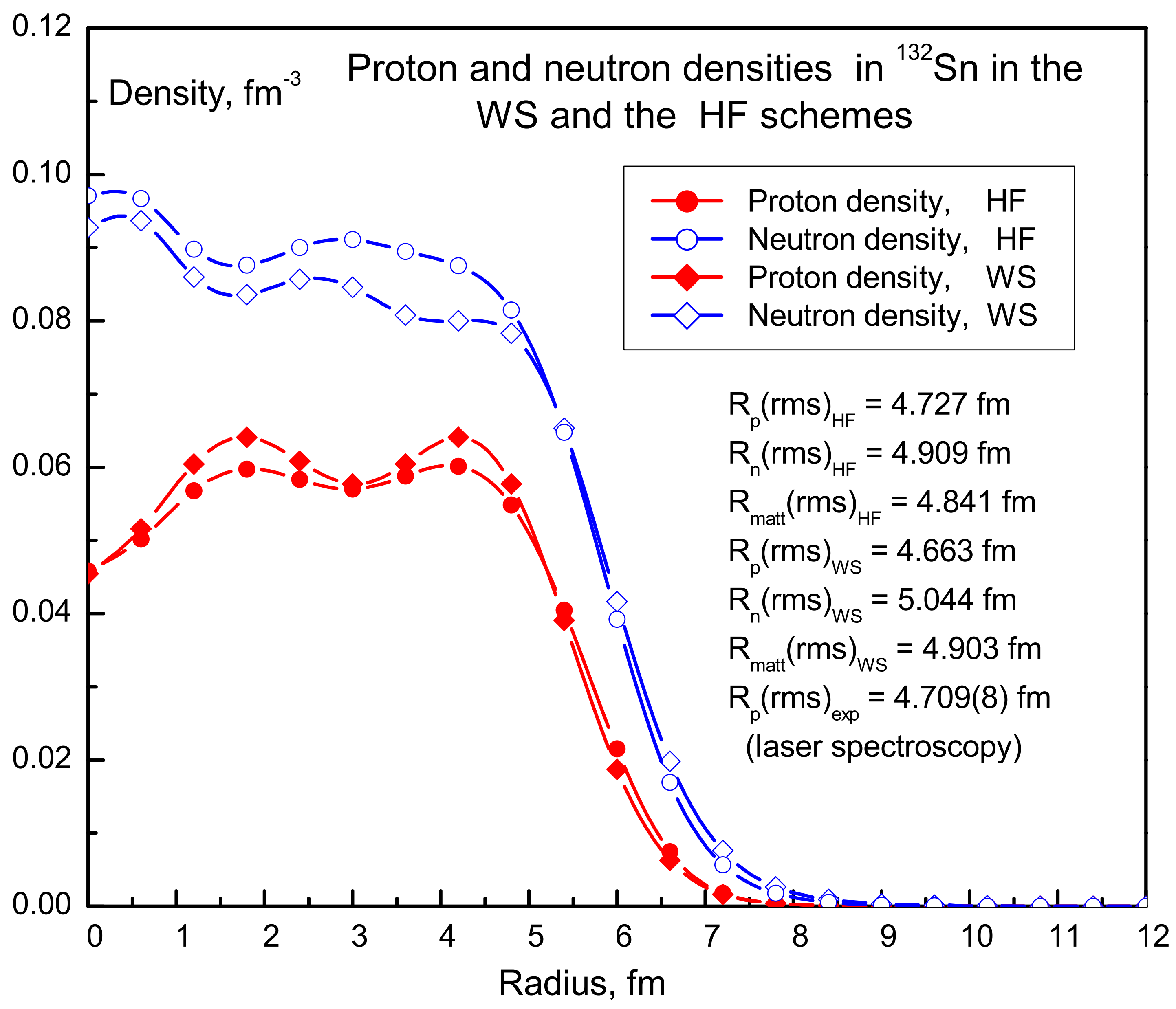
N	$R(p)_{th}$	$R(p)_{exp}$	$R(n)_{th}$	N	$R(p)_{th}$	$R(p)_{exp}$	$R(n)_{th}$
50	4.464 [4.469]	–	4.387 [4.392]	68	4.624 [4.628]	4.639(2)	4.728 [4.730]
52	4.484 [4.489]	–	4.434 [4.439]	70	4.639 [4.643]	4.651(2)	4.760 [4.762]
54	4.503 [4.508]	–	4.478 [4.482]	72	4.654 [4.658]	4.663(2)	4.791 [4.794]
56	4.521 [4.527]	–	4.519 [4.523]	74	4.668 [4.672]	4.673(3)	4.816 [4.819]
58	4.540 [4.545]	4.558(3)	4.557 [4.560]	76	4.683 [4.686]	4.683(4)	4.840 [4.844]
60	4.558 [4.563]	4.577(3)	4.594 [4.596]	78	4.696 [4.700]	4.692 (6)	4.863 [4.867]
62	4.575 [4.580]	4.594(2)	4.628 [4.630]	80	4.710 [4.714]	4.702(7)	4.884 [4.889]
64	4.592 [4.596]	4.609(2)	4.662 [4.664]	82	4.724 [4.727]	4.709(8)	4.905 [4.909]
66	4.609 [4.612]	4.624(2)	4.696 [4.697]	84	4.739 [4.742]	–	4.962 [4.950]



Proton and neutron densities in ^{100}Sn in the WS and the HF schemes
Density, fm^{-3}
Radius, fm
Proton density, HF
Neutron density, HF
Proton density, WS
Neutron density, WS
$R_p(rms)_{HF}$ = 4.469 fm
$R_n(rms)_{HF}$ = 4.392 fm
$R_{matt}(rms)_{HF}$ = 4.430 fm
$R_p(rms)_{WS}$ = 4.527 fm
$R_n(rms)_{WS}$ = 4.369 fm
$R_{matt}(rms)_{WS}$ = 4.449 fm
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0
1
2
3
4
5
6
7
8
9
10
11
12



Proton and neutron densities in 120Sn in the WSBCS and the HFBCS schemes
Density, fm-3
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0 1 2 3 4 5 6 7 8 9 10 11 12
Radius, fm
Rp(rms)HFBCS = 4.643 fm
Rn(rms)HFBCS = 4.762 fm
Rmatt(rms)HFBCS = 4.713 fm
Rp(rms)WSBCS = 4.597 fm
Rn(rms)WSBCS = 4.813 fm
Rmatt(rms)WSBCS = 4.724 fm
Rp(rms)exp = 4.655(2) fm
(2p → 1s muon transition);
Rp(rms)exp = 4.651(2) fm
(laser spectroscopy).
Proton density, HFBCS
Neutron density, HFBCS
Proton density, WSBCS
Neutron density, WSBCS



Proton and neutron densities in 132Sn in the WS and the HF schemes
Density, fm-3
Radius, fm
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0 1 2 3 4 5 6 7 8 9 10 11 12
Proton density, HF
Neutron density, HF
Proton density, WS
Neutron density, WS
Rp(rms)HF = 4.727 fm
Rn(rms)HF = 4.909 fm
Rmatt(rms)HF = 4.841 fm
Rp(rms)WS = 4.663 fm
Rn(rms)WS = 5.044 fm
Rmatt(rms)WS = 4.903 fm
Rp(rms)exp = 4.709(8) fm
(laser spectroscopy)

For the description of excited states and transition rates we used the QRPA approach with phenomenological mean field potential and effective finite range interaction, the same in the particle-particle, particle-hole and pairing channels

We represent the effective interaction $\hat{\vartheta}$ in the form

$$\hat{\vartheta} = (V + V_\sigma \vec{\sigma}_1 \vec{\sigma}_2 + V_T S_{12} + V_\tau \vec{\tau}_1 \vec{\tau}_2 + V_{\tau\sigma} \vec{\sigma}_1 \vec{\sigma}_2 \cdot \vec{\tau}_1 \vec{\tau}_2 + V_{\tau T}\, \vec{\tau}_1 \vec{\tau}_2\, S_{12}) \exp(-r_{12}^2/r_{00}^2) + \\ + \frac{e^2}{r_{12}} \left(\frac{1}{2} - \hat{t}_z(1) \right) \left(\frac{1}{2} - \hat{t}_z(2) \right),$$

We used the following values of the entering parameters: $V = -16.65$, $V_\sigma = 2.33$, $V_T = -3.00$, $V_\tau = 3.35$, $V_{\tau\sigma} = 4.33$, $V_{\tau T} = 3.00$ (all these values are in MeV), while $r_{00} = 1.75$ fm. For a system of only like particles this interaction coincides with the interaction introduced by K. Heyde and M. Waroquier, Nucl.Phys. A**167** (1971).

Using the standard procedure, we can pass to the quasiparticle basis, $a^+ \longrightarrow \xi^+$:

$$a_\alpha^+ = u_{|\alpha|} \xi_\alpha^+ - v_{|\alpha|} \varphi_\alpha \xi_{-\alpha}; \quad u_{|\alpha|}^2 + v_{|\alpha|}^2 = 1\,.$$

Supposing the presence of correlations in the true ground state $|\tilde{0}\rangle$ of an even–even nuclei, we define the creation operator $Q^+_{n,JM}$ of the one-phonon excited state $|\omega_n, JM\rangle$ with $|\omega_n, JM\rangle = Q^+_{n,JM}|\tilde{0}\rangle$ in the following way:

$$Q^+_{n,JM} = \sum_{a\geq b} X^{n,J}_{j_a j_b} \left[\xi^+_a \xi^+_b\right]_{JM} - \sum_{c\geq d} Y^{n,J}_{j_c j_d} \left[\xi_c \xi_d\right]_{JM} \ ,$$

where

$$\left[\xi^+_a \xi^+_b\right]_{JM} = \frac{1}{\sqrt{1+\delta_{j_a j_b}}} \sum_{m_a m_b} C^{JM}_{j_a m_a j_b m_b} \xi^+_{j_a m_a} \xi^+_{j_b m_b} \, ,$$

$$\left[\xi_c \xi_d\right]_{JM} = \frac{1}{\sqrt{1+\delta_{j_c j_d}}} \sum_{m_c m_d} C^{JM}_{j_c m_c j_d m_d} \xi_{j_c - m_c} \xi_{j_d - m_d} \cdot \varphi_c \varphi_d \ ,$$

$$X^{n,J}_{j_a j_b} = \langle \omega_n; JM| \left[\xi^+_a \xi^+_b\right]_{JM} |\tilde{0}\rangle, \ Y^{n,J}_{j_c j_d} = \langle \omega_n; JM| \left[\xi_c \xi_d\right]_{JM} |\tilde{0}\rangle \, .$$

One may obtain the set of the QRPA equations which define the amplitudes "X" and "Y" of the states $|\omega_n, JM\rangle$ and the eigenvalues ω_n. These equations have the form

$$\left\| \begin{array}{cc} [(E-\omega)I + A] & B \\ -B & -[(E+\omega)I + A] \end{array} \right\| \times \left(\begin{array}{c} X \\ Y \end{array} \right) = 0 \, .$$

Here, $E = E_{ab} = E_{j_a} + E_{j_b}$, $\; I_{cd,ab} = \delta_{j_a j_c}\delta_{j_b j_d}$, while the matrix elements of the sub-matrices A and B in the case of even–even nuclei are as follows:

$$
\begin{aligned}
A_{cd,ab} \equiv A^J_{j_c j_d, j_a j_b} = & \left(u_{j_c} u_{j_d} u_{j_a} u_{j_b} + v_{j_c} v_{j_d} v_{j_a} v_{j_b} \right) {}_a\langle j_c j_d; J | \hat{\vartheta} | j_a j_b; J \rangle_a + \\
& + \left(u_{j_c} v_{j_d} u_{j_a} v_{j_b} + v_{j_c} u_{j_d} v_{j_a} u_{j_b} \right) {}_a\langle j_c \bar{j}_d; J | \hat{\vartheta} | j_a \bar{j}_b; J \rangle_a + \\
& + (-1)^{j_a + j_b + J + 1} \left(v_{j_c} u_{j_d} u_{j_a} v_{j_b} + u_{j_c} v_{j_d} v_{j_a} u_{j_b} \right) {}_a\langle j_c \bar{j}_d; J | \hat{\vartheta} | j_b \bar{j}_a; J \rangle_a \, ;
\end{aligned}
$$

$$
\begin{aligned}
B_{cd,ab} \equiv B^J_{j_c j_d, j_a j_b} = & \left(u_{j_c} u_{j_d} v_{j_a} v_{j_b} + v_{j_c} v_{j_d} u_{j_a} u_{j_b} \right) {}_a\langle j_c j_d; J | \hat{\vartheta} | j_a j_b; J \rangle_a - \\
& - \left(u_{j_c} v_{j_d} v_{j_a} u_{j_b} + v_{j_c} u_{j_d} u_{j_a} v_{j_b} \right) {}_a\langle j_c \bar{j}_d; J | \hat{\vartheta} | j_a \bar{j}_b; J \rangle_a + \\
& + (-1)^{j_a + j_b + J} \left(v_{j_c} u_{j_d} v_{j_a} u_{j_b} + u_{j_c} v_{j_d} u_{j_a} v_{j_b} \right) {}_a\langle j_c \bar{j}_d; J | \hat{\vartheta} | j_b \bar{j}_a; J \rangle_a \, .
\end{aligned}
$$

Using an explicit form of the matrix equation, one may easily obtain the ortonormalization relation

$$
\left| \sum_{a \geq b} X^{n,J}_{j_a j_b} X^{m,J}_{j_a j_b} - \sum_{c \geq d} Y^{n,J}_{j_c j_d} Y^{m,J}_{j_c j_d} \right| = \delta_{mn} \, ,
$$

which in terms of the QRPA bosons corresponds to the condition

$$
\langle \omega_n, JM | \, \omega_m, JM \rangle = \langle \tilde{0} | Q_{n,JM} \cdot Q^+_{m,JM} | \tilde{0} \rangle = \delta_{mn} \, .
$$

We also have ${}_a\langle j_c j_d; J|\hat{\vartheta}|j_a j_b; J\rangle_a$ and ${}_a\langle j_c \bar{j}_d; J|\hat{\vartheta}|j_a \bar{j}_b; J\rangle_a$ are the antisymmetric matrix elements of the effective interaction $\hat{\vartheta}$ in the particle–particle and particle–hole channels with a given spin. They have the form

$$
\begin{aligned}
{}_a\langle j_c j_d; J|\hat{\vartheta}|j_a j_b; J\rangle_a \quad &= \quad \frac{1}{\sqrt{(1+\delta_{j_c j_d})(1+\delta_{j_a j_b})}}\Big[\langle j_c j_d; J|\hat{\vartheta}|j_a j_b; J\rangle + \\
&+ \quad (-1)^{j_a+j_b+J+1}\langle j_c j_d; J|\hat{\vartheta}|j_b j_a; J\rangle\Big] \,,
\end{aligned}
$$

$$
\begin{aligned}
{}_a\langle j_c \bar{j}_d; J|\hat{\vartheta}|j_a \bar{j}_b; J\rangle_a = -\frac{(-1)^{l_b+l_d}}{\sqrt{(1+\delta_{j_c j_d})(1+\delta_{j_a j_b})}} \sum_{J'} (2J'+1) W[j_b j_a j_c j_d; JJ'] \times \\
\times \left[\langle j_b j_c; J'|\hat{\vartheta}|j_d j_a; J'\rangle + (-1)^{j_d+j_a+J'+1}\langle j_b j_c; J'|\hat{\vartheta}|j_a j_d; J'\rangle\right] .
\end{aligned}
$$

Considering the transition rates, we must distinguish between two different cases, i.e., the phonon–phonon (between the two excited states) and the phonon–ground-state transitions. The latter transition is described by the reduced matrix element

$$\langle\tilde{0}\|\mathcal{M}(\lambda)\|\omega_n, J\rangle = (-1)^{\lambda}\delta(J,\lambda)\delta(\pi_n\pi_\lambda) \times$$

$$\times\left[\sum_{j_a\geq j_b} X^{n,J}_{j_aj_b}(u_{j_a}v_{j_b} \pm v_{j_a}u_{j_b})\frac{(-1)^{l_b}}{\sqrt{1+\delta_{j_aj_b}}}\langle j_a\|\hat{m}(\lambda)\|j_b\rangle - \right.$$

$$\left. - \sum_{j_a\geq j_b} Y^{n,J}_{j_aj_b}(v_{j_a}u_{j_b} \pm u_{j_a}v_{j_b})\frac{(-1)^{l_b}}{\sqrt{1+\delta_{j_aj_b}}}\langle j_a\|\hat{m}(\lambda)\|j_b\rangle\right],$$

where the upper signs refer to T-even (Eλ), while the lower ones to T-odd (Mλ) transitions.

One can show that the "phonon–phonon" matrix element has the form

$$\langle\omega_n, J'\|\hat{\mathcal{M}}(\lambda)\|\omega_m, J\rangle = [(2J+1)(2J'+1)]^{1/2}\cdot\sum_{j_a\geq j_b, j_c\geq j_d}\frac{\left[X^{m,J}_{j_aj_b}X^{n,J'}_{j_cj_d} \pm Y^{m,J}_{j_aj_b}Y^{n,J'}_{j_cj_d}\right]}{\sqrt{(1+\delta_{j_aj_b})(1+\delta_{j_cj_d})}}\times$$

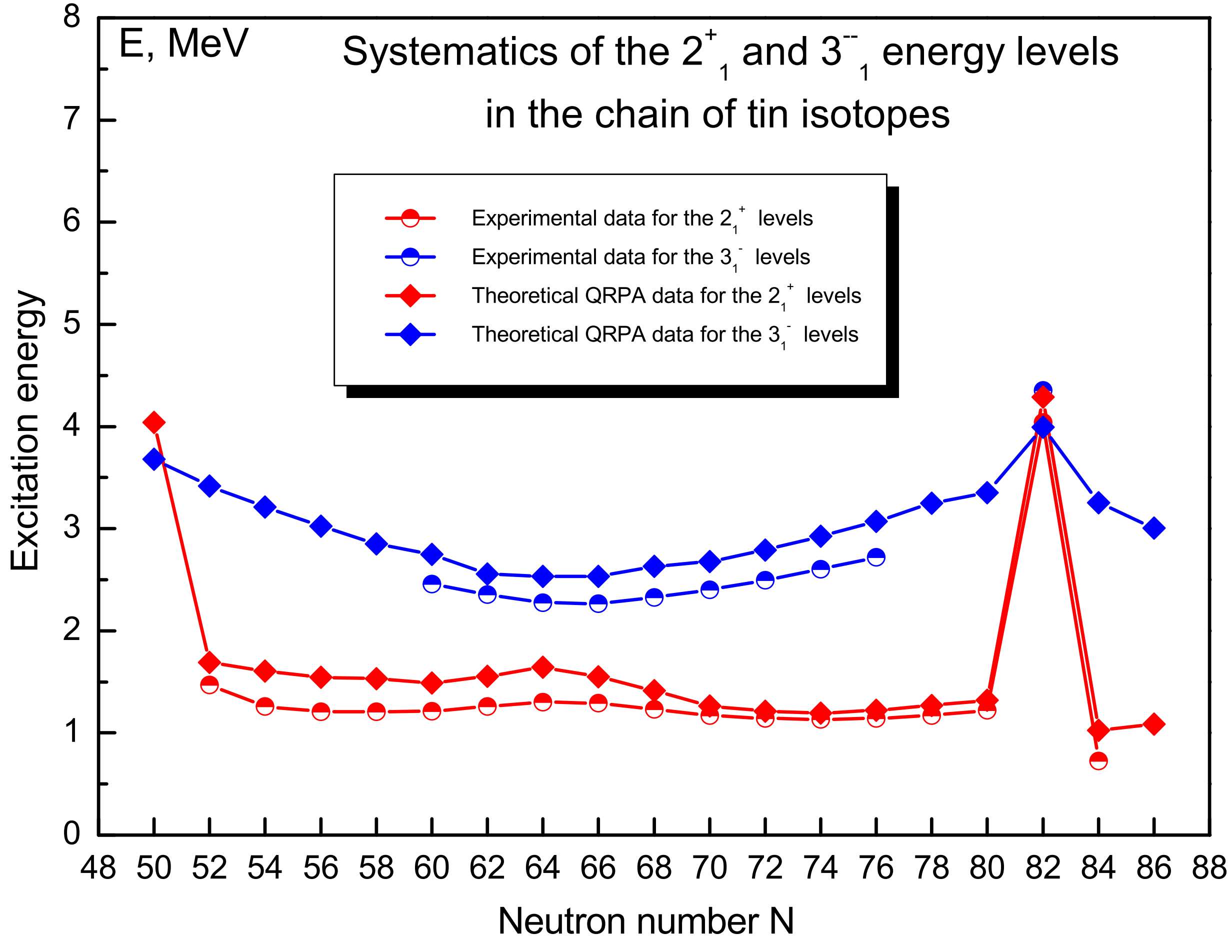
$$\times\left\{\delta_{j_bj_d}W[\lambda j_c J j_b; j_a J'](u_{j_c}u_{j_a} \mp v_{j_c}v_{j_a})\langle j_c\|\hat{m}(\lambda)\|j_a\rangle - \right.$$

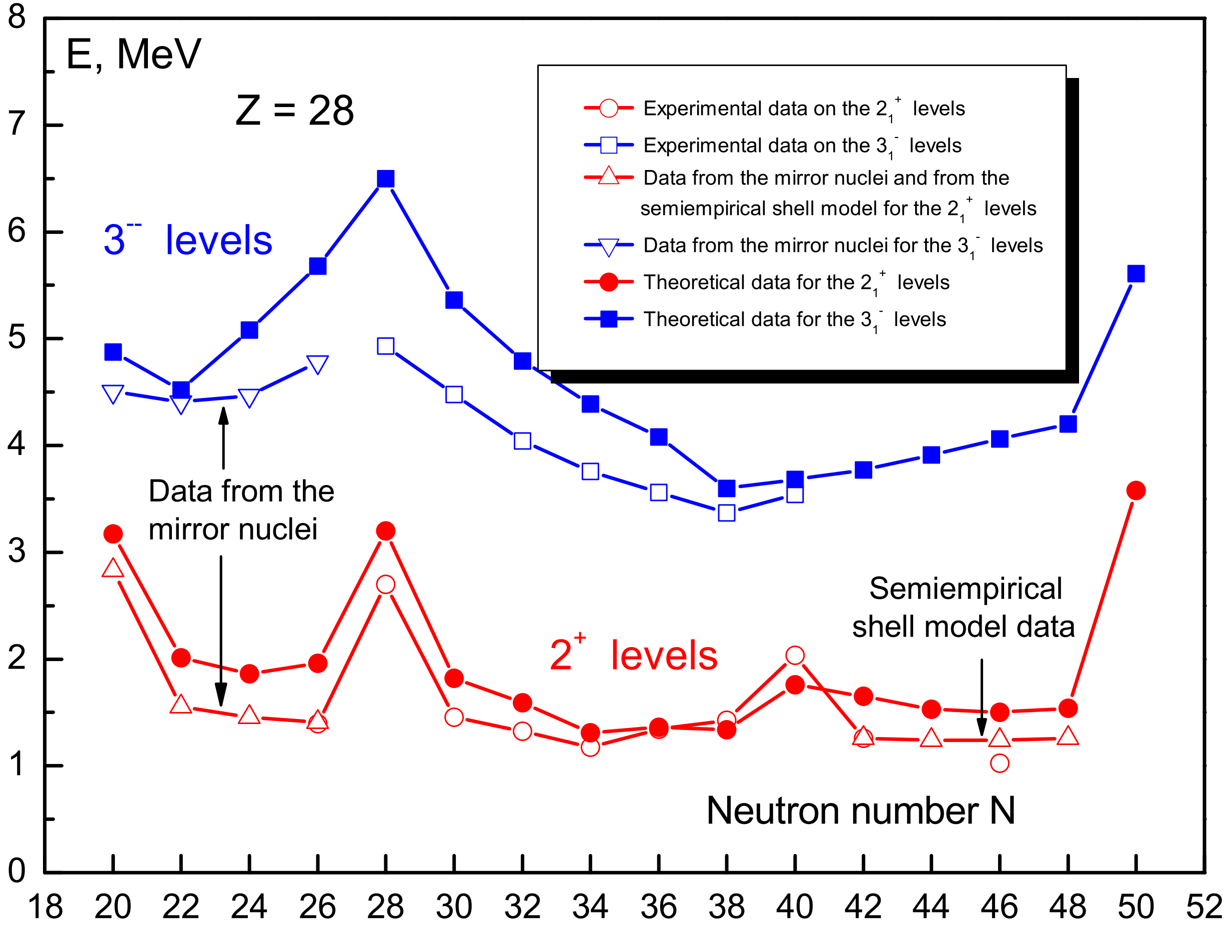
$$- (-1)^{j_c+j_d+J'}\delta_{j_bj_c}W[\lambda j_d J j_b; j_a J'](u_{j_d}u_{j_a} \mp v_{j_d}v_{j_a})\langle j_d\|\hat{m}(\lambda)\|j_a\rangle -$$

$$- (-1)^{j_a+j_b+J}\delta_{j_aj_d}W[\lambda j_c J j_a; j_b J'](u_{j_c}u_{j_b} \mp v_{j_c}v_{j_b})\langle j_c\|\hat{m}(\lambda)\|j_b\rangle +$$

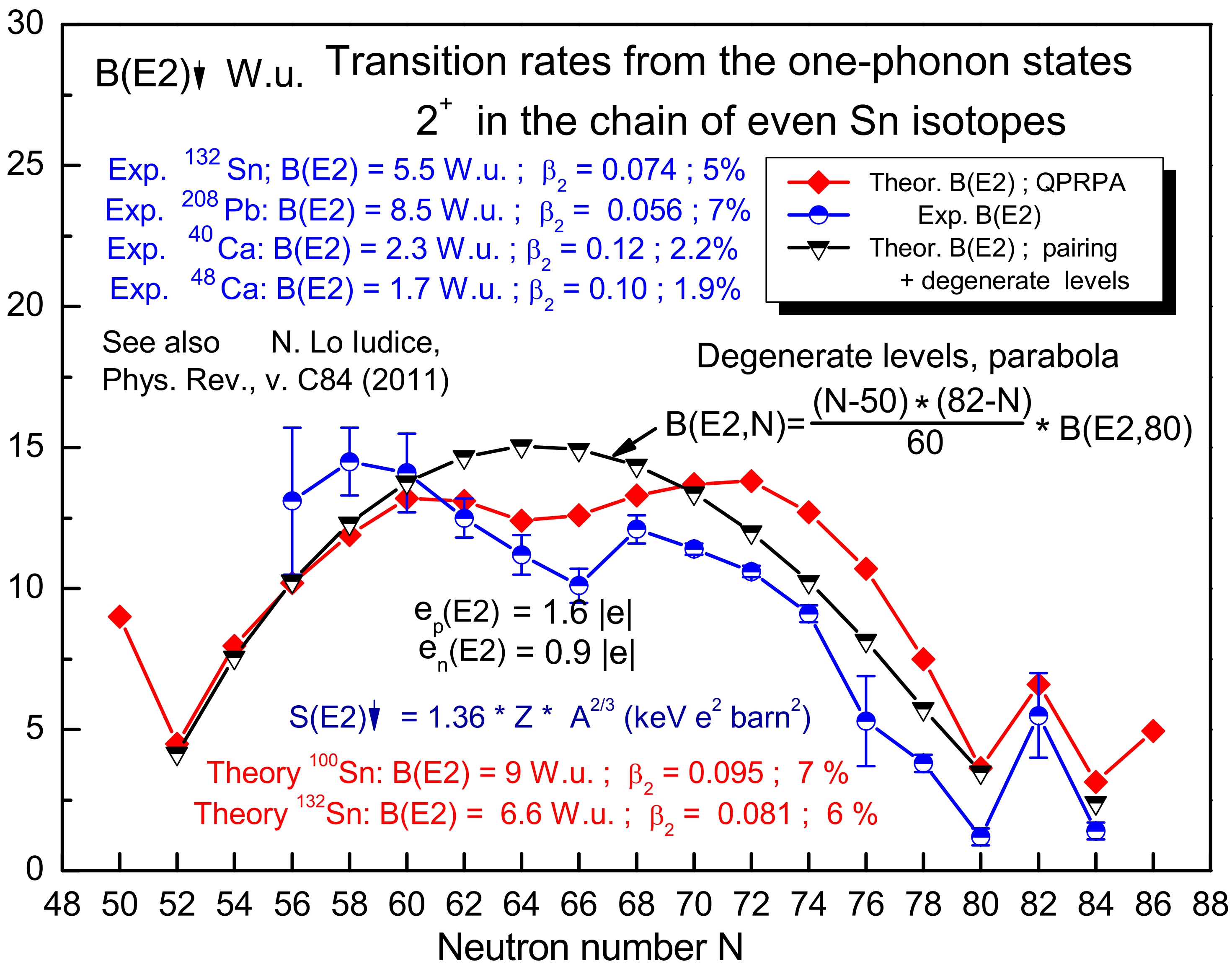
$$\left. + (-1)^{j_a+j_b+J+j_c+j_d+J'}\delta_{j_aj_c}W[\lambda j_d J j_a; j_b J'](u_{j_d}u_{j_b} \mp v_{j_d}v_{j_b})\langle j_d\|\hat{m}(\lambda)\|j_b\rangle\right\},$$

where the upper signs refer to Eλ, while the lower ones – to Mλ transitions.

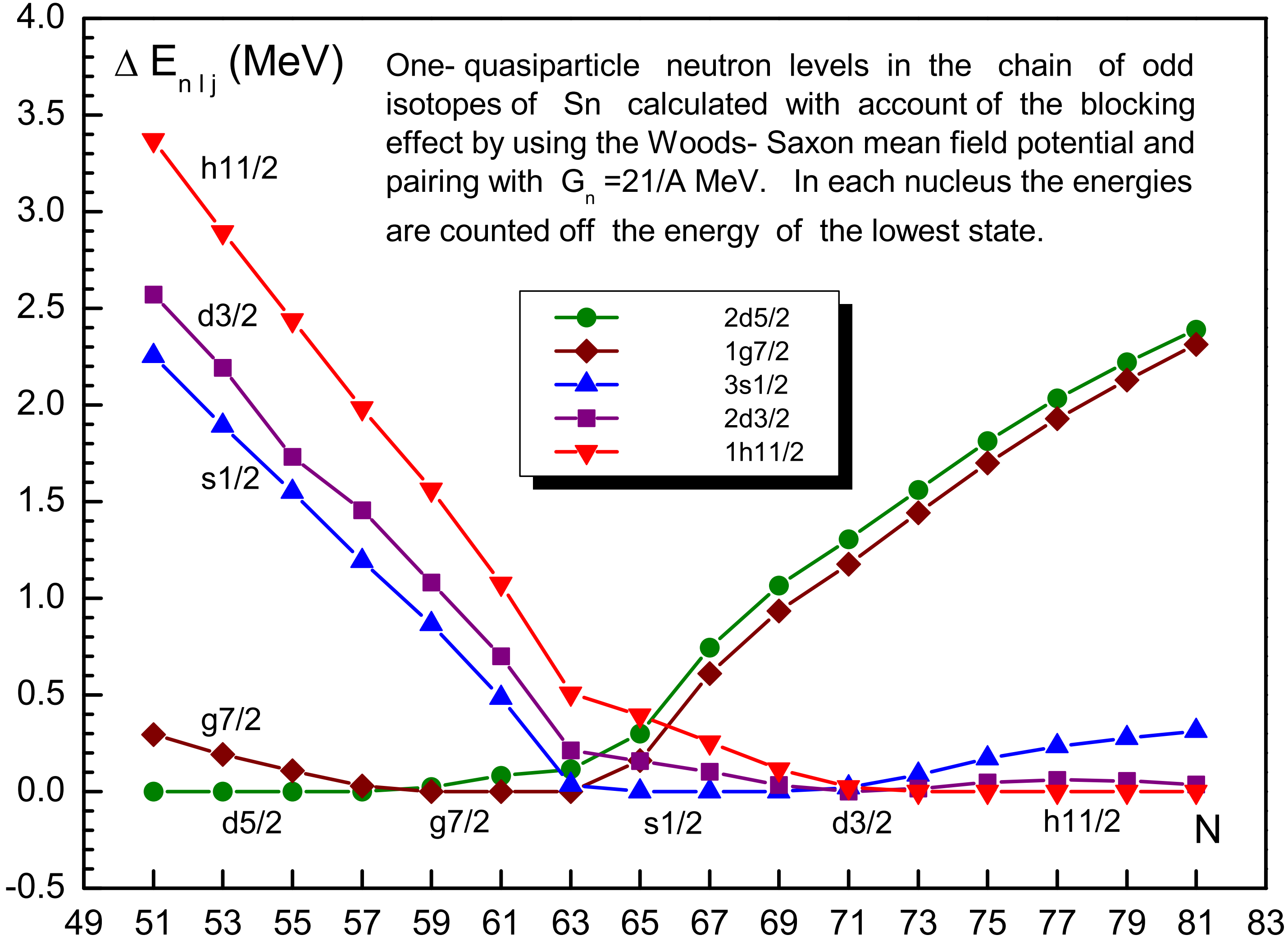

E, MeV
Systematics of the 2+1 and 3--1 energy levels
in the chain of tin isotopes
Experimental data for the 2+1 levels
Experimental data for the 3-1 levels
Theoretical QRPA data for the 2+1 levels
Theoretical QRPA data for the 3-1 levels
Excitation energy
Neutron number N
0
1
2
3
4
5
6
7
8
48
50
52
54
56
58
60
62
64
66
68
70
72
74
76
78
80
82
84
86
88



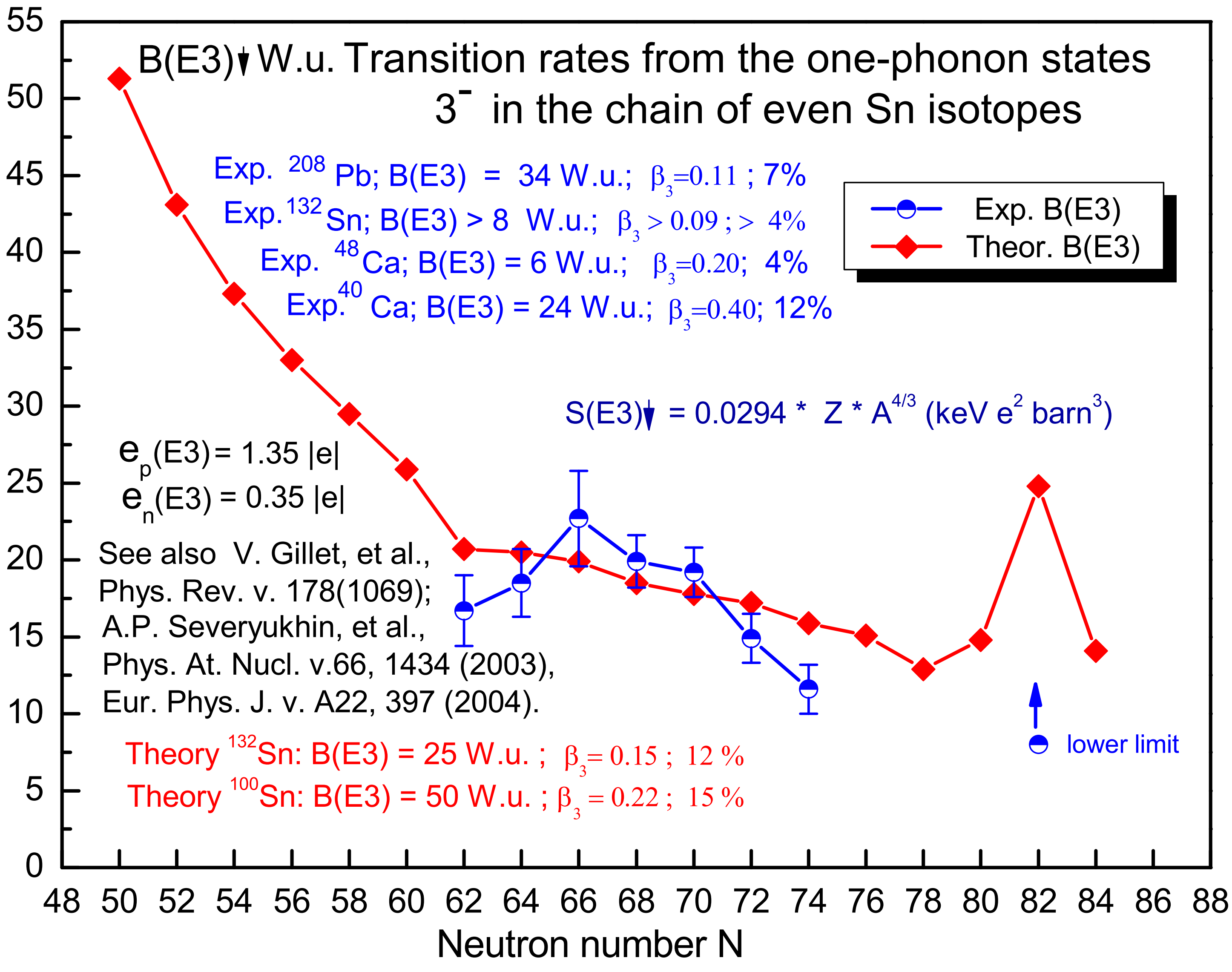
E, MeV
Z = 28
3⁻⁻ levels
2⁺ levels
Experimental data on the 2_1^+ levels
Experimental data on the 3_1^- levels
Data from the mirror nuclei and from the semiempirical shell model for the 2_1^+ levels
Data from the mirror nuclei for the 3_1^- levels
Theoretical data for the 2_1^+ levels
Theoretical data for the 3_1^- levels
Data from the mirror nuclei
Semiempirical shell model data
Neutron number N



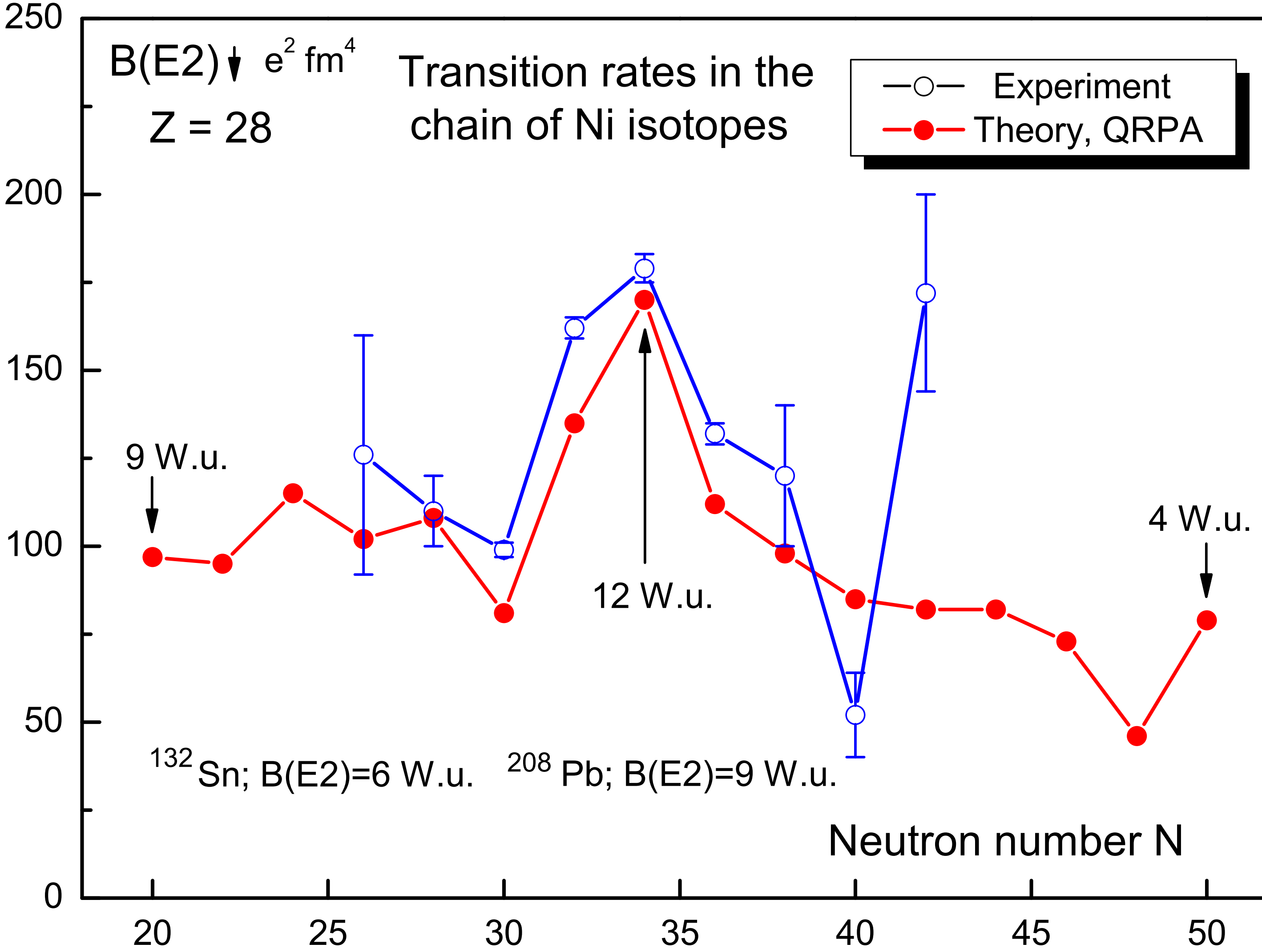
B(E2)↓ W.u.
Transition rates from the one-phonon states
2+ in the chain of even Sn isotopes
Exp. 132Sn; B(E2) = 5.5 W.u. ; β2 = 0.074 ; 5%
Exp. 208Pb: B(E2) = 8.5 W.u. ; β2 = 0.056 ; 7%
Exp. 40Ca: B(E2) = 2.3 W.u. ; β2 = 0.12 ; 2.2%
Exp. 48Ca: B(E2) = 1.7 W.u. ; β2 = 0.10 ; 1.9%
Theor. B(E2) ; QPRPA
Exp. B(E2)
Theor. B(E2) ; pairing + degenerate levels
See also N. Lo Iudice, Phys. Rev., v. C84 (2011)
Degenerate levels, parabola
B(E2,N)= (N-50) * (82-N) / 60 * B(E2,80)
ep(E2) = 1.6 |e|
en(E2) = 0.9 |e|
S(E2)↓ = 1.36 * Z * A2/3 (keV e2 barn2)
Theory 100Sn: B(E2) = 9 W.u. ; β2 = 0.095 ; 7 %
Theory 132Sn: B(E2) = 6.6 W.u. ; β2 = 0.081 ; 6 %
0
5
10
15
20
25
30
48 50 52 54 56 58 60 62 64 66 68 70 72 74 76 78 80 82 84 86 88
Neutron number N

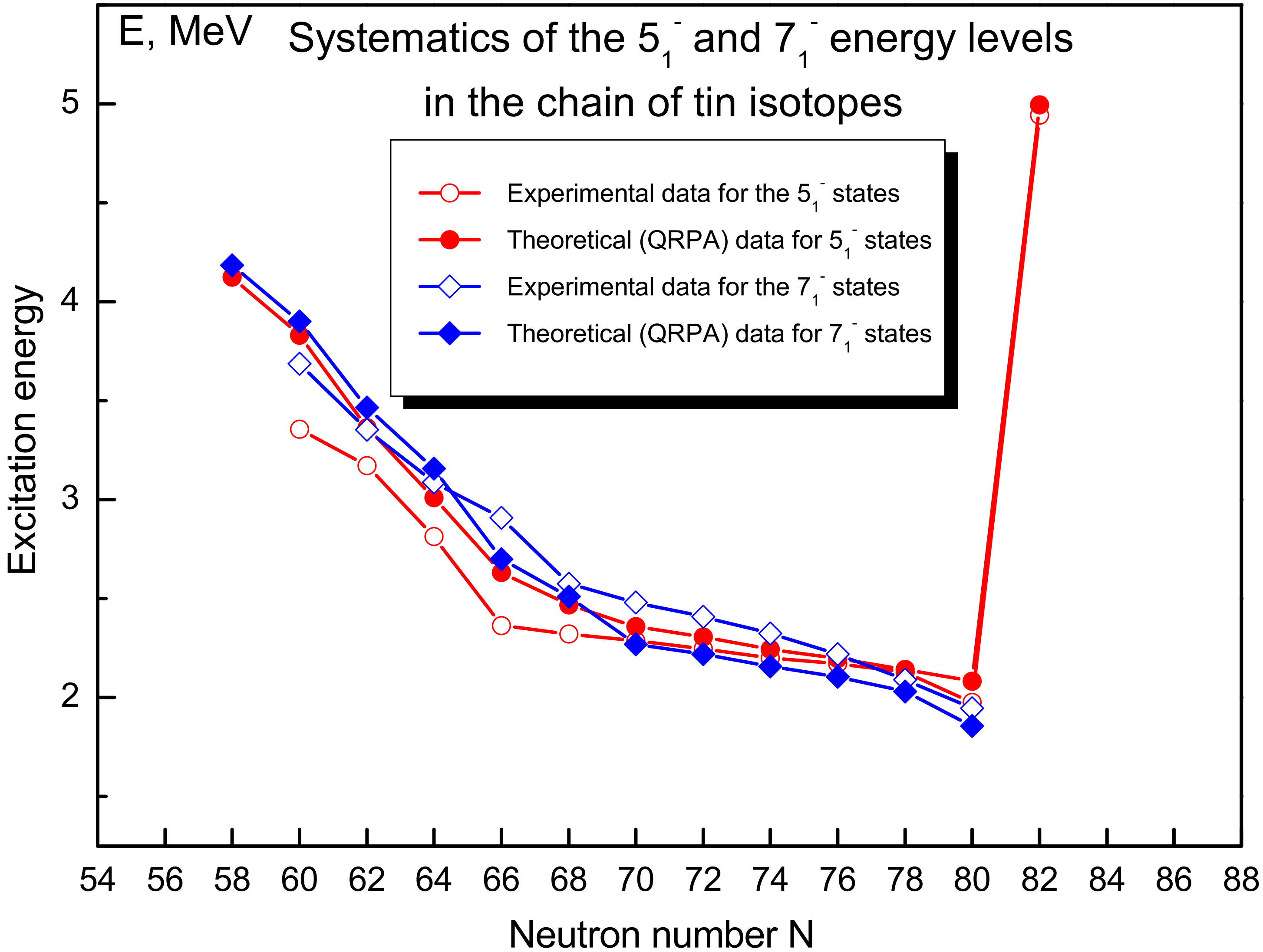


Δ $E_{n l j}$ (MeV)
One- quasiparticle neutron levels in the chain of odd isotopes of Sn calculated with account of the blocking effect by using the Woods- Saxon mean field potential and pairing with G_n =21/A MeV. In each nucleus the energies are counted off the energy of the lowest state.
2d5/2
1g7/2
3s1/2
2d3/2
1h11/2
h11/2
d3/2
s1/2
g7/2
d5/2
g7/2
s1/2
d3/2
h11/2
N
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
49
51
53
55
57
59
61
63
65
67
69
71
73
75
77
79
81
83

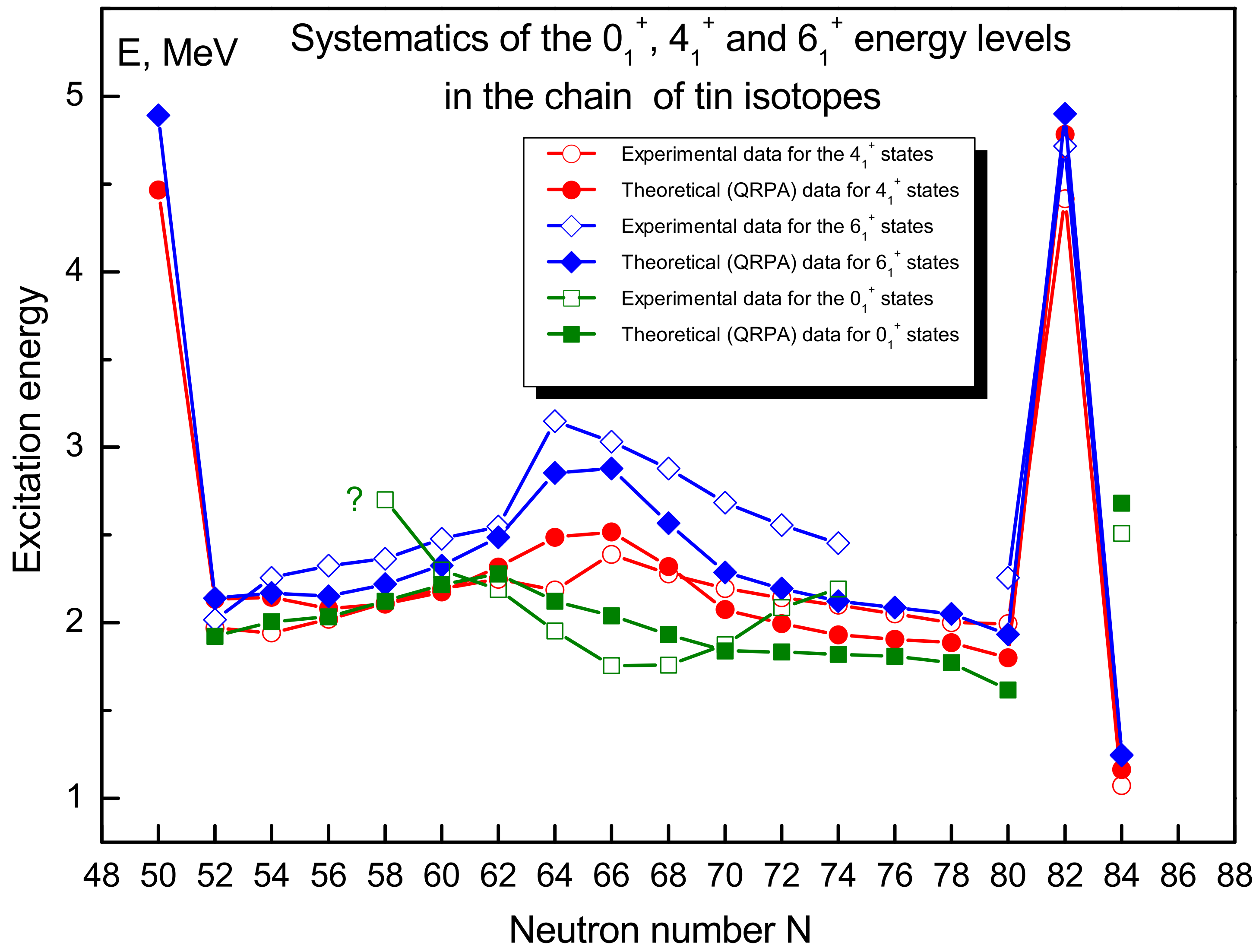


B(E3)↓W.u. Transition rates from the one-phonon states
3⁻ in the chain of even Sn isotopes
Exp. 208 Pb; B(E3) = 34 W.u.; β3=0.11 ; 7%
Exp. 132 Sn; B(E3) > 8 W.u.; β3 > 0.09 ; > 4%
Exp. 48 Ca; B(E3) = 6 W.u.; β3=0.20; 4%
Exp. 40 Ca; B(E3) = 24 W.u.; β3=0.40; 12%
Exp. B(E3)
Theor. B(E3)
S(E3)↓ = 0.0294 * Z * A4/3 (keV e2 barn3)
ep(E3)= 1.35 |e|
en(E3) = 0.35 |e|
See also V. Gillet, et al.,
Phys. Rev. v. 178(1069);
A.P. Severyukhin, et al.,
Phys. At. Nucl. v.66, 1434 (2003),
Eur. Phys. J. v. A22, 397 (2004).
lower limit
Theory 132Sn: B(E3) = 25 W.u. ; β3= 0.15 ; 12 %
Theory 100Sn: B(E3) = 50 W.u. ; β3 = 0.22 ; 15 %
0
5
10
15
20
25
30
35
40
45
50
55
48
50
52
54
56
58
60
62
64
66
68
70
72
74
76
78
80
82
84
86
88
Neutron number N

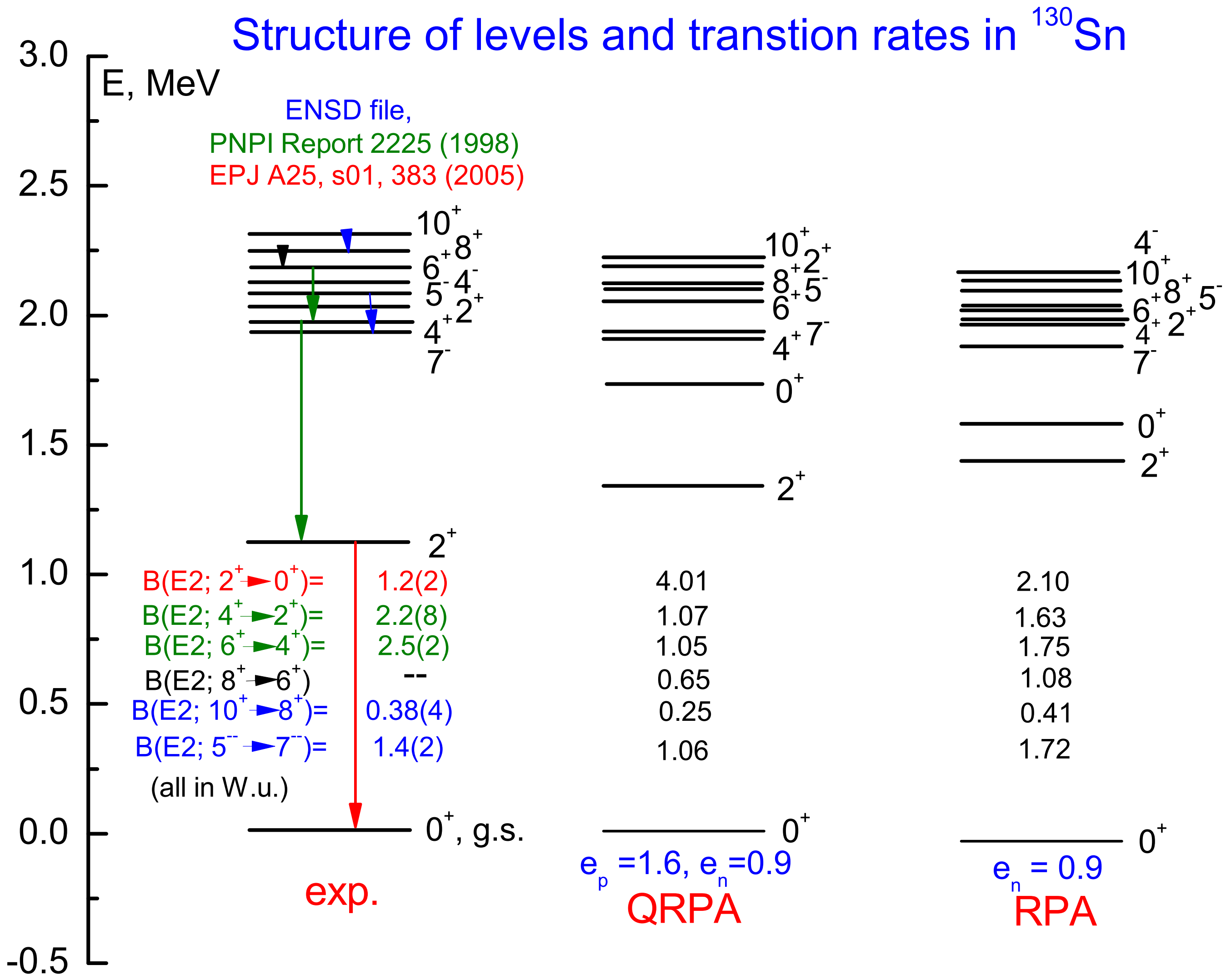

B(E2)↓ e² fm⁴
Z = 28
Transition rates in the chain of Ni isotopes
Experiment
Theory, QRPA
9 W.u.
12 W.u.
4 W.u.
132Sn; B(E2)=6 W.u.
208Pb; B(E2)=9 W.u.
Neutron number N
0
50
100
150
200
250
20
25
30
35
40
45
50



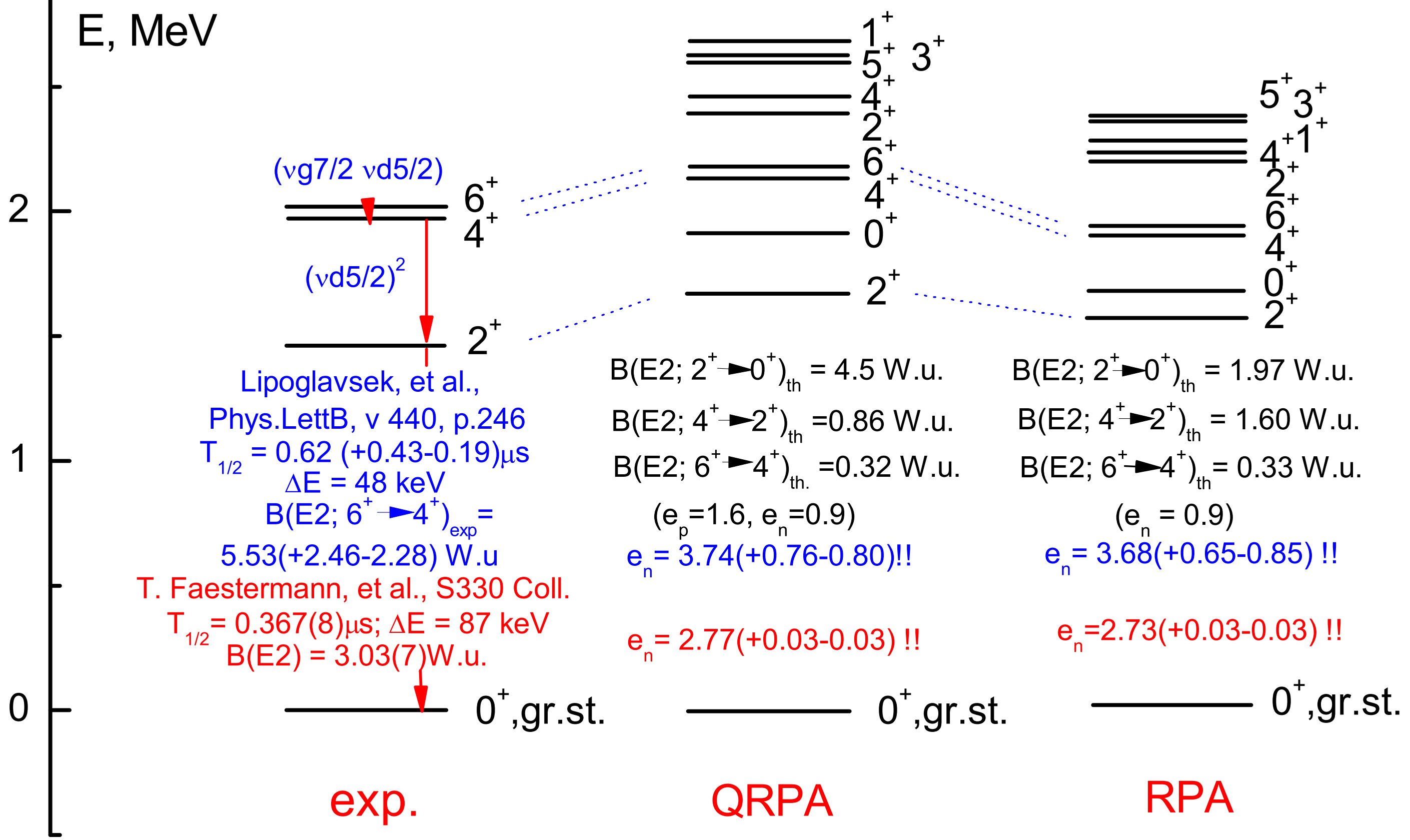
E, MeV
Systematics of the 5₁⁻ and 7₁⁻ energy levels
in the chain of tin isotopes
Experimental data for the 5₁⁻ states
Theoretical (QRPA) data for 5₁⁻ states
Experimental data for the 7₁⁻ states
Theoretical (QRPA) data for 7₁⁻ states
Excitation energy
5
4
3
2
54 56 58 60 62 64 66 68 70 72 74 76 78 80 82 84 86 88
Neutron number N



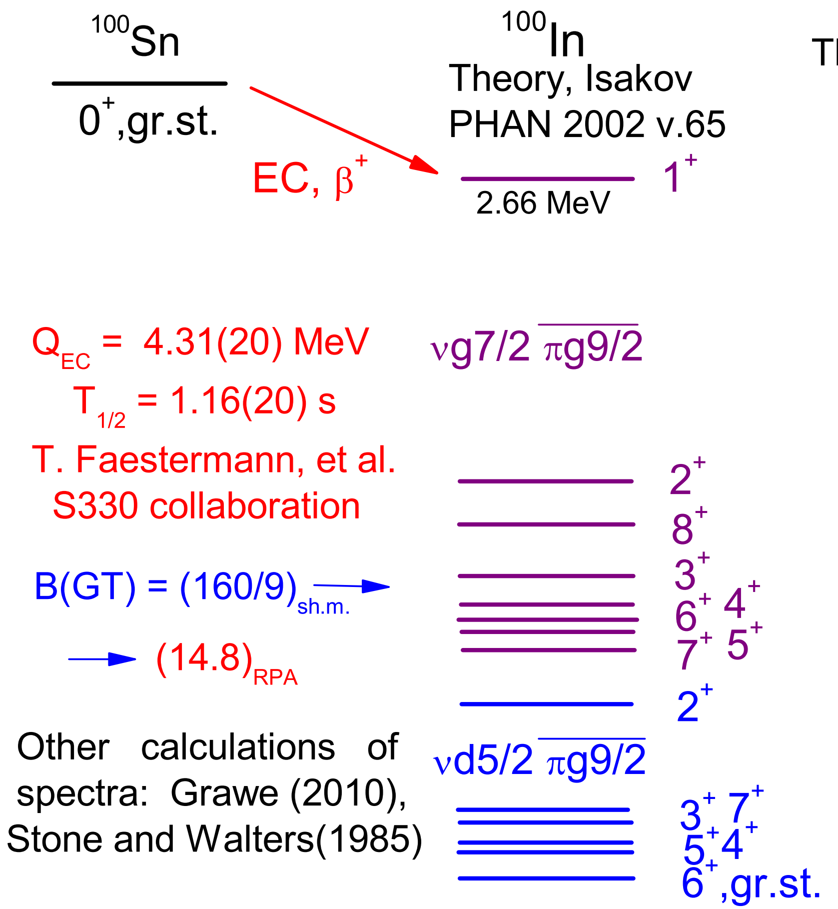
E, MeV
Systematics of the 0_1^+, 4_1^+ and 6_1^+ energy levels
in the chain of tin isotopes
Experimental data for the 4_1^+ states
Theoretical (QRPA) data for 4_1^+ states
Experimental data for the 6_1^+ states
Theoretical (QRPA) data for 6_1^+ states
Experimental data for the 0_1^+ states
Theoretical (QRPA) data for 0_1^+ states
?
Excitation energy
1
2
3
4
5
48 50 52 54 56 58 60 62 64 66 68 70 72 74 76 78 80 82 84 86 88
Neutron number N


Structure of levels and transtion rates in 130Sn
E, MeV
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
ENSD file,
PNPI Report 2225 (1998)
EPJ A25, s01, 383 (2005)
10+
8+
6+
4-
5-
2+
4+
7-
2+
0+, g.s.
B(E2; 2+→0+)= 1.2(2)
B(E2; 4+→2+)= 2.2(8)
B(E2; 6+→4+)= 2.5(2)
B(E2; 8+→6+) --
B(E2; 10+→8+)= 0.38(4)
B(E2; 5-→7-)= 1.4(2)
(all in W.u.)
exp.
4.01
1.07
1.05
0.65
0.25
1.06
ep =1.6, en =0.9
QRPA
2.10
1.63
1.75
1.08
0.41
1.72
en = 0.9
RPA

Structure of levels and transition rate in ^{102}Sn



Structure of levels and EC/β^+-transition rate in ^{100}In

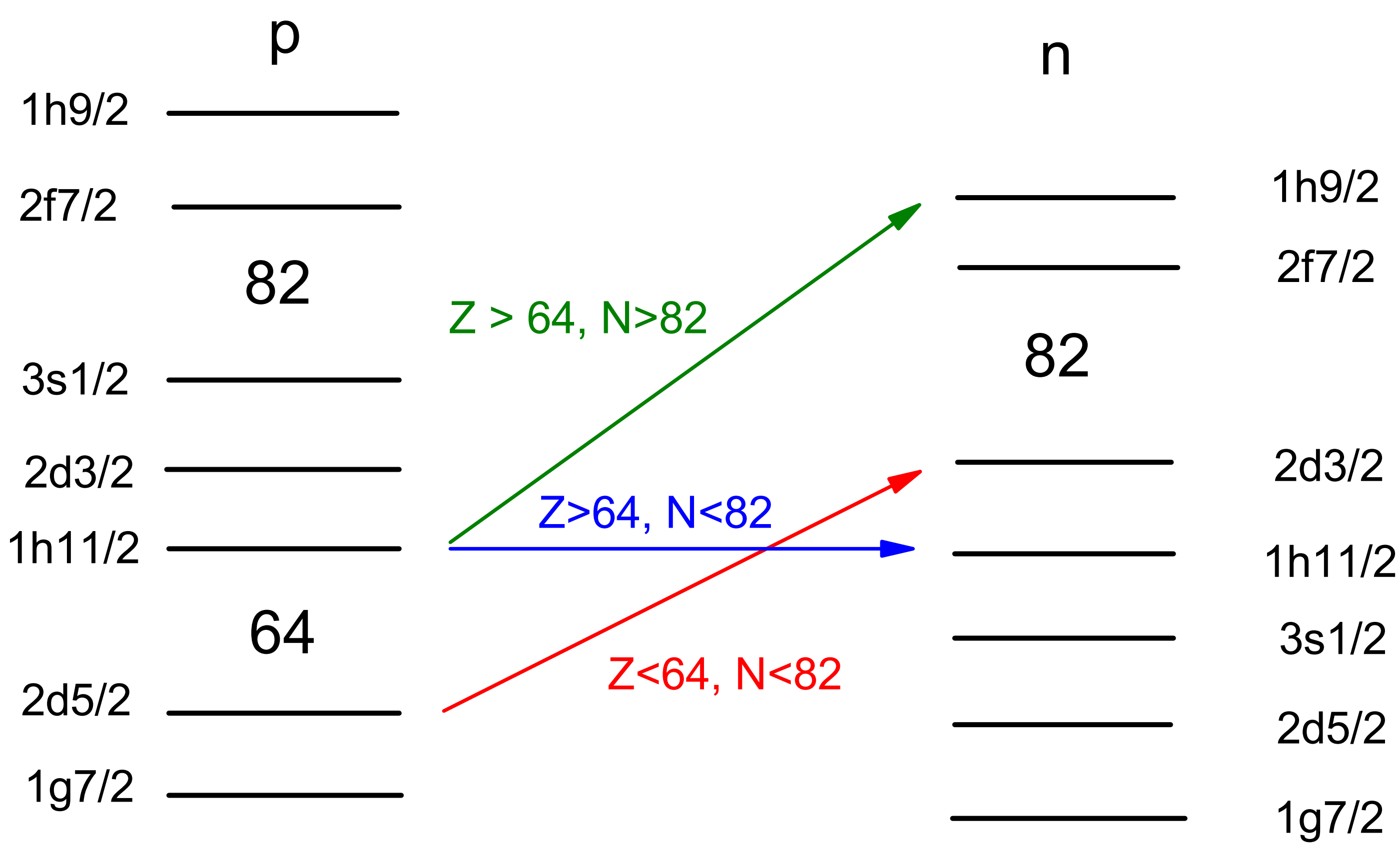


Theory, Isakov PHAN 2009

Final state	$\lvert G_A/G_V \rvert$
^{15}N($1/2^-$)	1.120(158)
^{17}O($5/2^+$)	1.153(181)
^{18}O (0^+)	1.040(122)
^{18}F (1^+)	1.041(110)
^{39}K ($3/2^+$)	1.134(210)
^{41}Ca($7/2^-$)	1.107(108)
^{42}Ca (6^+)	1.118(62)
^{54}Fe (6^+)	1.073(123)
^{55}Co($7/2^-$)	1.338(150)
^{57}Ni($3/2^-$)	0.942(165)
^{57}Ni($1/2^-$)	1.115(179)
^{100}In(1^+)	1.00(15)
Average	**1.097(148)**

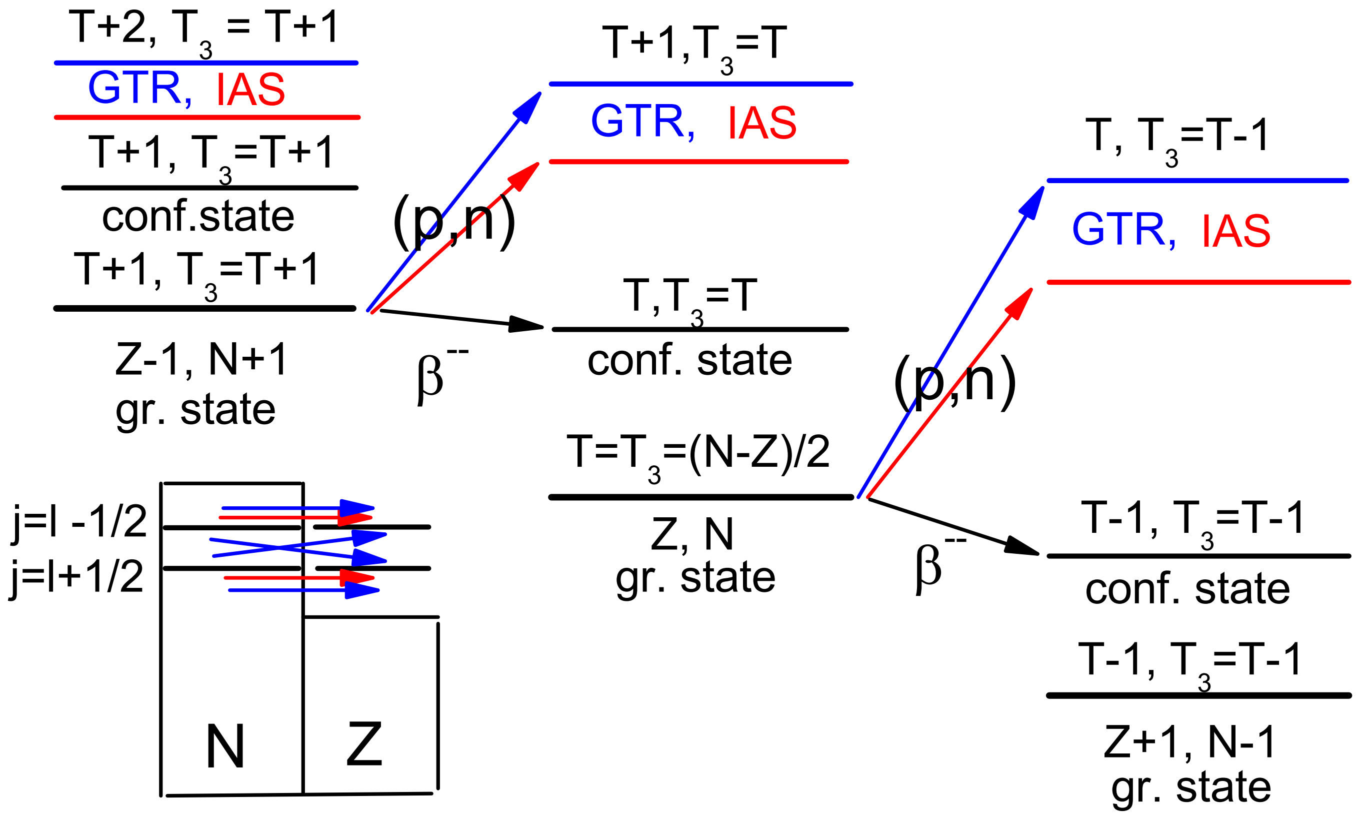
(1.24 for a free neutron)

Allowed β(+) transitions in neutron deficient nuclei



Alkhazov, Novikov, et al., G_A/G_V = 0.7 -- 0.9

Isobaric transition in N > Z nuclei



(p,n) reactions, Pyatov and Fayans, G_A/G_V = 0.8 -- 1.0

Conservation of weak vector current. Feynmann, Gell-Mann (1958).

$dV_\mu/dx_\mu = 0$ Conservation of electric charge in the neutral channel and of the vector weak constant in beta-decay.

Partially conserved axial current. Goldberger, Treiman (1958)

$dA_\mu/dx_\mu = C\,\phi$. Operator relationship.

<p|...|n> and <0|...|π> $\longrightarrow$ Bind constants of two weak decays:

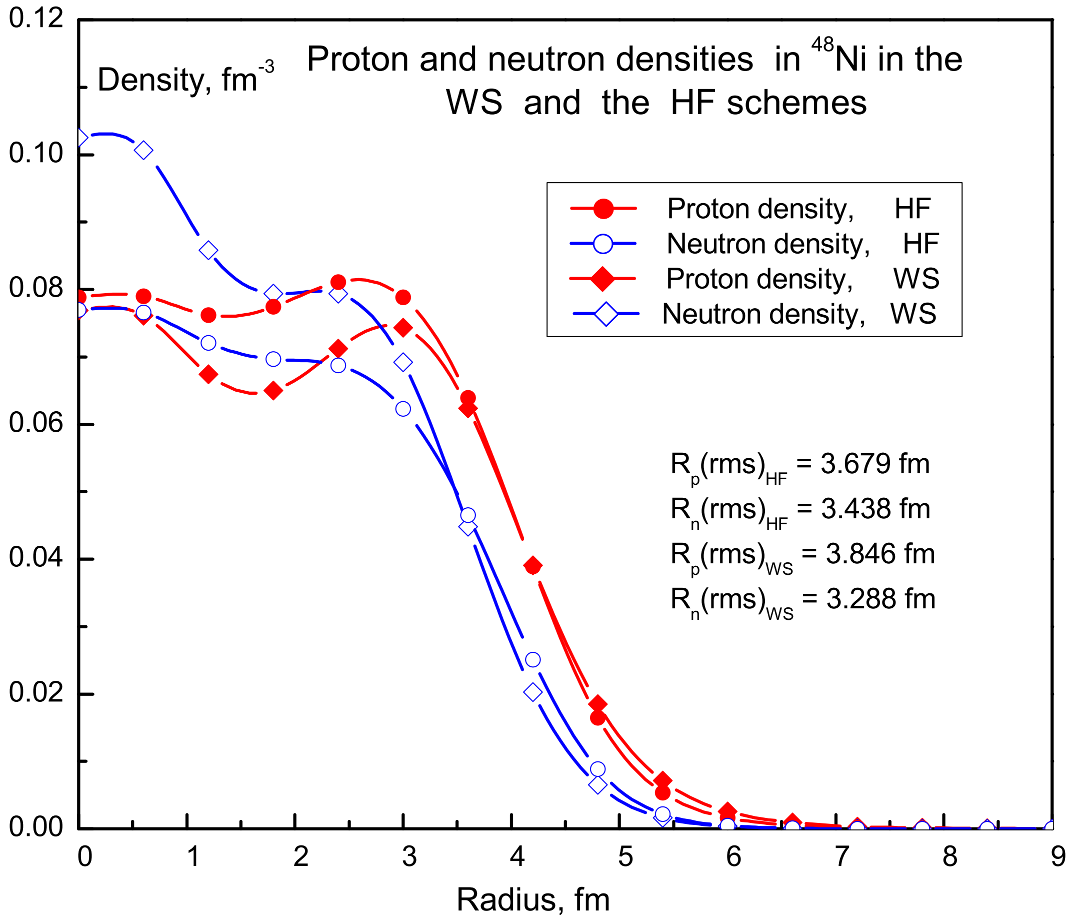
$n \longrightarrow p + e + \nu$ and $\pi \longrightarrow e + \nu$; 7% accuracy.

Adler, Weisberger (1965). PCAC + current algebra: $G_A/G_V = 1.24$ from the difference of cross sections of scattering of π^+ and π^- mesons on protons.

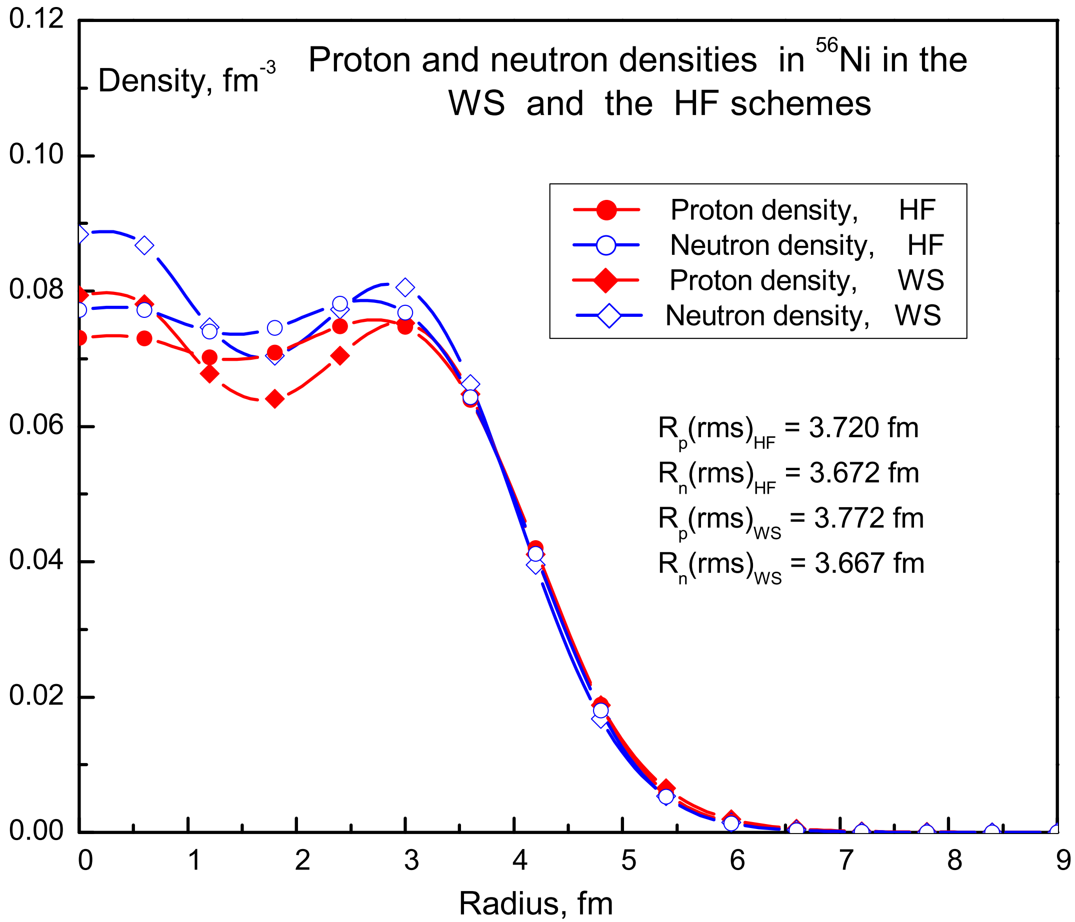
Ericson, Locher (1970). PCAC + dispersion relations for analysis of difference of (π^+, A) and (π^{--}, A) cross sections. $G_A/G_V = 1.0.$ (0.1).

Suppression of G_A due to the excitation of Δ isobars. Schematic model of valence quarks. $G_A/G_V = 1.0$.

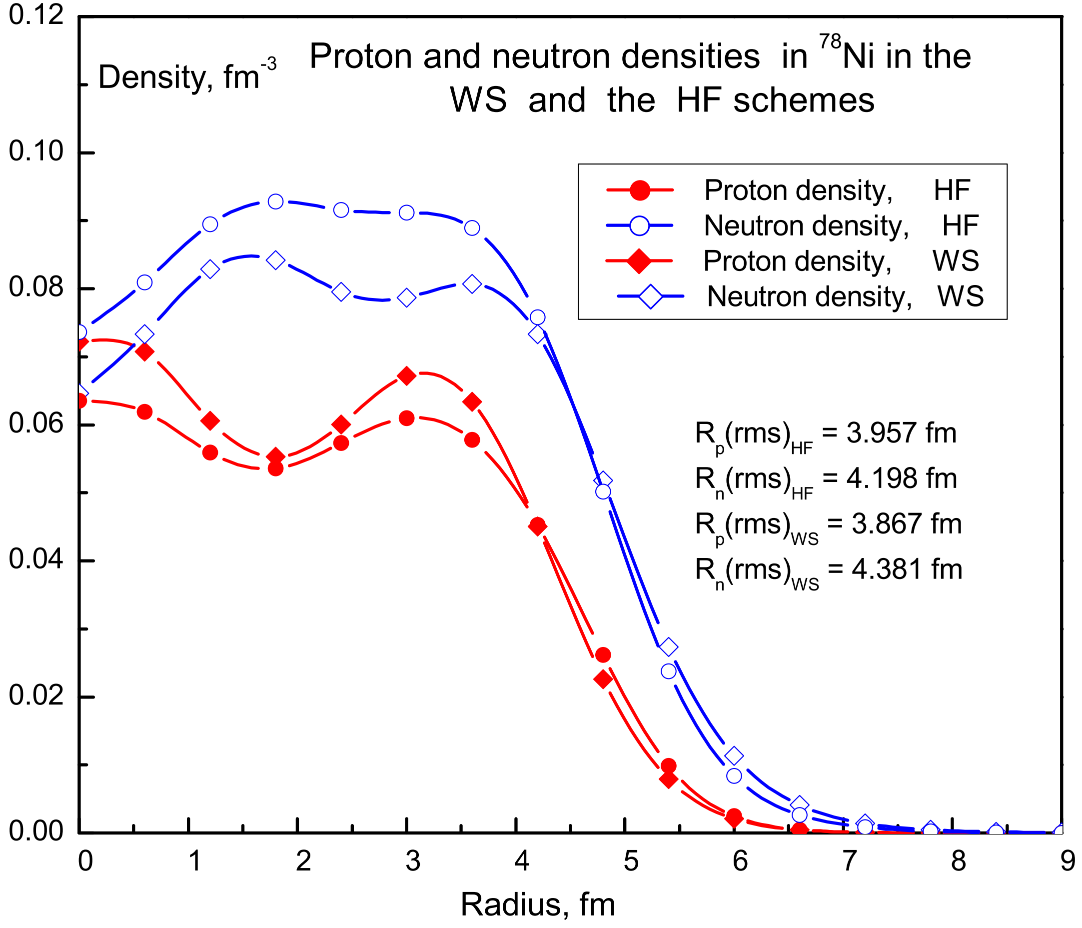
THANK YOU FOR ATTENTION



Proton and neutron densities in 48Ni in the WS and the HF schemes
Density, fm-3
Radius, fm
Proton density, HF
Neutron density, HF
Proton density, WS
Neutron density, WS
Rp(rms)HF = 3.679 fm
Rn(rms)HF = 3.438 fm
Rp(rms)WS = 3.846 fm
Rn(rms)WS = 3.288 fm
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0
1
2
3
4
5
6
7
8
9



Proton and neutron densities in ^{56}Ni in the WS and the HF schemes
Density, fm^{-3}
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0
1
2
3
4
5
6
7
8
9
Radius, fm
Proton density, HF
Neutron density, HF
Proton density, WS
Neutron density, WS
$R_p(rms)_{HF}$ = 3.720 fm
$R_n(rms)_{HF}$ = 3.672 fm
$R_p(rms)_{WS}$ = 3.772 fm
$R_n(rms)_{WS}$ = 3.667 fm



Proton and neutron densities in ^{78}Ni in the WS and the HF schemes
Density, fm^{-3}
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0
1
2
3
4
5
6
7
8
9
Radius, fm
Proton density, HF
Neutron density, HF
Proton density, WS
Neutron density, WS
$R_p(rms)_{HF}$ = 3.957 fm
$R_n(rms)_{HF}$ = 4.198 fm
$R_p(rms)_{WS}$ = 3.867 fm
$R_n(rms)_{WS}$ = 4.381 fm