THE CALCULATION METHOD AND OPTIMIZATION OF A PROTON BEAM LINE WITH COLLIMATORS BY USING COURANT–SNYDER FORMALISM

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There are many ways to optimize and calculate the lines of charged particle beams. Calculation methods based on the approximation of the phase ellipse of the beam are widely used. The beam is represented by a 6×6 matrix defining a hyperellipsoid in six-dimensional phase space. The projection of this hyperellipsoid on any two-dimensional plane is an ellipse that defines the boundary of the beam in that plane. This hyperellipsoid can be mathematically specified by Courant–Snyder (or Twiss) parameters [1].

However, this method is not designed for the use of limiting apertures or collimators in the beam line. Therefore, an artificial technique was employed. The effect of the collimator on the phase portrait of the beam was taken into account as follows.

Let the *z*-axis of the reference system is directed along the beam axis, *x* is the horizontal coordinate, and *y* is the vertical one. Let the collimator be a square 3×3 mm. The collimator acceptance is imposed on the *x* and *y* phase ellipses of the beam at the entrance to the transport path. The acceptance is represented by two straight lines mapped using the collimator aperture transition matrix $x, y \leq 3$ mm onto the entrance of the beam line.

According to Ref. [2],

$$\vec{\mathrm{X}} = M\vec{X}_0,$$

where $M = \begin{pmatrix} M_{11} & M_{12} & 0 & 0 \\ M_{21} & M_{22} & 0 & 0 \\ 0 & 0 & M_{33} & M_{34} \\ 0 & 0 & M_{43} & M_{44} \end{pmatrix}$, $\vec{\mathrm{X}} = \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}$, $\vec{\mathrm{X}}_0$ are the coordinates and angles of motion of the particle at the entrance to the beam line.

It is obvious that particles whose coordinates satisfy the following relations will pass through the collimator:

$$|x| = |M_{11}x_0 + M_{12}x'_0| \leq A,$$
$$|y| = |M_{33}y_0 + M_{34}y'_0| \leq A.$$

Here $|M_{11}x_0 + M_{12}x'_0| = A$ and $|M_{33}y_0 + M_{34}y'_0| = A$ are, respectively, parallel straight lines in the phase space of the beam *x*–*x*' and *y*–*y*', which are the mapping of the collimator acceptance to the beam line entrance.

Further, a new ellipse is inscribed in the part of the phase space bounded by the input beam ellipses (vertical and horizontal ones at the input of the beam line) and the straight lines of the reflected collimator (this allows one to remain in the TRACE-3*D* formalism working with beam ellipses). This procedure makes it possible to find the ellipse and emittance of the beam passing along the beam line and through the collimator without losses. The ratio of the area of the inscribed phase ellipse to the area of the original ellipse determines the intensity reduction factor.

For example, let us consider one of projects for a beam line for the C-80 cyclotron. Its scheme is shown in Fig. 1. The effect of the collimator on the phase portrait of the beam is shown in Fig. 2.

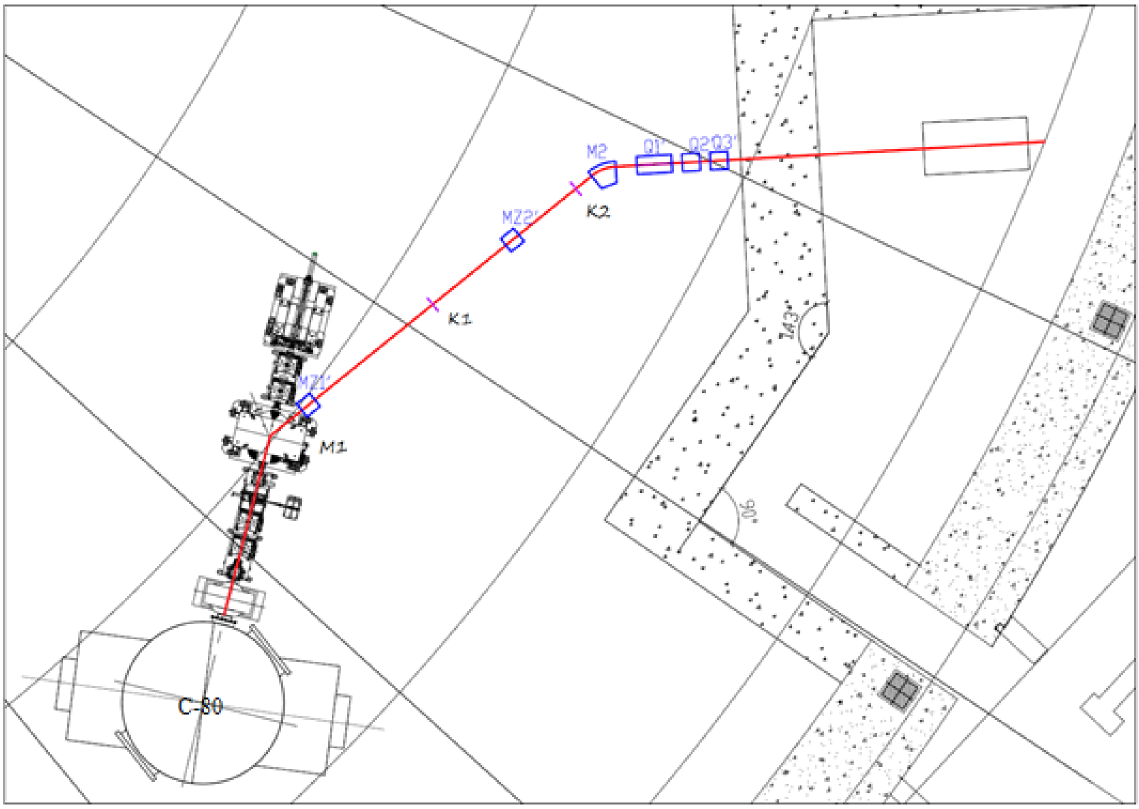

Fig. 1. Optimized beam transport path for ophthalmological needs. Here MZ1 and MZ2 are bending magnets in the vertical plane; M1 and M2 are bending magnets in the horizontal plane; K1 and K2 are collimators; Q1, Q2 and Q3 make up a triplet of quadrupole lenses

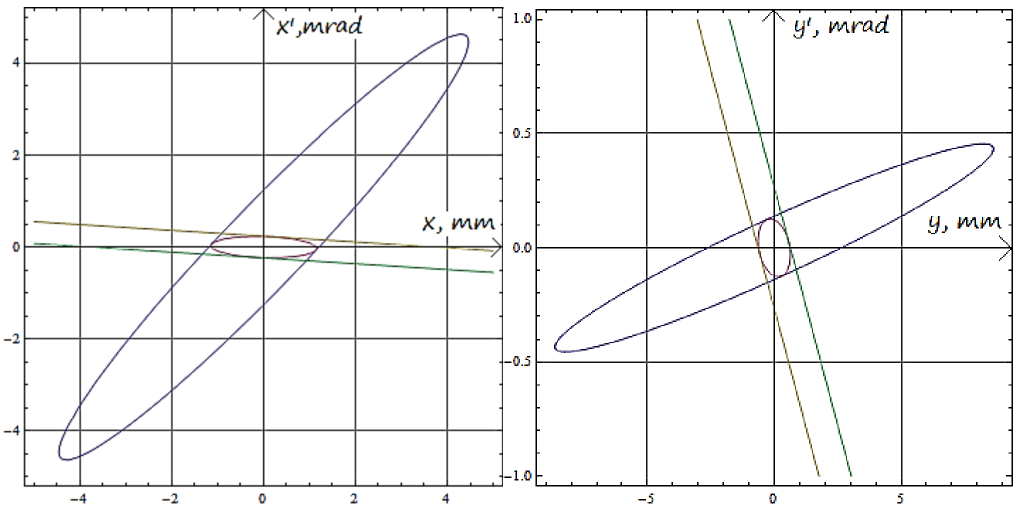

Fig. 2. Horizontal ellipse *x–x' (left*); vertical ellipse *y–y'* (*right*)

Figure 3 shows a screenshot of the result of the TRACE-3*D* software on optimizing the proposed beam line transportation. Above are the optimal values of the gradients of the magnetic fields of the quadrupole lenses. At the bottom of Fig. 4, the upper line corresponds to the horizontal beam envelope (in relation to the midline), and the lower line corresponds to the vertical beam envelope.

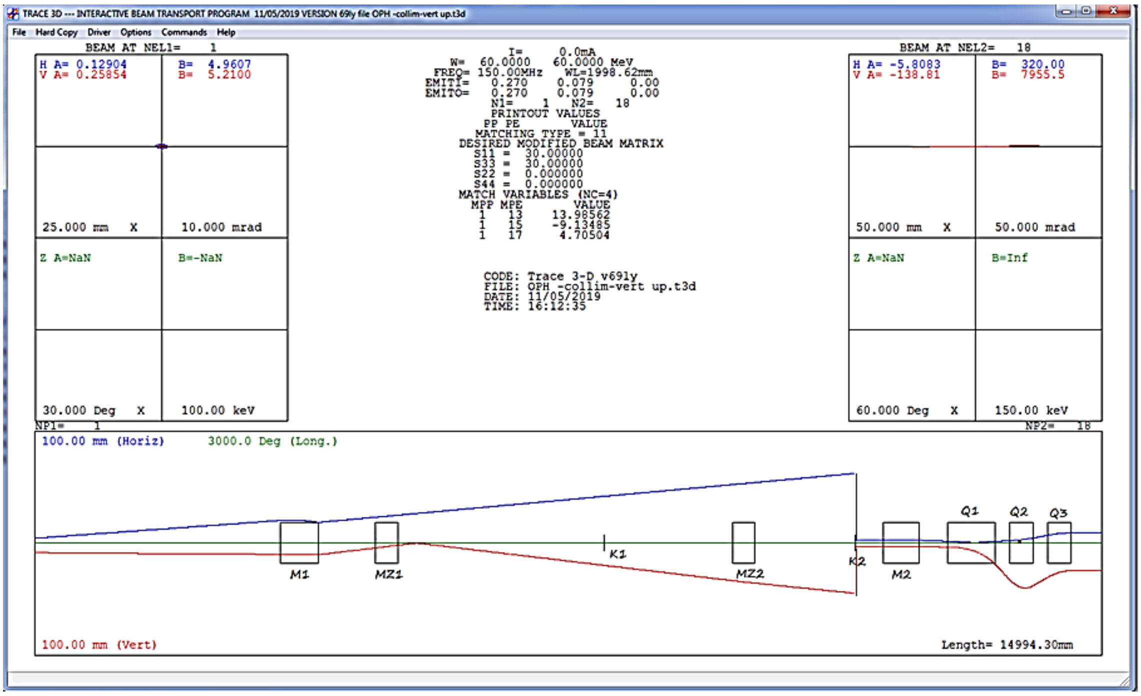

Fig. 4. TRACE-3*D* optimization calculation results and beam envelopes

References

1. K. Crandall, D. Rusthoi, TRACE 3-*D* Documentation, LA-UR-97-886, 106 (1997).
2. K.G. Steffen, High Energy Beam Optics (1965).