IMPORTANT FOCUSING PROPERTIES OF THE MAGNETIC STRUCTURE OF ISOCHRONOUS CYCLOTRONS WITH HIGH SPIRALING ANGLE OF THE POLE TIPS

D.A. Amerkanov, S.A. Artamonov, E.M. Ivanov, G.A. Riabov, V.A. Tonkikh

1. Introduction

Magnetic structures with a large spirality angle of pole tips were investigated in a number of works and are used in superconducting cyclotrons, H^- ion cyclotrons, *etc*. With the design and construction of an 80 MeV isochronous H^- cyclotron, such studies were continued and extended. In this work, a relatively simple approach for analysing the spiral structure is proposed.

The magnetic structure with a large spirality angle of the pole tips is used in cases when vertical focusing from the flutter (field difference in the valley and the sector) is insufficient and it is necessary to add the angle focusing. This situation is typical for superconducting cyclotrons and for cyclotrons that accelerate negative hydrogen ions. Moreover, at Joint Institute for Nuclear Research (JINR) in Dubna, such structures were investigated and a cyclotron and a synchrocyclotron with sectors in the form of an Archimedes spiral with a maximum spirality angle of up to 70 degrees were built. Numerical calculations of the magnetic field for an isochronous superconducting cyclotron with spiral sectors in the approximation of their uniform magnetization were carried out [1].

Two effects were noted: a decrease in the flutter in the central region with the introduction of spirality and a mismatch between the spirality of the sector iron and the magnetic field. However, calculations made for a specific geometry are not applicable in the case of a different design. With the construction of an isochronous cyclotron for accelerating H^- ions up to 40–80 MeV [2, 3], studies of the focusing properties of spiral structures were continued and expanded. Modern 3*D* software codes simplify the design of the magnetic field of any configuration by using trial and error. However, to speed up the procedure and to reduce the number of options for a 3*D* analysis, it is useful to first perform a simplified and visual analysis of the system and estimate the importance of various parameters in the framework of a simpler 2*D* approximation.

2. Development of two-dimensional approximation

2.1. Optimization of the magnet gaps

As a first approximation, the hill ($2g_h$) and valley ($2g_v$) gaps for each fixed sector thickness (h_s) were selected using 2*D* POISCR calculations based on the proposed new fill factor method. In this method, a 3*D* problem is reduced to a 2*D* one. The iron rings or the so-called shims mounted on the magnet poles and providing an isochronous rise in the field are calculated using a 2*D* program with a reduced value of the magnetic permeability $\mu_{\text{new}}(B) = \mu(B) \cdot C$. The permeability is reduced by a factor $C(r)$ – the so-called filling factor equal to the ratio of the azimuthal length of the sector to the length of the periodicity element at each radius *r*. The gap of the magnet obtained in this way corresponds to the gap of the hill, and there is no additional shim in the valley. Thus, two variants of the gaps of the main magnet and the gaps of the hills and the valleys were analysed. The parameters of these variants are presented in the caption to Fig. 1.

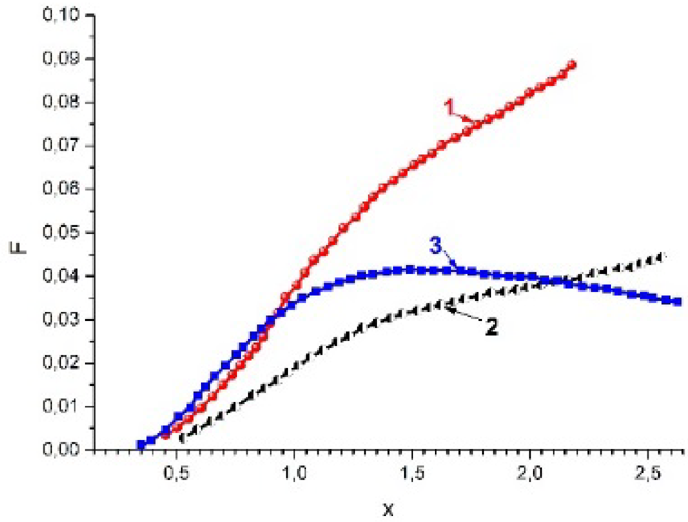

Fig. 1. The dependence of the flutter on the dimensionless parameter $x = r / N \cdot g_h$, where $N = 4$, for two options: *1* – $2g_v = 386$ mm, $2g_h = 170$ mm, $h_s = 108$ mm; *2* – $2g_v = 284$ mm, $2g_h = 145$ mm, $h_s = 69$ mm; *3* – a variant of uniform magnetization for the case of the geometry of the first variant

2.2. Flutter problem

The azimuthal variation of the magnetic field [1] is determined by the so-called flutter $F(r)$:

$$F(r) = <(B - <B>)^2>/<B>^2, \quad <\ldots> = (2\pi)^{-1} \int_0^{2\pi} \ldots d\theta.$$

The flutter can be represented as a Fourier harmonics expansion of the azimuthal variation of the magnetic field. The fundamental contribution to the expansion is made by the general focusing harmonic associated with the number of sectors and periodicity elements (in our case, $N = 4$). If we denote the value of the fundamental focusing harmonic $f = B_N / <B>$, then $F = f^2/2$.

An analytical calculation of the flutter is a complex and practically impossible problem, therefore, approximate methods were used. In particular, in Ref. [4], an expression was obtained for the general harmonic of the magnetic field variation in an isochronous cyclotron in the approximation of uniform magnetization of the sectors of the magnet,

$$B_N = 8M \sin(2\pi a/d) \exp(-2\pi g_h/d),$$

where $2a$ is the length of the sector along the azimuth for a given radius r; $2g_h$ is the gap in the hill; d is the period of the structure, equal to the total length of the hill and valley, $4\pi M = 21$ kG. It follows from this expression that for an isochronous cyclotron with a period of the magnetic field structure equal to $d = 2\pi r / N$, where N is the number of sectors, the flutter grows with increasing radius according to the law

$$F \sim \frac{B_N^2}{2} \sim \exp(-2/x), \quad x = r/N \cdot g_h. \tag{1}$$

Although this approximation is insufficient for obtaining accurate quantitative estimates, it allows, in a unified manner, to get an idea of the relationship between different parameters of the magnetic structure. Moreover, the introduction of the dimensionless parameter x enables the comparison of different variants of the structures. In particular, the flutter rises as the gap in the hill decreases and falls as the number of sectors increases.

The maximum flutter value corresponds to the case when the azimuthal extent of the sector and the valley are equal. In this case, $a = 0.25d$ and $\sin(\pi/2) = 1$. For the case when the parameter x becomes less than 0.5, the flutter drops very sharply, *i. e.*, at radii $r \leq 0.5Ng_h$, the azimuthal variation becomes ineffective, and focusing tends to zero. The results of these calculations are shown with curve *3* in Fig. 1.

The dependence of the flutter on the radius can be calculated using 3*D* programs. However, they are usually a commercial product. Therefore, in our case, to speed up and simplify the calculations, we used 2*D* approximate calculations. The flutter can be estimated using a 2*D* program if we replace the calculation of the edge effect along the azimuth with the calculation of the edge effect along the radius of the two-dimensional magnet with a gap in the form of teeth and valleys along the radius. In this case, the gap variation of a 2*D*-

magnet along the radius corresponds to the length and gaps of the sector and valley along the azimuth of the investigated 3*D* magnet.

Simple estimates carried out for the C-80 cyclotron show that the structure with straight sectors does not provide the necessary vertical focusing; therefore, it is necessary to use the following effect.

2.3. Influence of the spirality effect

As it is well known, the frequency of vertical oscillations, which determines the vertical focusing, can be expressed using the following approximation:

$$\nu_z^2 \approx -k + F \cdot S(r,\gamma), \quad S(r,\gamma) = 1 + 2tg^2\gamma, \tag{2}$$

where k is the rate index of the average magnetic field growth along the radius,

$$k = \left(\frac{r}{< B >}\right)\left(\frac{d < B >}{dr}\right) \approx \frac{2W}{E_0}.$$

Here W is the kinetic energy, E_0 = 938 MeV is the rest mass of the proton, γ is the spirality angle. The frequency of axial oscillations is determined by two oppositely acting terms – the defocusing of the isochronous rise of the mean field and the focusing action of the azimuthal variation of the field. The task is to select F and γ for each value of the energy W so that ν_z^2 remains greater than zero during acceleration. At the same time, to limit the dissociation of negative ions, it is necessary to strive for the lowest possible value of the magnetic field in the hill, *i. e.* to the minimum flutter value. The spiraling sector provides an increase in focusing force due to the non-perpendicular angle of particle entry into the sector region. Effectively, the flutter F is multiplied by the coefficient $S\ (r,\gamma)$; however, in a structure with a large spirality angle, the increase in focusing force is noticeably less [1] than could be expected from the above formula. This is due to a decrease of the flutter when the spirality angle is introduced and a mismatch between the iron and the magnetic spirality angles.

There is a simple geometric explanation for the first effect. With a large spirality angle, the difference between the sector length along the azimuth (*AB* in Fig. 2) and the width of the A_1B_1 sector determined from geometric considerations, becomes significant. In the case of straight sectors, the flutter is determined by the magnetic field difference in the hill and the valley. In this case, the field falls off along the azimuth. When a spiraling angle is introduced, the distance between the sectors along a line perpendicular to the centre-line of the sector is much smaller than the distance along the azimuth. This leads to a decrease in the effective length of the sector A_1B_1 and, accordingly, to a decrease in the length of the valley. From geometric considerations, we can conclude that the effective length of the A_1B_1 sector is approximately equal to the length of the sector along the azimuth *AB* multiplied by a factor equal to cosγ, *i. e.* $A_1B_1 \approx AB\cos\gamma$. With a decrease in the length of the sector, the effective length of the valley and the period of periodicity decrease. In this case, the dimensionless parameter x, introduced in expression (1) and related to the length of the periodicity period, will also decrease and becomes $x_{\text{eff}} = x\cos\gamma$. According to Fig. 1, a decrease in the value of the parameter $x_{\text{eff}} = x\cos\gamma$ leads to a shift along the graph towards lower flutter values. At intermediate and large radii, the spirality causes an increase in the edge focusing and a decrease in the flutter, and the total effect leads to an overall increase in focusing. However, at small radii, the introduction of spiraling reduces the effective sector length and this can lead to a very sharp drop in the flutter and a total decrease in focusing. The total effect of the introduction of the spiral sectors can be characterized by a parameter that is the product of two factors: the flutter F and S (r, γ). Since the flutter drops sharply at $x_{\text{eff}} < 0.5$, the introduction of spiraling in the central region leads to a decrease in focusing. For each radius, it is possible to calculate the limiting value of the spiraling angle, exceeding which spiraling does not give an increase in focusing. This value for each parameter x can be estimated by finding the solution for the following equation:

$$U(x,\gamma) = (F(x\cos\gamma)/F(x)) \cdot (1 + 2\text{tg}^2\gamma) - 1 = 0, \tag{3}$$

where $F(x)$ is a function of the type shown in Fig. 1.

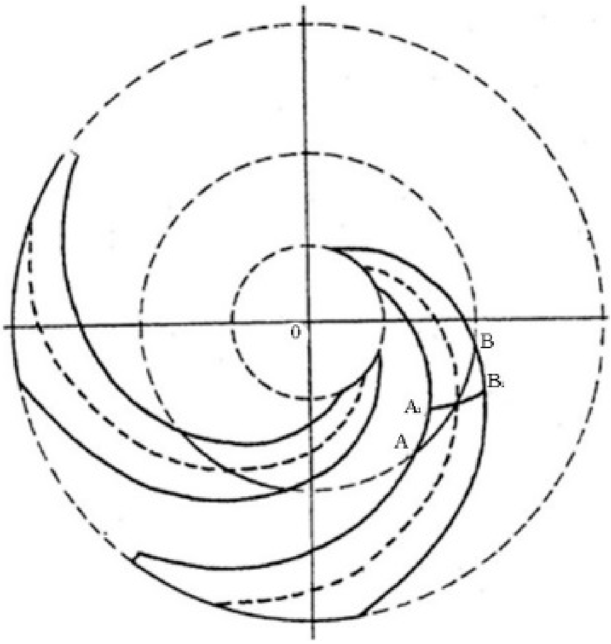

Fig. 2. There is a difference between the sector length along the azimuth *AB* and the "effective" length A_1B_1 at large spiraling angles. The effective sector width corresponding to its average line for a given *r* is equal to $A_1B_1 \approx AB\cos\gamma$

Figure 3 shows the limiting spiraling angle calculated by formula (3) as a function of the radius for the case of the C-80 cyclotron. According to Fig. 3, spiraling leads to decreasing of vertical focusing at radii smaller than 35 cm, and it is advisable to use a structure with a large spirality angle at radii greater than 35 cm. Thus, a structure with a large spiraling angle is effective only at radii larger that the hill gap value.

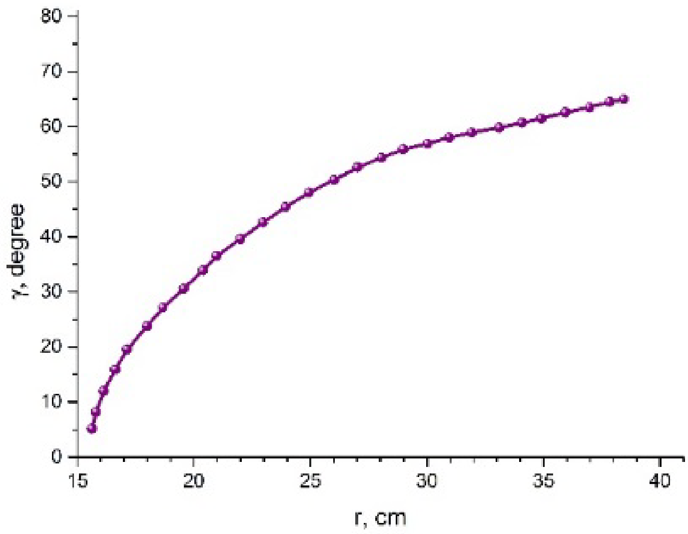

Fig. 3. Ultimate spirality angle for C-80 cyclotron in dependence on the radius for $2g_v$ = 386 mm, $2g_h$ = 170 mm, N = 4

3. 3*D* computation and experiment

When choosing the spiral angle in the cyclotron design, it is necessary to take into account the fact that the magnetic field does not completely repeat the iron sector geometry [5].

According to Fig. 4, there exists an effect of magnetic spirality "netration" into the region of straight sectors [6]. This effect leads to a decrease in vertical focusing at radii smaller than 35 cm. This explains the widespread use of direct sectors in the central region. It is also seen that at radii ~ $75 \leq r \leq 88$ cm there is a "lag" between the magnetic field spiraling angle and the geometrical one. The maximum lag reaches 7°, which at a spirality angle of 65° leads to a 30% decrease in focusing.

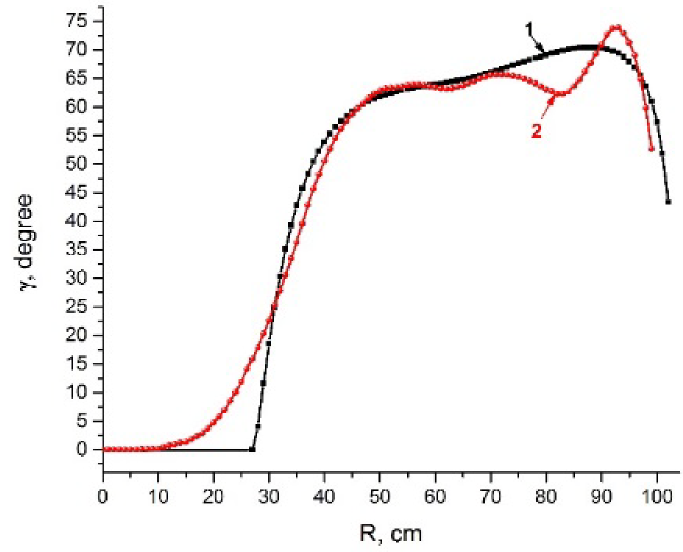

Fig. 4. The spirality angle γ (deg.) in dependence on the radius of the cyclotron: *1* – the data of the sector geometry spirality (*black*); *2* – the spirality of the fourth focusing harmonic of the magnetic field, obtained in 3*D* calculations and measurements of the magnetic field (*red*)

After preliminary assessments related to the choice of the parameters of the magnetic structure, the final variants were calculated in detail using the 3*D* MERMAID program.

In the calculations, to achieve the maximum accuracy, the magnetic structure was described using ~ 20.5 million straight prisms [7]. During the design process, two variants of the magnetic structure were considered at a finite radius: the flutter $F = 0.04$, the spirality angle $\gamma = 55°$, and $F = 0.025$ with $\gamma = 65°$, and the field variation amplitudes of 4.14 and 3.28 kG, respectively. Ultimately, the second option was adopted, providing a lower field in the hill, at which the loss of H^- ions due to electrodissociation does not exceed 2.6% [8].

4. Conclusion

The analysis of the structure of the magnetic field of a cyclotron with a high spirality angle, presented in this work, makes it possible to investigate the effect of different parameters of the structure and promptly compare various options. Such an approach provides the means to qualitatively analyse the effect of a decrease in the vertical focusing of the spiral structure at the centre of the cyclotron. It is expedient to use the structure with a large spiraling angle only at radii larger than the gap in the hill. In the central region, it is advisable to use direct sectors. The paper presents a technique that was used in the design of the magnetic structure of the C-80 cyclotron. The use of the limiting large values of the spirality angle in the C-80 cyclotron made it possible to obtain the limiting energy as high as 80 MeV in a magnet with a diameter of 2 m at an extraction radius of 0.9 m. The magnetic structure allows us to limit the negative ions electrodissociation to less than 2.6%.

References

1. H.G. Blosser, D.A. Jonson, Nucl. Instr. Meth. **121**, 301 (1974).
2. Yu.N. Gavrish *et al.*, in *Proc. 25th Rus. Part. Accelerator Conf. (RuPAC-2016)*, 180 (2016).
3. N.K. Abrossimov, S.A. Artamonov, V.A. Eliseev, G.A. Riabov, Preprint PNPI 2049, 1995.
4. V.I. Danilov, Preprint JINR-R-409 (1959).
5. S.A. Artamonov, E.M. Ivanov, G.A. Riabov, in *Proc. 26th Rus. Part. Accelerator Conf. (RuPAC-2018)*, 103 (2018).
6. D.A. Amerkanov, S.A. Artamonov, E.M. Ivanov *et al.*, in *Proc. 27th Rus. Part. Accelerator Conf. (RuPAC-2021)*, 219 (2021).
7. S.A. Artamonov, E.M. Ivanov, G.A. Riabov, N.A. Chernov, in *Proc. 23th Rus. Part. Accelerator Conf. (RuPAC-2012)*, 475 (2012).
8. S.A. Artamonov, D.A. Amerkanov, G.I. Gorkin *et al.*, in *Proc. of XX Int. Conf. Beam Dynamics & Optimization (BDO-2014)*, 18 (2014).