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Mean-field transport theory for the two-flavour NJL model

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Abstract. By making a decomposition of the Wigner function in spinor and isospin space we derive a set of kinetic equations for the quark distribution functions and the spin densities. A detailed analysis of the consequences imposed by the chiral invariance on the form of the transport equations is presented.

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I Introduction

The purpose of this paper is to generalize the already existing formulation of the mean-field transport theory for the Nambu – Jona-Lasinio (NJL) model [1,2]. The main aspect of this generalization is an extension of the simplified one-flavor approach [3] to the more realistic two-flavor case. Formally, this is achieved by applying the technique of decomposition of the Wigner function in both spinor and isospin space. In this way we can also extend and complement some of the earlier calculations done in the framework of QED [4] and QHD (quantum hadrondynamics) [5], where the Wigner function was decomposed only in spinor space.

The transport theory for the NJL model was initially developed by Zhang and Wilets [6]. They used the closed-time-path formalism together with the effective action method. Unfortunately, the explicit form of the gap equation derived by Zhang and Wilets is valid only in the mean-field approximation. This fact strongly restricts the possible applications of their advanced method. A further development in the transport theory for the NJL model was done in [3], where *chirally invariant* mean-field transport equations were derived for the one-flavour version of the model. In the present paper we generalize these results to the two-flavour case. We take into account all the coefficients in the spinor and isospin decomposition. Thus, although our formulation is restricted to the mean-field approximation it does not include any further simplifying assumptions concerning the structure of the Wigner function. This allows us to study the space-time evolution of the quark distribution functions and the dynamics of spin. In this way we complement the results of [6] where only spin saturated systems were considered. We include also the possibility of having nonzero pseudoscalar condensates, which is crucial in studies of the chiral invariance of the theory, and is of interest in connection with the possible creation of disoriented chiral condensates [7].

We concentrate in more details on two special cases. The first one corresponds to the exact chiral limit, i.e., to the situation when the current quark masses vanish. On the other hand, the second case corresponds to the physical situation when the current quark masses do not vanish and are (slightly) different from each other. Studying the chiral limit we investigate how the concepts of chiral invariance can be *explicitly* included in the formulation of the transport theory. In this case we follow the treatment of [3].

The study of the transport theory for the NJL model becomes interesting in the context of the ultra-relativistic heavy-ion collisions. One expects that these highly energetic processes offer the possibility of creation of a short-lived quark-gluon plasma (QGP) [8]. It is very likely that during such a deconfinement phase transition the chiral symmetry is restored. This is indicated by lattice simulations of QCD [9] which show that the two phase transitions occur at the same temperature. Transport theory based on the NJL model gives us the possibility to study phenomena connected with the chiral phase transition in systems out of thermodynamic and chemical equilibrium. This is an attractive feature of the model, since the methods for applying QCD directly to such situations have not been elaborated yet. The QCD transport theory has been formulated in papers by Heinz [10] and by Elze et al. [11] but this approach, with few exceptions [12], is not frequent in phenomenological applications.

In the mean-field approximation the NJL model includes only quark degrees of freedom. Therefore, this approach is not fully appropriate for the description of hadronic matter at low temperatures or densities and one has to go beyond the mean-field approximation in order to obtain the agreement of the low-temperature NJL results with chiral perturbation theory [13]. On the other hand, close to the deconfinement and chiral phase transitions the quark degrees of freedom become relevant. In this situation the mean-field approach is appropriate since it describes many important features of the chiral phase transition like, e.g., the decrease of the in-medium quark condensate. Consequently, the present formulation of the transport theory is most suitable for description of the phenomena happening in the neighborhood of the chiral phase transition [14–16]. Work concerning the inclusion of meson degrees of freedom into the transport theory for the NJL model (this is equivalent to the extension of the mean-field approach) is currently being carried out by the Heidelberg group [17].

The paper is organized as follows. In the next Section we define the model. In Sect. III we introduce the Wigner function, define its spinor and isospin decomposition, and discuss the chiral transformation rules. Section IV presents the quantum kinetic equations satisfied by the coefficients of the spinor and isospin decomposition. In Sect. V we do the classical approximation and derive the so-called constraint equations. This is done separately for massless and massive quarks. The classical kinetic equations for the quark distribution functions and for spin densities are derived in Sect. VI. We summarize the paper in Sect. VII.

II Definition of the model

In this paper we consider the Lagrangian

$$\mathcal{L} = \bar{\Psi}\,(i\,\partial\!\!\!/ - \hat{m})\,\Psi + \frac{G}{2}\left[(\bar{\Psi}\Psi)^2 + (\bar{\Psi}i\gamma_5\boldsymbol{\tau}\Psi)^2\right], \qquad (1)$$

where $\Psi = (\psi_u, \psi_d)$ is the doublet of the Dirac fields, G is the coupling constant, $\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3)$ are the Pauli isospin matrices, and $\hat{m}$ is the matrix containing current quark masses m_u and m_d. This matrix can be written in the form $\hat{m} = m_0 + \mathbf{m}\cdot\boldsymbol{\tau}$, where $m_0 = \frac{1}{2}(m_u + m_d)$, $m_1 = m_2 = 0$ and $m_3 = \frac{1}{2}(m_u - m_d)$. For simplicity, the color degrees of freedom of quarks have been neglected.

Using Lagrangian (1) in the mean-field approximation we find the following equation of motion for the field Ψ

$$\left[i\,\partial\!\!\!/ - \hat{m} - \sigma(x) - i\gamma_5\boldsymbol{\pi}(x)\cdot\boldsymbol{\tau}\right]\Psi(x) = 0. \qquad (2)$$

Here the mean fields $\sigma(x)$ and $\boldsymbol{\pi}(x)$ are defined through the expressions

$$\sigma(x) = -G\,\langle\,\bar{\Psi}(x)\Psi(x)\rangle = -G\ \mathrm{Tr}\left(\hat{\rho}\,\bar{\Psi}(x)\Psi(x)\right) \qquad (3)$$

and

$$\begin{aligned}\boldsymbol{\pi}(x) &= -G\,\langle\,\bar{\Psi}(x)i\gamma_5\boldsymbol{\tau}\Psi(x)\rangle \\ &= -G\ \mathrm{Tr}\left(\hat{\rho}\,\bar{\Psi}(x)i\gamma_5\boldsymbol{\tau}\Psi(x)\right),\end{aligned} \qquad (4)$$

where $\hat{\rho}$ is the density operator and Tr denotes the trace over the Fock space.

An important feature of the NJL model is invariance under several symmetries of QCD. The Lagrangian (1) is invariant under $\mathrm{U}_V(1)$ transformations, which leads to the conservation of the baryon current

$$\partial_\mu V^\mu(x) = 0, \quad V^\mu(x) = \langle\bar{\Psi}(x)\gamma^\mu\Psi(x)\rangle. \qquad (5)$$

In the isospin symmetric case, $m_u = m_d$, the Lagrangian (1) is in addition invariant under $\mathrm{SU}_V(2)$ transformations. This fact gives the conservation of the isospin current

$$\partial_\mu \mathbf{V}^\mu(x) = 0, \quad \mathbf{V}^\mu(x) = \langle\bar{\Psi}(x)\gamma^\mu\boldsymbol{\tau}\Psi(x)\rangle. \qquad (6)$$

In the chiral limit, $m_u = m_d = 0$, (1) does not change under $\mathrm{SU}_A(2)$ transformations. This invariance leads to the conservation of the axial current

$$\partial_\mu \mathbf{A}^\mu(x) = 0, \quad \mathbf{A}^\mu(x) = \langle\bar{\Psi}(x)\gamma^\mu\gamma_5\boldsymbol{\tau}\Psi(x)\rangle. \qquad (7)$$

We note that we have defined the conserved currents as the expectation values in the state characterized by the density matrix $\hat{\rho}$. In this case the conservation laws follow directly from (2)–(4).

Let us now introduce the Green function

$$S^<_{ij\,\alpha\beta}(x,y) = \langle\bar{\Psi}_{j\beta}(y)\Psi_{i\alpha}(x)\rangle, \qquad (8)$$

where i, j and α, β are isospin and spinor indices, respectively ($i, j = 1, 2$ and $\alpha, \beta = 1, ..., 4$). One can easily check that the Green function (8) fulfills the same equation as the field Ψ does, namely

$$\left[i\,\partial\!\!\!/ - \hat{m} - \sigma(x) - i\gamma_5\boldsymbol{\pi}(x)\cdot\boldsymbol{\tau}\right]S^<(x,y) = 0. \qquad (9)$$

Moreover, the mean fields $\sigma(x)$ and $\boldsymbol{\pi}(x)$ can be determined directly from $S^<(x,y)$ via relations

$$\begin{aligned}\sigma(x) &= -G\ \mathrm{tr}\,S^<(x,x), \\ \boldsymbol{\pi}(x) &= -G\ \mathrm{tr}\,i\gamma_5\boldsymbol{\tau}S^<(x,x),\end{aligned} \qquad (10)$$

with the trace tr taken over spinor and flavor indices. One can observe that (9) and (10) form a closed system of equations. It will be the subject of our studies in the following Chapters.

III Wigner function

A Decomposition in spinor and isospin space

Our aim is to derive a set of kinetic equations for the classical distribution functions. This can be achieved in the standard way by introducing the Wigner function

$$W_{ij\,\alpha\beta}(X,p) = \frac{1}{\hbar^4}\int d^4u\, e^{\frac{i}{\hbar}p\cdot u}\, S^<_{ij\,\alpha\beta}\left(X + \tfrac{1}{2}u, X - \tfrac{1}{2}u\right) \qquad (11)$$

and finding the equations of motion for this function in the limit $\hbar \to 0$. In Eq. (11) the quantity X is the center-of-mass coordinate, $X = \frac{1}{2}(x + y)$, and u is the relative coordinate, $u = x - y$.

Starting from (9) and using the well known results for the Wigner transform of the derivative of a two-point function, $\partial f(x,y)/\partial x^\mu$, and the Wigner transform of the product of a one-point function with the two-point one, $f(x)g(x,y)$, we find the following equation

$$\left[K^\mu\gamma_\mu - M(X) - \mathbf{m}\cdot\boldsymbol{\tau} + \frac{i\hbar}{2}\partial_\mu M(X)\,\partial_p^\mu \right. \tag{12}$$
$$\left. -i\gamma_5\,\boldsymbol{\pi}(X)\cdot\boldsymbol{\tau} - \frac{\hbar}{2}\gamma_5\,\partial_\mu\boldsymbol{\pi}(X)\cdot\boldsymbol{\tau}\,\partial_p^\mu\right] W(X,p) = 0,$$

where we use the standard notation $\partial^\mu = \partial/\partial X_\mu$, $\partial_p^\mu = \partial/\partial p_\mu$, $K^\mu = p^\mu + \frac{i\hbar}{2}\partial^\mu$ and $M(X) = \sigma(X) + m_0$. We note that in (12) higher order gradients have been neglected, so it is valid only for weakly inhomogeneous systems. In addition, there is no collision term on the right-hand side of (12), which is a consequence of the mean-field approximation. The effects of collisions have been recently studied in [16].

Since the Wigner function satisfies the condition $\overline{W}(X,p) = \gamma^0 W^\dagger(X,p)\gamma^0 = W(X,p)$, it can be represented by the following combination of Dirac tensors Γ

$$W = \hat{\mathcal{F}} + i\gamma_5\hat{\mathcal{P}} + \gamma^\mu\hat{\mathcal{V}}_\mu + \gamma^\mu\gamma_5\hat{\mathcal{A}}_\mu + \frac{1}{2}\sigma^{\mu\nu}\hat{\mathcal{S}}_{\mu\nu}. \tag{13}$$

In the decomposition (13) each coefficient $\hat{C}$ [i.e., the functions $\hat{\mathcal{F}}(X,p)$,$\hat{\mathcal{P}}(X,p)$,$\hat{\mathcal{V}}_\mu(X,p)$, $\hat{\mathcal{A}}_\mu(X,p)$ and $\hat{\mathcal{S}}_{\mu\nu}(X,p)$] is a hermitian two by two matrix. Thus, it can be further decomposed in the isospin space according to the rule

$$\hat{C} = C + \mathbf{C}\cdot\boldsymbol{\tau}. \tag{14}$$

In this way we generalize the usual approach, which does not take into account such structure. Similarly to (14) we introduce the quantity $\hat{M}(X)$ defined as $\hat{M}(X) = M(X) + \mathbf{m}\cdot\boldsymbol{\tau} = \sigma(x) + m_0 + \mathbf{m}\cdot\boldsymbol{\tau}$.

Many of the functions defined by the spinor and isospin decomposition, (13) and (14), have a direct physical interpretation. For example, the quantities $\mathcal{V}^\mu(X,p)$, $\boldsymbol{\mathcal{V}}^\mu(X,p)$ and $\boldsymbol{\mathcal{A}}^\mu(X,p)$ are the phase-space densities of the baryon, isospin and axial currents. Moreover, according to (10), the mean fields $\sigma(X)$ and $\boldsymbol{\pi}(X)$ are simply related to the functions $\mathcal{F}(X,p)$ and $\boldsymbol{\mathcal{P}}(X,p)$

$$\sigma(X) = -8G\int\frac{d^4p}{(2\pi)^4}\mathcal{F}(X,p) \tag{15}$$

and

$$\boldsymbol{\pi}(X) = 8G\int\frac{d^4p}{(2\pi)^4}\boldsymbol{\mathcal{P}}(X,p). \tag{16}$$

For the physical interpretation of the other components we refer the reader to [4,18].

B Chiral transformations

The most important feature of Lagrangian (1) is its exact chiral invariance in the case $\hat{m} = 0$. Studying the transport theory it is interesting to analyze the consequences of this symmetry for the transport phenomena. This has been already discussed in detail for the one-flavor version of the model [3]. In this paper we shall generalize these results to the two-flavor case.

The $\mathrm{SU}_A(2)$ chiral transformation of the field Ψ is defined as follows

$$\Psi \to \Psi' = \exp(-i\gamma_5\frac{\boldsymbol{\chi}\cdot\boldsymbol{\tau}}{2})\Psi = \left(\cos\frac{\chi}{2} - i\gamma_5\mathbf{n}\cdot\boldsymbol{\tau}\sin\frac{\chi}{2}\right)\Psi. \tag{17}$$

Here $\mathbf{n}$ is a unit vector in direction of $\boldsymbol{\chi}$ and χ is the length of $\boldsymbol{\chi}$. The property (17) induces the transformation rules:

$$\sigma \to \sigma' = \sigma\cos\chi - \boldsymbol{\pi}\cdot\mathbf{n}\sin\chi \tag{18}$$

for the scalar mean field and

$$\boldsymbol{\pi} \to \boldsymbol{\pi}' = \boldsymbol{\pi}\cos^2\frac{\chi}{2} - [2(\boldsymbol{\pi}\cdot\mathbf{n})\mathbf{n} - \boldsymbol{\pi}]\sin^2\frac{\chi}{2} + \sigma\mathbf{n}\sin\chi \tag{19}$$

for the pseudoscalar mean field, respectively. As expected, the combination $\sigma^2 + \boldsymbol{\pi}^2$ is an invariant of the chiral transformations. Another consequence of (17) is the transformation law for the Wigner function

$$W \to W' = \exp(-i\gamma_5\frac{\boldsymbol{\chi}\cdot\boldsymbol{\tau}}{2})\; W\; \exp(-i\gamma_5\frac{\boldsymbol{\chi}\cdot\boldsymbol{\tau}}{2}). \tag{20}$$

Equation (20) leads to a set of the transformation rules for the coefficients in the spinor and isospin decomposition. Their full form is listed in the Appendix. Below we give the prescriptions for the infinitesimal chiral transformations (denoting the infinitesimal value of $\boldsymbol{\chi}$ by $\delta\boldsymbol{\chi}$). Infinitesimal chiral transformations of the scalar and pseudoscalar coefficients are:

$$\mathcal{F} \to \mathcal{F}' = \mathcal{F} + \boldsymbol{\mathcal{P}}\cdot\mathbf{n}\;\delta\chi, \tag{21a}$$
$$\boldsymbol{\mathcal{F}} \to \boldsymbol{\mathcal{F}}' = \boldsymbol{\mathcal{F}} + \mathcal{P}\mathbf{n}\;\delta\chi, \tag{21b}$$
$$\mathcal{P} \to \mathcal{P}' = \mathcal{P} - \boldsymbol{\mathcal{F}}\cdot\mathbf{n}\;\delta\chi, \tag{21c}$$
$$\boldsymbol{\mathcal{P}} \to \boldsymbol{\mathcal{P}}' = \boldsymbol{\mathcal{P}} - \mathcal{F}\mathbf{n}\;\delta\chi, \tag{21d}$$

and of the vector and axial-vector coefficients:

$$\mathcal{V}_\mu \to \mathcal{V}'_\mu = \mathcal{V}_\mu, \tag{22a}$$
$$\boldsymbol{\mathcal{V}}_\mu \to \boldsymbol{\mathcal{V}}'_\mu = \boldsymbol{\mathcal{V}}_\mu - \mathbf{n}\times\boldsymbol{\mathcal{A}}_\mu\;\delta\chi, \tag{22b}$$
$$\mathcal{A}_\mu \to \mathcal{A}'_\mu = \mathcal{A}_\mu, \tag{22c}$$
$$\boldsymbol{\mathcal{A}}_\mu \to \boldsymbol{\mathcal{A}}'_\mu = \boldsymbol{\mathcal{A}}_\mu - \mathbf{n}\times\boldsymbol{\mathcal{V}}_\mu\;\delta\chi. \tag{22d}$$

The quantities $\mathcal{S}_{\mu\nu}, \boldsymbol{\mathcal{S}}_{\mu\nu}, \tilde{\mathcal{S}}_{\mu\nu}$ and $\tilde{\boldsymbol{\mathcal{S}}}_{\mu\nu}$ [the last two are the dual spin tensors defined below in (24)] transform in the same way as the functions $\mathcal{F}, \boldsymbol{\mathcal{F}}, \mathcal{P}$ and $\boldsymbol{\mathcal{P}}$. This property resembles the case of the one-flavor calculation (see (22) of [3]). In addition, similarly to the one-flavor case one finds that both $\mathcal{V}^\mu$ and $\mathcal{A}^\mu$ are chirally invariant.

IV Kinetic equations

Substituting formula (13) into (12) and comparing the coefficients appearing at the Dirac tensors we find a set of coupled equations

$$K^\mu \hat{\mathcal{V}}_\mu - \hat{M}\hat{\mathcal{F}} + \boldsymbol{\pi}\cdot\boldsymbol{\tau}\hat{\mathcal{P}} = -\frac{i\hbar}{2}\left(\partial_\nu M \partial_p^\nu \hat{\mathcal{F}} - \partial_\nu \boldsymbol{\pi}\cdot\boldsymbol{\tau}\partial_p^\nu \hat{\mathcal{P}}\right), \tag{23a}$$

$$-iK^\mu \hat{\mathcal{A}}_\mu - \hat{M}\hat{\mathcal{P}} - \boldsymbol{\pi}\cdot\boldsymbol{\tau}\hat{\mathcal{F}} = -\frac{i\hbar}{2}\left(\partial_\nu M \partial_p^\nu \hat{\mathcal{P}} + \partial_\nu \boldsymbol{\pi}\cdot\boldsymbol{\tau}\partial_p^\nu \hat{\mathcal{F}}\right), \tag{23b}$$

$$K_\mu \hat{\mathcal{F}} + iK^\nu \hat{\mathcal{S}}_{\nu\mu} - \hat{M}\hat{\mathcal{V}}_\mu + i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\hat{\mathcal{A}}_\mu = -\frac{i\hbar}{2}\left(\partial_\nu M \partial_p^\nu \hat{\mathcal{V}}_\mu - i\partial_\nu \boldsymbol{\pi}\cdot\boldsymbol{\tau}\partial_p^\nu \hat{\mathcal{A}}_\mu\right), \tag{23c}$$

$$iK^\mu \hat{\mathcal{P}} - K_\nu \hat{\tilde{\mathcal{S}}}^{\nu\mu} - \hat{M}\hat{\mathcal{A}}^\mu + i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\hat{\mathcal{V}}^\mu = -\frac{i\hbar}{2}\left(\partial_\nu M \partial_p^\nu \hat{\mathcal{A}}^\mu - i\partial_\nu \boldsymbol{\pi}\cdot\boldsymbol{\tau}\partial_p^\nu \hat{\mathcal{V}}^\mu\right), \tag{23d}$$

$$i(K^\mu \hat{\mathcal{V}}^\nu - K^\nu \hat{\mathcal{V}}^\mu) - \varepsilon^{\mu\nu\alpha\beta} K_\alpha \hat{\mathcal{A}}_\beta - \boldsymbol{\pi}\cdot\boldsymbol{\tau}\hat{\tilde{\mathcal{S}}}^{\mu\nu} + \hat{M}\hat{\mathcal{S}}^{\mu\nu} = \frac{i\hbar}{2}(\partial_\gamma M \partial_p^\gamma \hat{\mathcal{S}}^{\mu\nu} - \partial_\gamma \boldsymbol{\pi}\cdot\boldsymbol{\tau}\partial_p^\gamma \hat{\tilde{\mathcal{S}}}^{\mu\nu}). \tag{23e}$$

In (23d) and (23e) we have introduced the dual spin tensor $\hat{\tilde{\mathcal{S}}}^{\mu\nu}$ defined through relation

$$\hat{\tilde{\mathcal{S}}}^{\mu\nu} = \tfrac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\hat{\mathcal{S}}_{\alpha\beta}. \tag{24}$$

Equations (23) represent the *spinor* decomposition of (12). It is a generalization of the set of equations (24) – (28) from [3] where a one-flavor formulation has been analyzed. In the two-flavor case each of the equations in (IV) has a matrix form. In order to put the equations in a more transparent form, we perform an *isospin* decomposition. This procedure allows us to deal only with real quantities, which represent physical observables. At the first stage of the isospin decomposition we insert expressions of the form (14) into (23). Subsequently, we calculate the sum and the difference of the initial matrix equation and its adjoint for all the formulas appearing in (23). In the resulting five pairs of equations one compares the coefficients at the Pauli isospin matrices. In doing so, it is convenient to use two relations

$$\mathbf{z}\cdot\{\boldsymbol{\tau}, \hat{C}\} = 2\,\mathbf{z}\cdot\mathbf{C} + 2C\,\mathbf{z}\cdot\boldsymbol{\tau} \tag{25}$$

and

$$\mathbf{z}\cdot[\boldsymbol{\tau}, \hat{C}] = 2i\,(\mathbf{z}\times\mathbf{C})\,\boldsymbol{\tau}, \tag{26}$$

where $\mathbf{z}$ is an arbitrary three-vector and [,] ({ , }) denotes the commutator (anticommutator). Finally, we get five groups of equations.

Scalar equations:

$$p^\mu \mathcal{V}_\mu - M\mathcal{F} - \mathbf{m}\cdot\boldsymbol{\mathcal{F}} + \boldsymbol{\pi}\cdot\boldsymbol{\mathcal{P}} = 0, \tag{27a}$$

$$p^\mu \boldsymbol{\mathcal{V}}_\mu - M\boldsymbol{\mathcal{F}} - \mathbf{m}\mathcal{F} + \boldsymbol{\pi}\mathcal{P} = -\frac{\hbar}{2}\left(\partial_\nu \boldsymbol{\pi}\times\partial_p^\nu \boldsymbol{\mathcal{P}}\right), \tag{27b}$$

$$\frac{\hbar}{2}\partial^\mu \mathcal{V}_\mu = -\frac{\hbar}{2}\partial_\nu M \partial_p^\nu \mathcal{F} + \frac{\hbar}{2}\partial_\nu \boldsymbol{\pi}\cdot\partial_p^\nu \boldsymbol{\mathcal{P}}, \tag{27c}$$

$$\frac{\hbar}{2}\partial^\mu \boldsymbol{\mathcal{V}}_\mu - \mathbf{m}\times\boldsymbol{\mathcal{F}} + \boldsymbol{\pi}\times\boldsymbol{\mathcal{P}} = -\frac{\hbar}{2}\partial_\nu M \partial_p^\nu \boldsymbol{\mathcal{F}} + \frac{\hbar}{2}\partial_\nu \boldsymbol{\pi}\partial_p^\nu \mathcal{P}. \tag{27d}$$

Pseudoscalar equations:

$$\frac{\hbar}{2}\partial^\mu \mathcal{A}_\mu - M\mathcal{P} - \mathbf{m}\cdot\boldsymbol{\mathcal{P}} - \boldsymbol{\pi}\cdot\boldsymbol{\mathcal{F}} = 0, \tag{28a}$$

$$\frac{\hbar}{2}\partial^\mu \boldsymbol{\mathcal{A}}_\mu - M\boldsymbol{\mathcal{P}} - \mathbf{m}\mathcal{P} - \boldsymbol{\pi}\mathcal{F} = \frac{\hbar}{2}\partial_\nu \boldsymbol{\pi}\times\partial_p^\nu \boldsymbol{\mathcal{F}}, \tag{28b}$$

$$p^\mu \mathcal{A}_\mu = \frac{\hbar}{2}\partial_\nu M \partial_p^\nu \mathcal{P} + \frac{\hbar}{2}\partial_\nu \boldsymbol{\pi}\cdot\partial_p^\nu \boldsymbol{\mathcal{F}}, \tag{28c}$$

$$\mathbf{m}\times\boldsymbol{\mathcal{P}} + \boldsymbol{\pi}\times\boldsymbol{\mathcal{F}} + p^\mu \boldsymbol{\mathcal{A}}_\mu = \frac{\hbar}{2}\partial_\nu M \partial_p^\nu \boldsymbol{\mathcal{P}} + \frac{\hbar}{2}\partial_\nu \boldsymbol{\pi}\partial_p^\nu \mathcal{F}. \tag{28d}$$

Vector equations:

$$p_\mu \mathcal{F} - \frac{\hbar}{2}\partial^\nu \mathcal{S}_{\nu\mu} - M\mathcal{V}_\mu - \mathbf{m}\cdot\boldsymbol{\mathcal{V}}_\mu = -\frac{\hbar}{2}\partial_\nu \boldsymbol{\pi}\cdot\partial_p^\nu \boldsymbol{\mathcal{A}}_\mu, \tag{29a}$$

$$-\boldsymbol{\pi}\times\boldsymbol{\mathcal{A}}_\mu + p_\mu \boldsymbol{\mathcal{F}} - \frac{\hbar}{2}\partial^\nu \boldsymbol{\mathcal{S}}_{\nu\mu} - M\boldsymbol{\mathcal{V}}_\mu - \mathbf{m}\mathcal{V}_\mu = -\frac{\hbar}{2}\partial_\nu \boldsymbol{\pi}\partial_p^\nu \mathcal{A}_\mu, \tag{29b}$$

$$\frac{\hbar}{2}\partial_\mu \mathcal{F} + p^\nu \mathcal{S}_{\nu\mu} + \boldsymbol{\pi}\cdot\boldsymbol{\mathcal{A}}_\mu = -\frac{\hbar}{2}\partial_\nu M \partial_p^\nu \mathcal{V}_\mu, \tag{29c}$$

$$-\mathbf{m}\times\boldsymbol{\mathcal{V}}_\mu + \frac{\hbar}{2}\partial_\mu \boldsymbol{\mathcal{F}} + p^\nu \boldsymbol{\mathcal{S}}_{\nu\mu} + \boldsymbol{\pi}\mathcal{A}_\mu = -\frac{\hbar}{2}\partial_\nu M \partial_p^\nu \boldsymbol{\mathcal{V}}_\mu - \frac{\hbar}{2}\partial_\nu \boldsymbol{\pi}\times\partial_p^\nu \boldsymbol{\mathcal{A}}_\mu. \tag{29d}$$

Axial-vector equations:

$$-\frac{\hbar}{2}\partial^\mu \mathcal{P} - p_\nu \tilde{\mathcal{S}}^{\nu\mu} - M\mathcal{A}^\mu - \mathbf{m}\cdot\boldsymbol{\mathcal{A}}^\mu = -\frac{\hbar}{2}\partial_\nu \boldsymbol{\pi}\cdot\partial_p^\nu \boldsymbol{\mathcal{V}}^\mu, \tag{30a}$$

$$-\boldsymbol{\pi}\times\boldsymbol{\mathcal{V}}^\mu - \frac{\hbar}{2}\partial^\mu \boldsymbol{\mathcal{P}} - p_\nu \tilde{\boldsymbol{\mathcal{S}}}^{\nu\mu} - M\boldsymbol{\mathcal{A}}^\mu - \mathbf{m}\mathcal{A}^\mu = -\frac{\hbar}{2}\partial_\nu \boldsymbol{\pi}\cdot\partial_p^\nu \mathcal{V}^\mu, \tag{30b}$$

$$p^\mu \mathcal{P} - \frac{\hbar}{2}\partial_\nu \tilde{\mathcal{S}}^{\nu\mu} + \boldsymbol{\pi}\cdot\boldsymbol{\mathcal{V}}^\mu = -\frac{\hbar}{2}\partial_\nu M\,\partial_p^\nu \mathcal{A}^\mu, \tag{30c}$$

$$-\mathbf{m}\times\boldsymbol{\mathcal{A}}^\mu + p^\mu \boldsymbol{\mathcal{P}} - \frac{\hbar}{2}\partial_\nu \tilde{\boldsymbol{\mathcal{S}}}^{\nu\mu} + \boldsymbol{\pi}\mathcal{V}^\mu = -\frac{\hbar}{2}\partial_\nu M\,\partial_p^\nu \boldsymbol{\mathcal{A}}^\mu - \frac{\hbar}{2}\partial_\nu \boldsymbol{\pi}\times\partial_p^\nu \boldsymbol{\mathcal{V}}^\mu. \tag{30d}$$

Tensor equations:

$$-\frac{\hbar}{2}(\partial^\mu \mathcal{V}^\nu - \partial^\nu \mathcal{V}^\mu) - \varepsilon^{\mu\nu\alpha\beta} p_\alpha \mathcal{A}_\beta - \boldsymbol{\pi}\cdot\tilde{\boldsymbol{\mathcal{S}}}^{\mu\nu} + M\mathcal{S}^{\mu\nu} + \mathbf{m}\cdot\boldsymbol{\mathcal{S}}^{\mu\nu} = 0, \tag{31a}$$

$$-\frac{\hbar}{2}(\partial^\mu \boldsymbol{\mathcal{V}}^\nu - \partial^\nu \boldsymbol{\mathcal{V}}^\mu) - \varepsilon^{\mu\nu\alpha\beta} p_\alpha \boldsymbol{\mathcal{A}}_\beta - \boldsymbol{\pi}\tilde{\mathcal{S}}^{\mu\nu} + M\boldsymbol{\mathcal{S}}^{\mu\nu} + \mathbf{m}\mathcal{S}^{\mu\nu} = \frac{\hbar}{2}\partial_\gamma \boldsymbol{\pi} \times \partial_p^\gamma \tilde{\boldsymbol{\mathcal{S}}}^{\mu\nu}, \tag{31b}$$

$$p^\mu \mathcal{V}^\nu - p^\nu \mathcal{V}^\mu - \frac{\hbar}{2}\varepsilon^{\mu\nu\alpha\beta}\partial_\alpha \mathcal{A}_\beta = \frac{\hbar}{2}\left(\partial_\gamma M \partial_p^\gamma \mathcal{S}^{\mu\nu} - \partial_\gamma \boldsymbol{\pi}\cdot\partial_p^\gamma \tilde{\boldsymbol{\mathcal{S}}}^{\mu\nu}\right), \tag{31c}$$

$$-\boldsymbol{\pi}\times\tilde{\boldsymbol{\mathcal{S}}}^{\mu\nu} + \mathbf{m}\times\boldsymbol{\mathcal{S}}^{\mu\nu} + p^\mu \boldsymbol{\mathcal{V}}^\nu - p^\nu \boldsymbol{\mathcal{V}}^\mu - \frac{\hbar}{2}\varepsilon^{\mu\nu\alpha\beta}\partial_\alpha \boldsymbol{\mathcal{A}}_\beta = \frac{\hbar}{2}\left(\partial_\gamma M \partial_p^\gamma \boldsymbol{\mathcal{S}}^{\mu\nu} - \partial_\gamma \boldsymbol{\pi}\partial_p^\gamma \tilde{\mathcal{S}}^{\mu\nu}\right). \tag{31d}$$

The first two equations in each set of formulas (27)–(31) can be regarded as the "hermitian" parts of (23). They correspond to (31)–(35) of the one-flavor approach [3]. The other equations in formulas (27)–(31) are the "anti-hermitian" parts of (23) and correspond to (37)–(41) of [3]. In the case $\hat{m} = 0$, using expressions (21) and (22) defining infinitesimal chiral transformations, one can check that (27)–(31) are chirally invariant.

Note that (27c), integrated over four-momentum p, leads to baryon current conservation (5). Analogously, one finds using (27d) and (16)

$$\hbar\, \partial_\mu \mathbf{V}^\mu(X) = 16\,\mathbf{m}\times\int \frac{d^4p}{(2\pi)^4}\, \boldsymbol{\mathcal{F}}(X,p), \tag{32}$$

which yields the conservation of the isospin current in the symmetric case $m_u = m_d$ [compare (6)]. Finally, (28b), (15) and (16) lead to the expression

$$\hbar\, \partial_\mu \mathbf{A}^\mu(X) = -\frac{2m_0}{G}\boldsymbol{\pi}(X) - 16\,\mathbf{m}\int \frac{d^4p}{(2\pi)^4}\, \mathcal{P}(X,p), \tag{33}$$

which reduces to the axial conservation law, (7), in the chiral limit $m_u = m_d = 0$. Thus we see that after making the gradient expansion, the conservation laws are still included in the transport equations.

V Constraint equations

A Classical Approximation

In order to obtain classical transport equations one makes an expansion of the functions $\hat{\mathcal{F}}(X,p)$, $\hat{\mathcal{P}}(X,p)$, $\hat{\mathcal{V}}_\mu(X,p)$, $\hat{\mathcal{A}}_\mu(X,p)$, $\hat{\mathcal{S}}_{\mu\nu}(X,p)$, $\sigma(X)$ and $\boldsymbol{\pi}(X)$ in powers of $\hbar$. In this way each coefficient in the decomposition (13) as well as the functions σ and $\boldsymbol{\pi}$ can be represented as a series

$$\hat{C} = \hat{C}_{(0)} + \hbar \hat{C}_{(1)} + \hbar^2 \hat{C}_{(2)} + \ldots \quad . \tag{34}$$

Inserting expressions of the form (34) into the kinetic equations (27)–(31) and comparing the terms appearing in the leading (zeroth) order of $\hbar$ we find a set of constraint equations, which connect different leading order terms of the coefficients $\hat{C}$.

Scalar constraint equations:

$$p^\mu \mathcal{V}^{(0)}_\mu - M_{(0)}\mathcal{F}_{(0)} - \mathbf{m}\cdot\boldsymbol{\mathcal{F}}_{(0)} + \boldsymbol{\pi}_{(0)}\cdot\boldsymbol{\mathcal{P}}_{(0)} = 0, \tag{35a}$$

$$p^\mu\, \boldsymbol{\mathcal{V}}^{(0)}_\mu - M_{(0)}\, \boldsymbol{\mathcal{F}}_{(0)} - \mathbf{m}\, \mathcal{F}_{(0)} + \boldsymbol{\pi}_{(0)}\, \mathcal{P}_{(0)} = 0, \tag{35b}$$

$$-\mathbf{m}\times\boldsymbol{\mathcal{F}}_{(0)} + \boldsymbol{\pi}_{(0)}\times\boldsymbol{\mathcal{P}}_{(0)} = 0. \tag{35c}$$

Pseudoscalar constraint equations:

$$M_{(0)}\mathcal{P}_{(0)} + \mathbf{m}\cdot\boldsymbol{\mathcal{P}}_{(0)} + \boldsymbol{\pi}_{(0)}\cdot\boldsymbol{\mathcal{F}}_{(0)} = 0, \tag{36a}$$

$$M_{(0)}\, \boldsymbol{\mathcal{P}}_{(0)} + \mathbf{m}\, \mathcal{P}_{(0)} + \boldsymbol{\pi}_{(0)}\, \mathcal{F}_{(0)} = 0, \tag{36b}$$

$$p^\mu \mathcal{A}^{(0)}_\mu = 0, \tag{36c}$$

$$\mathbf{m}\times\boldsymbol{\mathcal{P}}_{(0)} + \boldsymbol{\pi}_{(0)}\times\boldsymbol{\mathcal{F}}_{(0)} + p^\mu \boldsymbol{\mathcal{A}}^{(0)}_\mu = 0. \tag{36d}$$

Vector constraint equations:

$$p_\mu \mathcal{F}_{(0)} - M_{(0)}\mathcal{V}^{(0)}_\mu - \mathbf{m}\cdot\boldsymbol{\mathcal{V}}^{(0)}_\mu = 0, \tag{37a}$$

$$-\boldsymbol{\pi}_{(0)}\times\boldsymbol{\mathcal{A}}^{(0)}_\mu + p_\mu\, \boldsymbol{\mathcal{F}}_{(0)} - M_{(0)}\, \boldsymbol{\mathcal{V}}^{(0)}_\mu - \mathbf{m}\, \mathcal{V}^{(0)}_\mu = 0, \tag{37b}$$

$$p^\nu \mathcal{S}^{(0)}_{\nu\mu} + \boldsymbol{\pi}_{(0)}\cdot\boldsymbol{\mathcal{A}}^{(0)}_\mu = 0, \tag{37c}$$

$$-\mathbf{m}\times\boldsymbol{\mathcal{V}}^{(0)}_\mu + p^\nu \boldsymbol{\mathcal{S}}^{(0)}_{\nu\mu} + \boldsymbol{\pi}_{(0)}\mathcal{A}^{(0)}_\mu = 0. \tag{37d}$$

Axial-vector constraint equations:

$$p_\nu \tilde{\mathcal{S}}^{\nu\mu}_{(0)} + M_{(0)}\mathcal{A}^\mu_{(0)} + \mathbf{m}\cdot\boldsymbol{\mathcal{A}}^\mu_{(0)} = 0, \tag{38a}$$

$$\boldsymbol{\pi}_{(0)}\times\boldsymbol{\mathcal{V}}^\mu_{(0)} + p_\nu \tilde{\boldsymbol{\mathcal{S}}}^{\nu\mu}_{(0)} + M_{(0)}\boldsymbol{\mathcal{A}}^\mu_{(0)} + \mathbf{m}\, \mathcal{A}^\mu_{(0)} = 0, \tag{38b}$$

$$p^\mu \mathcal{P}_{(0)} + \boldsymbol{\pi}_{(0)}\cdot\boldsymbol{\mathcal{V}}^\mu_{(0)} = 0, \tag{38c}$$

$$-\mathbf{m}\times\boldsymbol{\mathcal{A}}^\mu_{(0)} + p^\mu \boldsymbol{\mathcal{P}}_{(0)} + \boldsymbol{\pi}_{(0)}\mathcal{V}^\mu_{(0)} = 0. \tag{38d}$$

Tensor constraint equations:

$$-\varepsilon^{\mu\nu\alpha\beta}p_\alpha \mathcal{A}^{(0)}_\beta - \boldsymbol{\pi}_{(0)}\cdot\tilde{\boldsymbol{\mathcal{S}}}^{\mu\nu}_{(0)} + M_{(0)}\mathcal{S}^{\mu\nu}_{(0)} + \mathbf{m}\cdot\boldsymbol{\mathcal{S}}^{\mu\nu}_{(0)} = 0, \tag{39a}$$

$$-\varepsilon^{\mu\nu\alpha\beta}p_\alpha \boldsymbol{\mathcal{A}}^{(0)}_\beta - \boldsymbol{\pi}_{(0)}\tilde{\mathcal{S}}^{\mu\nu}_{(0)} + M_{(0)}\boldsymbol{\mathcal{S}}^{\mu\nu}_{(0)} + \mathbf{m}\, \mathcal{S}^{\mu\nu}_{(0)} = 0, \tag{39b}$$

$$p^\mu \mathcal{V}^\nu_{(0)} - p^\nu \mathcal{V}^\mu_{(0)} = 0, \tag{39c}$$

$$-\boldsymbol{\pi}_{(0)}\times\tilde{\boldsymbol{\mathcal{S}}}^{\mu\nu}_{(0)} + \mathbf{m}\times\boldsymbol{\mathcal{S}}^{\mu\nu}_{(0)} + p^\mu \boldsymbol{\mathcal{V}}^\nu_{(0)} - p^\nu \boldsymbol{\mathcal{V}}^\mu_{(0)} = 0. \tag{39d}$$

The constraint equations written above have to be supplemented by the expressions for the mean fields to leading order in $\hbar$

$$\sigma_{(0)}(X) = -8G\int \frac{d^4p}{(2\pi)^4}\mathcal{F}_{(0)}(X,p) \tag{40}$$

and

$$\boldsymbol{\pi}_{(0)}(X) = 8G\int \frac{d^4p}{(2\pi)^4}\boldsymbol{\mathcal{P}}_{(0)}(X,p). \tag{41}$$

B Chiral limit

In this subsection we shall analyze the form of the constraint equations (35)–(39) in the limit $m_0 = \mathbf{m} = 0$ [in this case $M_{(0)}(X) = \sigma_{(0)}(X)$]. Having in mind the previous studies relying on the spinor decomposition [3–5], we expect that quantities $\mathcal{F}_{(0)}$, $\boldsymbol{\mathcal{F}}_{(0)}$, $\mathcal{A}^\mu_{(0)}$ and $\boldsymbol{\mathcal{A}}^\mu_{(0)}$ can be used as the fundamental variables in construction of the transport theory: $\mathcal{F}_{(0)}$ describes the quark space-time distribution, $\boldsymbol{\mathcal{F}}_{(0)}$ specifies the isospin of quarks, $\mathcal{A}^\mu_{(0)}$ describes the quark spin density, and $\boldsymbol{\mathcal{A}}^\mu_{(0)}$ determines the spin density of quarks with different isospin components. We note that the "classical" isospin of quarks is described here by a three-vector. This fact is connected with the form of the decomposition (14) of the two by two, quantum mechanical density matrix. We also note that (36c) and (36d) indicate that only three out of four Lorentz components of $\mathcal{A}^\mu_{(0)}$ and $\boldsymbol{\mathcal{A}}^\mu_{(0)}$ are independent. Therefore, similarly to the "classical" isospin, the "classical" spin of quarks is also described by a three-vector.

Let us now express the quantities $\mathcal{P}_{(0)}, \boldsymbol{\mathcal{P}}_{(0)}, \mathcal{V}^\mu_{(0)}, \boldsymbol{\mathcal{V}}^\mu_{(0)}$, $\mathcal{S}^{\mu\nu}_{(0)}$, $\tilde{\mathcal{S}}^{\mu\nu}_{(0)}$, $\boldsymbol{\mathcal{S}}^{\mu\nu}_{(0)}$ and $\tilde{\boldsymbol{\mathcal{S}}}^{\mu\nu}_{(0)}$ as functions of $\mathcal{F}_{(0)}$, $\boldsymbol{\mathcal{F}}_{(0)}$, $\mathcal{A}^\mu_{(0)}$ and $\boldsymbol{\mathcal{A}}^\mu_{(0)}$. First of all, it is easy to notice that (36a) and (36b) define the pseudoscalar densities in terms of $\mathcal{F}_{(0)}$ and $\boldsymbol{\mathcal{F}}_{(0)}$

$$\mathcal{P}_{(0)} = -\frac{\boldsymbol{\pi}_{(0)}\cdot\boldsymbol{\mathcal{F}}_{(0)}}{\sigma_{(0)}}, \tag{42}$$

$$\boldsymbol{\mathcal{P}}_{(0)} = -\frac{\boldsymbol{\pi}_{(0)}\mathcal{F}_{(0)}}{\sigma_{(0)}}. \tag{43}$$

In the similar way (37a) and (37b) define $\mathcal{V}^\mu_{(0)}$ and $\boldsymbol{\mathcal{V}}^\mu_{(0)}$ in terms of $\mathcal{F}_{(0)}$, $\boldsymbol{\mathcal{F}}_{(0)}$ and $\boldsymbol{\mathcal{A}}^\mu_{(0)}$

$$\mathcal{V}^\mu_{(0)} = p^\mu \frac{\mathcal{F}_{(0)}}{\sigma_{(0)}}, \tag{44}$$

$$\boldsymbol{\mathcal{V}}^\mu_{(0)} = p^\mu \frac{\boldsymbol{\mathcal{F}}_{(0)}}{\sigma_{(0)}} - \frac{\boldsymbol{\pi}_{(0)}\times\boldsymbol{\mathcal{A}}^\mu_{(0)}}{\sigma_{(0)}}. \tag{45}$$

After a few algebraic manipulations (39a) and (39b) allow us to express the spin tensors $\mathcal{S}^{\mu\nu}_{(0)}, \boldsymbol{\mathcal{S}}^{\mu\nu}_{(0)}$ and the dual spin tensors $\tilde{\mathcal{S}}^{\mu\nu}_{(0)}, \tilde{\boldsymbol{\mathcal{S}}}^{\mu\nu}_{(0)}$ as functions of $\mathcal{A}^\mu_{(0)}$ and $\boldsymbol{\mathcal{A}}^\mu_{(0)}$

$$\mathcal{S}^{\mu\nu}_{(0)} = -\frac{1}{\mathcal{M}^2}\left[p^\mu\boldsymbol{\pi}_{(0)}\cdot\boldsymbol{\mathcal{A}}^\nu_{(0)} - p^\nu\boldsymbol{\pi}_{(0)}\cdot\boldsymbol{\mathcal{A}}^\mu_{(0)}\right] + \frac{\sigma_{(0)}}{\mathcal{M}^2}\varepsilon^{\mu\nu\alpha\beta}p_\alpha\mathcal{A}^{(0)}_\beta, \tag{46}$$

$$\tilde{\mathcal{S}}^{\mu\nu}_{(0)} = -\frac{\sigma_{(0)}}{\mathcal{M}^2}\left[p^\mu\mathcal{A}^\nu_{(0)} - p^\nu\mathcal{A}^\mu_{(0)}\right] - \varepsilon^{\mu\nu\alpha\beta}p_\alpha\frac{\boldsymbol{\pi}_{(0)}\cdot\boldsymbol{\mathcal{A}}^{(0)}_\beta}{\mathcal{M}^2}, \tag{47}$$

$$\boldsymbol{\mathcal{S}}^{\mu\nu}_{(0)} = -\frac{\boldsymbol{\pi}_{(0)}}{\mathcal{M}^2}\left[p^\mu\mathcal{A}^\nu_{(0)} - p^\nu\mathcal{A}^\mu_{(0)} + \varepsilon^{\mu\nu\alpha\beta}p_\alpha\frac{\boldsymbol{\pi}_{(0)}\cdot\boldsymbol{\mathcal{A}}^{(0)}_\beta}{\sigma_{(0)}}\right] + \varepsilon^{\mu\nu\alpha\beta}p_\alpha\frac{\boldsymbol{\mathcal{A}}^{(0)}_\beta}{\sigma_{(0)}} \tag{48}$$

and

$$\tilde{\boldsymbol{\mathcal{S}}}^{\mu\nu}_{(0)} = \frac{\boldsymbol{\pi}_{(0)}}{\mathcal{M}^2}\left[p^\mu\frac{\boldsymbol{\pi}_{(0)}\cdot\boldsymbol{\mathcal{A}}^\nu_{(0)}}{\sigma_{(0)}} - p^\nu\frac{\boldsymbol{\pi}_{(0)}\cdot\boldsymbol{\mathcal{A}}^\mu_{(0)}}{\sigma_{(0)}} - \varepsilon^{\mu\nu\alpha\beta}p_\alpha\mathcal{A}^{(0)}_\beta\right] - \frac{1}{\sigma_{(0)}}\left(p^\mu\boldsymbol{\mathcal{A}}^\nu_{(0)} - p^\nu\boldsymbol{\mathcal{A}}^\mu_{(0)}\right). \tag{49}$$

Here we have introduced the chirally invariant mass

$$\mathcal{M}^2(X) = \sigma^2_{(0)}(X) + \pi^2_{(0)}(X). \tag{50}$$

Let us now take into consideration the other constraint equations appearing in the leading order of $\hbar$. Substituting expressions (42)–(45) into (35a) and (35b) one finds two mass-shell conditions

$$[p^2 - \mathcal{M}^2(X)]\mathcal{F}_{(0)}(X,p) = 0 \tag{51}$$

and

$$[p^2 - \mathcal{M}^2(X)]\boldsymbol{\mathcal{F}}_{(0)}(X,p) = 0. \tag{52}$$

The third scalar constraint equation is automatically fulfilled, since $\boldsymbol{\mathcal{P}}_{(0)}$ and $\boldsymbol{\pi}_{(0)}$ are parallel [see (35c) and (43)].

Substituting expression (46) in (37c) and using condition (36d) one finds

$$[p^2 - \mathcal{M}^2(X)]\,\boldsymbol{\pi}_{(0)}\cdot\boldsymbol{\mathcal{A}}^\mu_{(0)} = 0. \tag{53}$$

Analogously, by substituting (48) in (37d) and using condition (36c) we get

$$[p^2 - \mathcal{M}^2(X)]\mathcal{A}^\mu_{(0)} = 0. \tag{54}$$

This formula follows also from Eqs. (38a), (47) and (36b). On the other hand, (38b), (49), (36c) and (53) lead to the mass-shell constraint

$$[p^2 - \mathcal{M}^2(X)]\,\boldsymbol{\mathcal{A}}^\mu_{(0)} = 0. \tag{55}$$

To complete our discussion of the constraint equations in the chiral limit we should discuss an important property of formulas (40) and (41). Using (43) in (41) we find that (40) and (41) are not independent — they do not determine separately the values of $\sigma_{(0)}(X)$ and $\boldsymbol{\pi}_{(0)}(X)$. This fact is a consequence of the chiral symmetry. It indicates that only the invariant mass $\mathcal{M}(X)$ has a physical significance.

C Massive quarks

In this subsection we are going to consider the case $\mathbf{m} \neq 0$. Calculating $\boldsymbol{\mathcal{P}}_{(0)}$ from (36b) and substituting into (41) we find that $\boldsymbol{\pi}_{(0)}$ and $\mathbf{m}$ must be parallel in this case. Using again (36b) we find that $\boldsymbol{\mathcal{P}}_{(0)}$ must also be parallel to $\mathbf{m}$. Further inspection of (35)–(39) indicates that all quantities $\mathbf{C}_{(0)}$ are parallel to $\mathbf{m}$ [i.e., only their third component is different from zero, $\mathbf{C}_{(0)} = (0, 0, C_{(0)\,3})$]. In this situation it is convenient to introduce the combinations

$$C_{(0)\,u} = C_{(0)} + C_{(0)\,3}, \quad C_{(0)\,d} = C_{(0)} - C_{(0)\,3}, \tag{56}$$

which describe up and down quarks, respectively. Using this notation, we can write

$$\mathcal{P}_{(0)\,u} = -\pi_{(0)\,3}\;\frac{\mathcal{F}_{(0)\,u}}{M_{(0)}+m_3}, \qquad \mathcal{P}_{(0)\,d} = \pi_{(0)\,3}\;\frac{\mathcal{F}_{(0)\,d}}{M_{(0)}-m_3} \tag{57}$$

and

$$\mathcal{V}^{\mu}_{(0)\,u} = \frac{p^\mu \mathcal{F}_{(0)\,u}}{M_{(0)}+m_3}, \quad \mathcal{V}^{\mu}_{(0)\,d} = \frac{p^\mu \mathcal{F}_{(0)\,d}}{M_{(0)}-m_3}. \tag{58}$$

In the massive case the gap equations (40) and (41) take the form

$$1+4G\int \frac{d^4p}{(2\pi)^4}\left(\frac{\mathcal{F}_{(0)\,u}(X,p)}{\sigma_{(0)}(X)}+\frac{\mathcal{F}_{(0)\,d}(X,p)}{\sigma_{(0)}(X)}\right)=0 \tag{59}$$

and

$$\pi_{(0)\,3}(X)\left[1+4G\int \frac{d^4p}{(2\pi)^4} \times \left(\frac{\mathcal{F}_{(0)\,u}(X,p)}{\sigma_{(0)}(X)+m_u}+\frac{\mathcal{F}_{(0)\,d}(X,p)}{\sigma_{(0)}(X)+m_d}\right)\right]=0. \tag{60}$$

These two equations lead us to the condition $\pi_{(0)\,3}(X) = 0$. Thus, in the case when the chiral symmetry is explicitly broken, the leading contribution to the pseudoscalar condensate should vanish (similarly as in the one-flavor case, see (86) of [3]).

Substituting expressions (58) into the scalar constraint equations (35a) and (35b), we find the mass-shell conditions

$$\left[p^2-(M^f(X))^2\right]\mathcal{F}_{(0)\,f}(X,p)=0 \qquad (f=u,d), \tag{61}$$

where we have introduced the notation

$$M^u(X) = M_{(0)}(X)+m_3, \qquad M^d(X) = M_{(0)}(X)-m_3. \tag{62}$$

We turn now to the discussion of the spin dynamics. First of all, (36c) and (36d) give us the condition

$$p_\mu \mathcal{A}^{\mu}_{(0)\,f}=0. \tag{63}$$

The tensor constraint equations (39a) and (39b) can be used to find the expressions for the spin tensors, namely

$$\mathcal{S}^{\mu\nu}_{(0)\,f} = \frac{1}{M^f}\varepsilon^{\mu\nu\alpha\beta}p_\alpha \mathcal{A}^{(0)\,f}_{\beta}. \tag{64}$$

The dual spin tensors are obtained by contracting this expression with the Levi-Civita tensor as in (24). Using now (63), (64), (38a) and (38b) we find

$$\left[p^2-(M^f(X))^2\right]\mathcal{A}^{\mu}_{(0)\,f}(X,p)=0. \tag{65}$$

VI Classical transport equations

In this Section we derive classical transport equations. This requires the study of (27)–(31) up to first order in $\hbar$. Since the calculations in the chiral limit have different characteristics from those done for massive current quarks, we discuss these two cases separately.

A Chiral limit

Substituting (43) and (44) into (27c) we find

$$p^\mu\partial_\mu F+\mathcal{M}\partial_\nu\mathcal{M}\partial^\nu_p F=0, \tag{66}$$

where $F(X,p)=\mathcal{F}_{(0)}(X,p)/\sigma_{(0)}(X)$ is the chirally invariant quark distribution function (compare (60) of [3]). In the similar way, substituting (42), (45) and the formula for $\boldsymbol{\mathcal{P}}(X,p)$ obtained from (28b) into (27d) one gets

$$p^\mu\partial_\mu\mathbf{F}+\mathcal{M}\partial_\nu\mathcal{M}\partial^\nu_p\mathbf{F}=\partial_\mu\left(\frac{\boldsymbol{\pi}_{(0)}}{\sigma_{(0)}}\right)\times\boldsymbol{\mathcal{A}}^{\mu}_{(0)}, \tag{67}$$

where $\mathbf{F}(X,p) = \boldsymbol{\mathcal{F}}_{(0)}(X,p)/\sigma_{(0)}(X)$. In contrast to $F(X,p)$ the function $\mathbf{F}(X,p)$ is not chirally invariant [under infinitesimal chiral transformations $\mathbf{F}\to\mathbf{F}' = \mathbf{F}+\boldsymbol{\pi}_{(0)}\times(\mathbf{F}\times\mathbf{n})\left(\delta\chi/\sigma_{(0)}\right)$]. However, one can check that the *form* of Eq. (67) is chirally invariant.

In order to find the kinetic equations satisfied by the functions $\mathcal{A}^{\mu}_{(0)}(X,p)$ and $\boldsymbol{\mathcal{A}}^{\mu}_{(0)}(X,p)$ we use (30c) and (30d), respectively. Calculating $\mathcal{P}(X,p)$ from (28a) and $\boldsymbol{\mathcal{V}}^{\mu}_{(0)}(X,p)$ from (29b), and substituting these two expressions into (30c) we find (to first order in $\hbar$)

$$p^\mu\partial_\nu\mathcal{A}^{\nu}_{(0)}+\mathcal{M}\partial_\nu\mathcal{M}\partial^\nu_p\mathcal{A}^{\mu}_{(0)}+\sigma_{(0)}\partial_\nu\tilde{\mathcal{S}}^{\mu\nu}_{(0)} +\boldsymbol{\pi}_{(0)}\cdot\partial_\nu\boldsymbol{\mathcal{S}}^{\mu\nu}_{(0)}=0. \tag{68}$$

Analogously, by computing $\boldsymbol{\mathcal{P}}(X,p)$ from (28b) and $\mathcal{V}^\mu(X,p)$ from (29a), and inserting these expressions into (30d) we find (again to first order in $\hbar$)

$$p^\mu\partial_\nu\boldsymbol{\mathcal{A}}^{\nu}_{(0)}+\mathcal{M}\partial_\nu\mathcal{M}\partial^\nu_p\boldsymbol{\mathcal{A}}^{\mu}_{(0)}+\sigma_{(0)}\partial_\nu\tilde{\boldsymbol{\mathcal{S}}}^{\mu\nu}_{(0)} +\boldsymbol{\pi}_{(0)}\partial_\nu\mathcal{S}^{\mu\nu}_{(0)}=0. \tag{69}$$

Now, using expressions (46)–(49), and defining the leading parts of the spin tensors, we find the desired equations

$$p^\nu\partial_\nu\mathcal{A}^{\mu}_{(0)}+\mathcal{M}\partial_\nu\mathcal{M}\partial^\nu_p\mathcal{A}^{\mu}_{(0)}+\frac{\partial_\nu\mathcal{M}}{\mathcal{M}}\left[p^\mu\mathcal{A}^{\nu}_{(0)}-p^\nu\mathcal{A}^{\mu}_{(0)}\right] -\varepsilon^{\mu\nu\alpha\beta}p_\alpha\frac{\mathcal{M}}{\sigma_{(0)}}\partial_\nu\left(\frac{\boldsymbol{\pi}_{(0)}}{\mathcal{M}}\right)\cdot\boldsymbol{\mathcal{A}}^{(0)}_{\beta}=0 \tag{70}$$

and

$$p^\nu\partial_\nu\boldsymbol{\mathcal{A}}^{\mu}_{(0)}+\mathcal{M}\partial_\nu\mathcal{M}\partial^\nu_p\boldsymbol{\mathcal{A}}^{\mu}_{(0)}+\frac{\sigma_{(0)}}{\mathcal{M}^2}\partial_\nu\left(\frac{\boldsymbol{\pi}_{(0)}}{\sigma_{(0)}}\right)\times \left[p^\mu\boldsymbol{\pi}_{(0)}\cdot\boldsymbol{\mathcal{A}}^{\nu}_{(0)}-p^\nu\boldsymbol{\pi}_{(0)}\cdot\boldsymbol{\mathcal{A}}^{\mu}_{(0)}\right]+ \frac{\partial_\nu\sigma_{(0)}}{\sigma_{(0)}}\left[p^\mu\boldsymbol{\mathcal{A}}^{\nu}_{(0)}-p^\nu\boldsymbol{\mathcal{A}}^{\mu}_{(0)}\right]-\varepsilon^{\mu\nu\alpha\beta}p_\alpha\frac{\sigma^2_{(0)}}{\mathcal{M}^2}\partial_\nu\times \left(\frac{\boldsymbol{\pi}_{(0)}}{\sigma_{(0)}}\right)\mathcal{A}^{(0)}_{\beta}=\sigma_{(0)}\partial^\mu\boldsymbol{\pi}_{(0)}\times\mathbf{F}. \tag{71}$$

Several comments are in order now:

(a) One can check that (70) and (71) are consistent with the conditions $p^\mu \mathcal{A}^{(0)}_\mu = 0$ and $p^\mu \boldsymbol{\mathcal{A}}^{(0)}_\mu = \boldsymbol{\mathcal{F}}_{(0)} \times \boldsymbol{\pi}_{(0)}$. One can also check by a straightforward but tedious calculation that (70) and (71) are chirally invariant.

(b) The kinetic equations (66), (67), (70) and (71) together with the mass-shell constraints (51), (52), (54) and (55) describe the evolution of quarks in the *given* scalar and pseudoscalar fields. The calculations within the NJL model require that these fields are obtained self-consistently from the quark distribution functions. However, the discussed set of equations can also be used in the cases where we are interested in the dynamics of quarks in *external* scalar and pseudoscalar fields. For that reason, the results presented in this Section are an extension of a part of the earlier results obtained in [19,20]. An interesting novel feature of the present approach is the coupling between the spin and isospin degrees of freedom.

(c) In the NJL model, one evaluates the mean fields from the self-consistent equations (15) and (16). However, due to the chiral symmetry of the model [see discussion following (55)] only a chirally invariant combination $\mathcal{M}^2 = \boldsymbol{\pi}^2_{(0)} + \sigma^2_{(0)}$ can be calculated from the knowledge of the quark distribution function. Thus, the discussed system of equations is closed (and can be solved) only in the case when the ratio $\boldsymbol{\pi}_{(0)}(X)/\sigma_{(0)}(X)$ is fixed and independent of the space-time position coordinate X. Nevertheless, this ratio can be arbitrary, as required by the chiral invariance of the system.

(d) Since the ratio $\boldsymbol{\pi}_{(0)}(X)/\sigma_{(0)}(X)$ is fixed and arbitrary, in practical calculations we can always set it to zero (this is like choosing the convenient gauge in QED-type calculations). In this case the form of the kinetic equations (67), (70) and (71) simplifies substantially and reduces to that known from the one-flavor approach (compare (62) and (68) of [3]).

(e) Equation (71) does not guarantee the conservation of the axial current, see (7) and (33). This fact can be seen in the special case, $\boldsymbol{\pi}_{(0)}(X)/\sigma_{(0)}(X) = 0$, by using similar arguments to those presented in the one flavor study [3]. Consequently, solutions of Eq. (71) are *constrained* by the condition of the axial current conservation. Only those solutions which satisfy (7) are valid.

B Massive quarks

Using (28b) and presenting similar arguments to those used in Section V, we find that the cross product $\boldsymbol{\pi} \times \boldsymbol{\mathcal{P}}$ vanishes up to the first order in $\hbar$. In this situation (27c) and (27d) lead to the kinetic equations satisfied by the distribution functions of the up and down quarks. They have the following form

$$p^\mu \partial_\mu F^f(X,p) + M^f(X)\partial_\mu M^f(X)\partial^\mu_p F^f(X,p) = 0 \quad (f = u,d), \tag{72}$$

where $F^f(X,p) = \mathcal{F}_{(0)\,f}(X,p)/M^f(X)$.

The kinetic equations for the spin densities follow from Eqs. (30c) and (30d). Using the formulas for the pseudoscalar densities $\mathcal{P}$ and $\mathcal{P}_3$, calculated from (28a) and (28b) up to the first order in $\hbar$, we can rewrite (30c) and (30d) in the form

$$\tfrac{1}{2}p^\mu \partial^\nu \mathcal{A}^{(0)}_\nu - m_3\left(p^\mu \mathcal{P}_{(1)\,3} + \pi_{(1)\,3}\mathcal{V}^\mu_{(0)}\right) + \tfrac{1}{2}M_{(0)}\partial_\nu \tilde{\mathcal{S}}^{\mu\nu}_{(0)} = -\tfrac{1}{2}M_{(0)}\partial_\nu M_{(0)}\partial^\nu_p \mathcal{A}^\mu_{(0)} \tag{73}$$

and

$$\tfrac{1}{2}p^\mu \partial^\nu \mathcal{A}^{(0)\,3}_\nu - m_3\left(p^\mu \mathcal{P}_{(1)} + \pi_{(1)\,3}\mathcal{V}^\mu_{(0)\,3}\right) + \tfrac{1}{2}M_{(0)}\partial_\nu \tilde{\mathcal{S}}^{\mu\nu}_{(0)\,3} = -\tfrac{1}{2}M_{(0)}\partial_\nu M_{(0)}\partial^\nu_p \mathcal{A}^\mu_{(0)\,3}. \tag{74}$$

Equations (28a) and (28b) give us additionally two conditions

$$M^u \mathcal{P}_{(1)\,u} = \tfrac{1}{2}\partial_\nu \mathcal{A}^\nu_{(0)\,u} - \pi_{(1)\,3}\mathcal{F}_{(0)\,u} \tag{75}$$

and

$$M^d \mathcal{P}_{(1)\,d} = \tfrac{1}{2}\partial_\nu \mathcal{A}^\nu_{(0)\,d} + \pi_{(1)\,3}\mathcal{F}_{(0)\,d}. \tag{76}$$

Substituting (58), (75) and (76) into the sum of (73) and (74), and using formulas for the dual spin tensors we find

$$p^\nu \partial_\nu \mathcal{A}^\mu_{(0)\,u} + M^u \partial_\nu M^u \partial^\nu_p \mathcal{A}^\mu_{(0)\,u} + \frac{\partial_\nu M^u}{M^u}\left(p^\mu \mathcal{A}^\nu_{(0)\,u} - p^\nu \mathcal{A}^\mu_{(0)\,u}\right) = 0. \tag{77}$$

Similarly, by substituting (58), (75) and (76) into the difference of (73) and (74) one finds

$$p^\nu \partial_\nu \mathcal{A}^\mu_{(0)\,d} + M^d \partial_\nu M^d \partial^\nu_p \mathcal{A}^\mu_{(0)\,d} + \frac{\partial_\nu M^d}{M^d}\left(p^\mu \mathcal{A}^\nu_{(0)\,d} - p^\nu \mathcal{A}^\mu_{(0)\,d}\right) = 0. \tag{78}$$

Equations (77) and (78) are the analogs of the spin kinetic equation derived for the first time in [3].

The mass-shell conditions allow us to express $F^{u,d}(X,p)$ as the sum of the quark and antiquark distribution functions $f^+_{u,d}(X,\mathbf{p})$ and $f^-_{u,d}(X,\mathbf{p})$

$$F^{u,d}(X,p) = 2\pi\left\{\frac{\delta\left(p^0 - E^{u,d}_p(X)\right)}{2E^{u,d}_p(X)} f^+_{u,d}(X,\mathbf{p}) + \frac{\delta\left(p^0 + E^{u,d}_p(X)\right)}{2E^{u,d}_p(X)}\left[f^-_{u,d}(X,-\mathbf{p}) - 1\right]\right\} \tag{79}$$

where

$$E^{u,d}_p(X) = \sqrt{(M^{u,d}(X))^2 + \mathbf{p}^2}. \tag{80}$$

Substituting this formula into (72), and integrating over p^0 gives

$$p^\mu \partial_\mu f^\pm_{u,d}(X,\mathbf{p}) + M^{u,d}(X)\partial_\mu M^{u,d}(X)\partial^\mu_p f^\pm_{u,d}(X,\mathbf{p}) = 0. \tag{81}$$

Using expression (79) we can also rewrite the gap equation in the more familiar form

$$M_{(0)} - m_0 = -2G \int \frac{d^3p}{(2\pi)^3} \frac{M^u}{E_p^u(X)} \times \left[f_u^+(X,\mathbf{p}) + f_u^-(X,\mathbf{p}) - 1 \right] -2G \int \frac{d^3p}{(2\pi)^3} \frac{M^d}{E_p^d(X)} \times \left[f_d^+(X,\mathbf{p}) + f_d^-(X,\mathbf{p}) - 1 \right]. \quad (82)$$

The last results indicate that the form of the kinetic equations for the two-flavor approach (with massive current quarks) is analogous to that of the one-flavour model. The up and down quarks, as well as their spins, evolve in the mean fields M^u and M^d, respectively. This allows to introduce the spin up and spin down densities as the appropriate combinations of the functions $F^f(X,p)$ and $\mathcal{A}^{\mu}_{(0)\,f}(X,p)$ (compare (69) and (70) of [3]). The only interplay between the two flavors occurs via the gap equation, which determines the common part of the mean fields M^u and M^d.

VII Summary

In this paper we have derived and analyzed the mean-field transport equations for the two-flavor NJL model. In this way we have extended the previous work [3], restricted to only one flavor. By decomposing the Wigner function in both spinor and isospin space, we could investigate the *most general* form of the mean-field classical transport equations.

We have discussed in detail the case of the chiral limit, studying the invariance properties of the transport equations. Our analysis shows the limitations in the applications of the chirally invariant kinetic equations. This sort of restrictions is already known from the one-flavor considerations: the ratio $\boldsymbol{\pi}_{(0)}(X)/\sigma_{(0)}(X)$ must be constant and the spin dynamics is constrained by axial current conservation.

An additional result of our approach are the transport equations for quark matter moving in the *externally given* scalar and pseudoscalar fields. They follow from our analysis in the case when the self-consistency required in the evaluation of the meson mean fields is relaxed. These equations describe non-trivial couplings between the spin and isospin degrees of freedom.

If the current quarks are massive, the difficulties connected with the requirement of chiral invariance are not present. The space-time evolution of the up and down quark distribution functions is determined by the mean fields $M^u(X) = M_{(0)}(X) + \frac{1}{2}(m_u - m_d)$ and $M^d(X) = M_{(0)}(X) - \frac{1}{2}(m_u - m_d)$, respectively. The common part of the mean fields, $M_{(0)}(X)$, is obtained from the self-consistent gap equation.

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Appendix

In this Appendix we list the chiral transformations rules obeyed by the coefficients appearing in the spinor and isospin decomposition of the Wigner function (13).

Scalar and pseudoscalar functions:

$$\mathcal{F} \rightarrow \mathcal{F}' = \mathcal{F} \cos\chi + \boldsymbol{\mathcal{P}}\cdot\mathbf{n} \sin\chi, \quad \text{(A1a)}$$

$$\boldsymbol{\mathcal{F}} \rightarrow \boldsymbol{\mathcal{F}}' = \boldsymbol{\mathcal{F}} \cos^2\frac{\chi}{2} - [2(\boldsymbol{\mathcal{F}}\cdot\mathbf{n})\mathbf{n} - \boldsymbol{\mathcal{F}}] \sin^2\frac{\chi}{2} + \mathcal{P}\mathbf{n} \sin\chi, \quad \text{(A1b)}$$

$$\mathcal{P} \rightarrow \mathcal{P}' = \mathcal{P} \cos\chi - \boldsymbol{\mathcal{F}}\cdot\mathbf{n} \sin\chi, \quad \text{(A1c)}$$

$$\boldsymbol{\mathcal{P}} \rightarrow \boldsymbol{\mathcal{P}}' = \boldsymbol{\mathcal{P}} \cos^2\frac{\chi}{2} - [2(\boldsymbol{\mathcal{P}}\cdot\mathbf{n})\mathbf{n} - \boldsymbol{\mathcal{P}}] \sin^2\frac{\chi}{2} - \mathcal{F}\mathbf{n} \sin\chi. \quad \text{(A1d)}$$

Vector and axial-vector functions:

$$\mathcal{V}_\mu \rightarrow \mathcal{V}'_\mu = \mathcal{V}_\mu, \quad \text{(A2a)}$$

$$\boldsymbol{\mathcal{V}}_\mu \rightarrow \boldsymbol{\mathcal{V}}'_\mu = \boldsymbol{\mathcal{V}}_\mu \cos^2\frac{\chi}{2} + [2(\boldsymbol{\mathcal{V}}_\mu\cdot\mathbf{n})\mathbf{n} - \boldsymbol{\mathcal{V}}_\mu] - \mathbf{n} \times \boldsymbol{\mathcal{A}}_\mu \sin\chi, \quad \text{(A2b)}$$

$$\mathcal{A}_\mu \rightarrow \mathcal{A}'_\mu = \mathcal{A}_\mu, \quad \text{(A2c)}$$

$$\boldsymbol{\mathcal{A}}_\mu \rightarrow \boldsymbol{\mathcal{A}}'_\mu = \boldsymbol{\mathcal{A}}_\mu \cos^2\frac{\chi}{2} + [2(\boldsymbol{\mathcal{A}}_\mu\cdot\mathbf{n})\mathbf{n} - \boldsymbol{\mathcal{A}}_\mu] - \mathbf{n} \times \boldsymbol{\mathcal{V}}_\mu \sin\chi, \quad \text{(A2d)}$$

Tensor functions:

$$\mathcal{S}_{\mu\nu} \rightarrow \mathcal{S}'_{\mu\nu} = \mathcal{S}_{\mu\nu} \cos\chi + \tilde{\boldsymbol{\mathcal{S}}}_{\mu\nu}\cdot\mathbf{n} \sin\chi, \quad \text{(A3a)}$$

$$\boldsymbol{\mathcal{S}}_{\mu\nu} \rightarrow \boldsymbol{\mathcal{S}}'_{\mu\nu} = \boldsymbol{\mathcal{S}}_{\mu\nu} \cos^2\frac{\chi}{2} - [2(\boldsymbol{\mathcal{S}}_{\mu\nu}\cdot\mathbf{n})\mathbf{n} - \boldsymbol{\mathcal{S}}_{\mu\nu}] \sin^2\frac{\chi}{2} + \tilde{\mathcal{S}}_{\mu\nu}\mathbf{n} \sin\chi, \quad \text{(A3b)}$$

$$\tilde{\mathcal{S}}_{\mu\nu} \rightarrow \tilde{\mathcal{S}}'_{\mu\nu} = \tilde{\mathcal{S}}_{\mu\nu} \cos\chi - \boldsymbol{\mathcal{S}}_{\mu\nu}\cdot\mathbf{n} \sin\chi, \quad \text{(A3c)}$$

$$\tilde{\boldsymbol{\mathcal{S}}}_{\mu\nu} \rightarrow \tilde{\boldsymbol{\mathcal{S}}}'_{\mu\nu} = \tilde{\boldsymbol{\mathcal{S}}}_{\mu\nu} \cos^2\frac{\chi}{2} - \left[2(\tilde{\boldsymbol{\mathcal{S}}}_{\mu\nu}\cdot\mathbf{n})\mathbf{n} - \tilde{\boldsymbol{\mathcal{S}}}_{\mu\nu}\right] \sin^2\frac{\chi}{2} - \mathcal{S}_{\mu\nu}\mathbf{n} \sin\chi. \quad \text{(A3d)}$$

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