Eur. Phys. J. A **1**, 257–260 (1998)

THE EUROPEAN PHYSICAL JOURNAL A

Magnetic dipole response in rare earth nuclei[1]

R.R. Hilton, W. Höhenberger, P. Ring

Physikdepartment, Technische Universität München,D-85747 Garching, Germany

Received: 14 July 1997 / Revised version: 20 October 1997

Abstract. Experimental observations in certain rare earth nuclei have established the presence of sizeable B(M1) strength of two peak structure lying in the 5-10 MeV region. The character of the states concerned, studied within a self-consistent Random Phase Approximation using Skyrme forces, are identified to be that of proton and neutron giant spin-flip resonances.

PACS. 21.10.Re Collective levels – 21.60.Jz Hartree-Fock and random phase transitions – 24.30.Cz Giant resonances

The response of the nucleus to leptonic and hadronic probes is a fruitful source of information about the nuclear system. With the availability of modern facilities, it is becoming possible to use them to explore electric and magnetic excitations of the nucleus with a precision not formerly feasible. Extensive experimental efforts have been put into determining the nature of the B(M1) strength found in deformed rare earth nuclei in the 5-10 MeV region [1–4], with recent investigations based upon photoexcitation studies [5]. They reveal an M1 distribution having a two peak structure ostensibly independent of the nucleus. These observations, together with the unexpected splitting of the B(M1) strength observed, have excited lively interest [6–10].

Highly contrasting interpretations of these states have arisen, which may be associated with a variety of inherent features of these calculations, such as the use of rather simplified separable interactions, the neglect of self-consistency or the use of truncated model spaces. We feel it is now opportune to present results of a fully self-consistent investigation of this problem using realistic density dependent interactions. Here we report on the first calculations on these states of this type in axially deformed nuclei. A preliminary account has been previously reported [11]

This study sheds light on some questions which have naturally arisen, namely; a) what is the nature of these states, b) what is the mechanism generating the double hump structure and finally, c) are they evidence of new giant resonances in heavy deformed nuclei?

The investigation of spin flip M1 strength in nuclei near closed shells has been a subject of interest for some time [12,13]. However, it is only comparatively recently that data has become available for heavy deformed nuclei in the rare earth region. In (p,p') experiments performed at TRIUMF [2,3], substantial M1 strength lying in the range 5-10 MeV has been found in agreement with earlier hints coming from (e,e') investigations [1]. The observations show an M1 strength distribution having a double hump structure, as shown in the top figures of Fig. 1, in which the maxima are seen to lie at around 6 and 8 MeV. A total summed spin flip M1 strength of around $10.6\mu_N^2$ is indicated. Very similar results have been found in a number of deformed rare earths [4].

A description of the fragmented structure of 1^+ states observed in the rare earth nuclei requires a formalism in which the interplay between the single particle and collective aspects of the system may be treated properly. The Random Phase Approximation offers such a microscopic formalism. If the effects of pairing correlations are also to be incorporated, the formalism must be extended and investigations undertaken within the framework of a self-consistent Quasi-Particle Random Phase Approximation (QRPA). The need for self-consistency is of paramount importance in these calculations as its absence leads invariably to contamination of the physical states by the spurious rotational state which carries the same quantum numbers as the states under investigation, and can be severe [14]. Only within the framework of a proper self-consistent formalism can this problem be solved satisfactorily, so that the spurious states decouple from all physical 1^+ states.

The form for an effective nucleon-nucleon interaction is a vexed question. The decision in favour of the Skyrme force is a result of both its successful ability to fit a large body of nuclear data, and at the same time offer numerically tractable solutions within the microscopic formalism we wish to use. For a spin saturated time reflection invariant system the Skyrme interaction has been shown to give good fits to binding energies, radii and shapes for a large range of nuclei over the periodic table. In the rare earth region in which we have particular interest, the Skyrme III parameter set [15] has been found to be very satisfactory

[1] Dedicated to Professor Richard Lemmer on the occasion of his 65th birthday

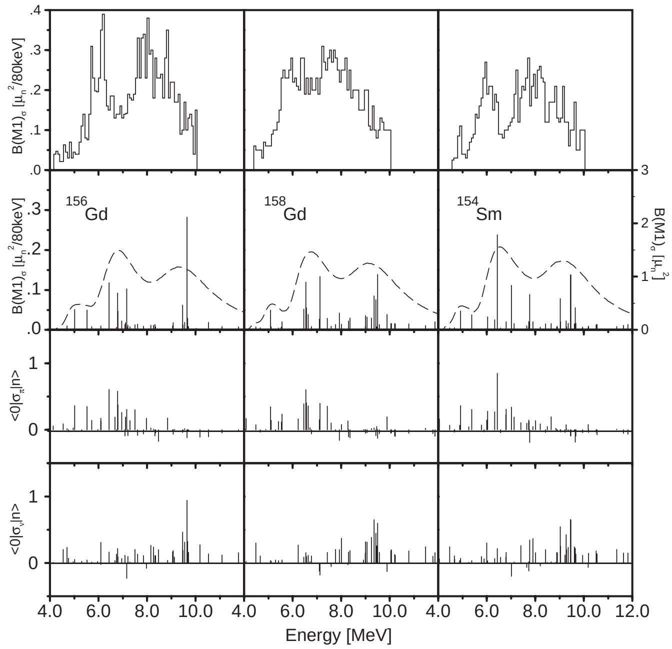

Fig. 1. Shown in the top row are the M1 strengths experimentally observed. The second row shows the calculated B(M1) spin-flip strength distribution for ^{156}Gd, ^{158}Gd, and ^{154}Sm when using some 2000 q.p. pairs (labelled on right), together with the Gaussian smeared spectrum of the M1 spin-flip strength. The third and fourth rows represent the proton and neutron spin-flip amplitudes respectively. The neutron amplitude is drawn showing its relative phase to that of the proton amplitude, in which overall arbitrary phases are assigned so that the total spin-flip matrix element is always positive

for structure calculations. The force used in the present calculation takes the form

$$
\begin{aligned}
V = & t_0(1 + x_0 P^\sigma)\delta(r) + \\
& \frac{1}{2} t_1[\delta(r)\mathbf{k}^2 + \mathbf{k}^2\delta(r)] + t_2 \mathbf{k}\delta(r)\mathbf{k} + \\
& iW_0(\sigma^i + \sigma^j)\mathbf{k}_\wedge \delta(r)\mathbf{k} + \frac{1}{6} t_3(1 + P^\sigma)\delta(r)\rho,
\end{aligned} \tag{1}
$$

with $\mathbf{k} = -i(\nabla_1 - \nabla_2)/2$.

The Hartree Fock equations generated by the Skyrme force are solved by expanding the single particle wave functions in a deformed oscillator basis which in turn determine the proton and neutron single particle energies. Pairing, as is known, must be introduced supplementarily, as the Skyrme force is repulsive for S states. The gap parameters are chosen for each nucleus so that the Belyaev moment of inertia expression [16] accords with that of the experimentally observed value. The QRPA equations for a density dependent interaction are established by obtaining the energy functional $E(\mathcal{R})$ using a generalized Slater determinant of the Hartree-Fock-Bogoliubov type $|HFB>$:

$$E(\mathcal{R}) \; = \; E(\rho, \kappa) \; = \; <HFB|H|HFB> \tag{2}$$

in which $\mathcal{R}$ is the generalized density

$$\mathcal{R} \; = \; \begin{pmatrix} \rho & \kappa \\ -\kappa & 1 - \rho^* \end{pmatrix}. \tag{3}$$

with $\rho_{kk'}$ and $\kappa_{kk'}$ taking the form

$$
\begin{aligned}
\rho_{kk'} &= <HFB|c^+_{k'} c_k|HFB>, \\
\kappa_{kk'} &= <HFB|c_{k'} c_k|HFB> .
\end{aligned} \tag{4}
$$

The ground state wave function $|HFB>$, and corresponding quasiparticle-operators α^+_k, are secured from a minimization of this functional with respect to variations of the generalized density $\mathcal{R}$. Within the RPA approximation, excited states $|\nu>$ are found as small amplitude oscillations $\delta\mathcal{R}^\nu$ around this minimum, given by solutions to the QRPA equations

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X^\nu_{1^+} \\ Y^\nu_{1^+} \end{pmatrix} = W^\nu_{1^+} \begin{pmatrix} X^\nu_{1^+} \\ Y^\nu_{1^+} \end{pmatrix} \tag{5}$$

in which the RPA amplitudes $\delta\mathcal{R} = (X^\nu_{kk'}, Y^\nu_{kk'})$ are given by,

$$
\begin{aligned}
X^\nu_{kk'} &= <0|\alpha_{k'}\alpha_k|\nu>, \\
Y^\nu_{kk'} &= <0|\alpha^+_k \alpha^+_{k'}|\nu> .
\end{aligned} \tag{6}
$$

where $|0>$ represents the correlated RPA ground state. As we are looking for 1^+ states, a search is made for quasiparticle pair states having $K = 1$ in which both q.p. pairs possess identical parity, so that quasi-boson operators are sought of the form,

$$Q^+_{\nu 1^+} = \sum_{k<k'} [X^{\nu 1^+}_{kk'} (\alpha^+_k \alpha^+_{k'})_{1^+} - Y^{\nu 1^+}_{kk'} (\alpha_{k'}\alpha_k)_{1^+}] \tag{7}$$

The Hermitian matrix A and the symmetric matrix B appearing in 5 can be expressed in terms of the quasi-particle energies and the second derivatives of the functional $E(\mathcal{R})$ with respect to appropriate matrix elements of the generalized density $\mathcal{R}$ using the Skyrme force. Further details of our notation and formalism are given in [16].

These investigations have needed extensive computational effort, necessitated by the fact that we are involved in the dual task of using both realistic interactions and embedding them within a self-consistent QRPA formalism. To be assured of the full convergence of the QRPA solutions and gain adequate proof that self-consistency had been achieved, calculations were pursued to some 2000 q.p. pairs. The spin-flip strength $B(M1)_\sigma$ for each solution of the QRPA equations was established from the expression

$$B(M1, 0 \rightarrow 1^+)_\sigma \; = \; 2|<0|M1_\sigma|1^+>|^2 \tag{8}$$

for all energy eigenvalues $W^\nu_{1^+}$ between 5-12 MeV, using the Skyrme III force.

In all calculations presented, renormalisation effects due to sub-nucleonic degrees of freedom have been provided for by the now accepted practice of reducing the spin gyromagnetic factor by 30%. The results obtained for the total spin flip strength are shown in the second row of Fig. 1, for the nuclei ^{156}Gd, ^{158}Gd, and ^{154}Sm. Our investigations disclose a dominant spin flip strength in this energy region with very little orbital M1 strength present, $(B(M1)_l^{tot} \approx 0.1 B(M1)_\sigma^{tot})$, in line with previous findings established within a single particle model description [17].

In an effort to simulate, phenomenologically, mechanisms outside the QRPA formalism which would lead to line broadening, we also show the results of smearing the lines exhibited in the second row of figures of Fig. 1 with a Gaussian, having a width which varies linearly between 0.1 and 1.2 MeV in the energy range 4-12 MeV [6]. This enables us to present our calculations in the same form as is experimentally displayed. The $B(M1)_\sigma$ distribution established shows a double hump shape with two peaks lying at around 6.5 and 9 MeV with a total summed spin flip strength in the region 5 to 12 MeV of between 10.6-10.8 μ_N^2 for the nuclei considered, in very good agreement with observation.

Of considerable interest is the nature and origin of the splitting of the M1 strength. In an effort to establish the physical character of these states, the separate proton and neutron spin-flip contributions were investigated. Displayed in the middle and lower portions of Fig. 1 are the respective proton and neutron spin-flip amplitudes obtained together with their relative phase. The arbitrary phase of each wave function is chosen in such a way that the spin matrix element $< 0|\sigma_p + \sigma_n|\nu >$ is positive. In these figures pure T=0 spin-flip states would manifest themselves as proton or neutron matrix elements $< 0|\sigma_p|\nu >$, $< 0|\sigma_n|\nu >$ at the same energy of equal size and sign, whereas pure T=1 states would possess matrix elements of equal size and opposite sign. The results, however, indicate that the double structure behaviour is due to the fact that the protons and neutrons deliver their principal spin-flip strength at different energies. The lower peak receives its dominant contribution from proton, the higher from neutron, spin-flip processes, in agreement with [7].

Examination of the overlaps with respect to various potential pictures shows that, in both the proton and neutron regions, at least one, and sometimes several dominant components concentrated within a relatively small energy interval, are of a collective spin-flip nature and demonstrate large overlaps with respect to a properly normalised state generated from the ground state by the total proton (or neutron) spin operator σ_τ with τ= p or n, i.e. $\sigma_\tau|0>$. As seen in Table 1, for these nuclei the main proton states lie in the energy interval 5.0-7.1 MeV with the strongest components enjoying overlaps of between 38-52%, and others reaching values 19-25%. For the neutrons the strength is more distributed, the strongest states achieving overlaps in the range 32-48%, with further states having values between 16-26%. The dominant proton states tend, in general, to cluster closer together in energy than is the case for the neutrons.

We have interpreted these as "flag leaders" with which to judge the other neighbouring weaker states. Our results would thus indicate the two peaks to be the fragmented structure of two highly collective states, namely, proton and neutron giant spin-flip resonances. Quite analogous collective character has been established using the SkI and SkII forces. The main differences of SkII w.r.t. SkIII appear as an upward displacement of the lower proton bump by around 0.5 MeV, a tendency to increase the double bump splitting together with somewhat larger total M1 strengths ($\simeq 11.4\mu_N^2$). For SkI, rather smaller M1 strengths are obtained ($\simeq 7.4\mu_N^2$) with the proton bump lowered in energy by some 0.5 MeV and a decrease in the splitting by 0.3 MeV. The essential collective features established here, however, seem not to be tied to the specific (isospin) characteristics of the SkIII force. Our collective interpretation of the double bump structure are in contrast to some previous investigations which find the M1 strength in this region to be either essentially of purely 2qp spin-flip nature [8], or to be in part of collective orbital-spin-flip combination origin [6], or to attribute an isovector spin-flip character to these states [9]. Our findings are much more in line with the sentiments expressed in ref [7] which interpreted the essentially 2qp spin-flip character obtained in their calculation as evidence for the birth of collective spin-flip states. The present calculation offers for the first time unequivocal evidence of the collective nature of these states, in which the large components of the fragmented collective states manifested allow one to pinpoint uniquely their physical nature.

Table 1. Overlaps of the dominant states in the 5-12 MeV region with respect to normalised $\sigma_\tau|0>$. The top, middle and bottom blocks of 5 (or 4) states correspond to the nuclei ^{156}Gd, ^{158}Gd and ^{154}Sm respectively

Energy MeV	Neutron Spin-Flip Overlap %	Energy MeV	Proton Spin-Flip Overlap %
9.35	33.16	6.54	38.00
9.50	30.37	6.57	25.37
9.42	22.18	7.12	25.27
9.25	16.64	6.46	24.54
8.02	18.96		
9.45	31.79	6.43	52.23
9.47	31.30	4.92	22.31
9.02	26.38	7.01	21.04
9.27	20.61	6.80	19.23
7.89	17.94	5.38	18.79
9.65	47.47	6.44	38.38
9.46	23.50	6.79	36.86
9.68	16.41	6.80	23.75
9.55	15.77	5.02	22.97
		5.52	22.06

To summarize, we see that the double hump B(M1)$_\sigma$ strength distribution observed in the 5-10 MeV region is reflected rather well in these calculations, providing a good description of both the shape and total strength found, although the established splitting is somewhat larger than observed, the specific proton and neutron maxima lying upward energy shifted by some 0.5 and 1 MeV respectively. Even the distinct fall-off in the M1 strength observed for ^{154}Sm in recent γ, γ' experiments [5] in the en-

Table 2. B(M1) strength distribution between 6.5-8 MeV in ^{154}Sm for SkIII using 1986 qp basis. Due to the energetic displacement the area 6.5-7.5 MeV in our calculations should be considered against those observations in the 6-7 MeV region

Energy [MeV]	$BM(1)_{orb}$ $[\mu_n^2]$	$B(M1)_\sigma$ $[\mu_n^2]$	$B(M1)_{tot}$ $[\mu_n^2]$
6.438	0.101	0.888	0.389
6.618	0.007	0.028	0.007
6.677	0.031	0.016	0.002
6.683	0.063	0.101	0.004
6.741	0.123	0.045	0.019
6.787	0.025	0.691	0.453
6.798	0.041	0.351	0.153
6.956	0.012	0.170	0.272
7.086	0.004	0.104	0.066
7.111	0.000	0.134	0.133
7.168	0.041	0.771	0.457
7.190	0.002	0.040	0.059
7.211	0.000	0.086	0.097
7.218	0.001	0.031	0.046
7.283	0.004	0.035	0.064
7.297	0.000	0.060	0.055
7.311	0.001	0.002	0.005
7.347	0.001	0.002	0.005
7.485	0.001	0.001	0.003
7.502	0.076	0.095	0.001
7.610	0.066	0.110	0.345
7.777	0.005	0.009	0.029
7.793	0.001	0.010	0.017
7.850	0.000	0.067	0.077
7.893	0.003	0.036	0.059
7.913	0.007	0.047	0.089
7.977	0.202	0.192	0.000
8.000	0.006	0.013	0.035

ergy region in between the two bumps, and attributed to an orbital-spin-flip destructive interference effect, we find echoed in our calculations cf. Table II. This agreement is satisfying and clearly lends support to our microscopic formalism and to the appropriateness of the interaction used which, it must be remembered, has in no way been fitted to present data. Indeed, the present investigation shows for the first time that Skyrme III force may be used to describe states with spin, and with better quantitative success than in previous investigations. Two new collective excitation modes of the nuclear system have been identified, which are rather unambiguously interpretable in the calculation as a proton and a neutron giant spin-flip resonance. For the rare earth region these are the first states of this nature to be found, which play an analogous role [4] to the spin-flip resonance discovered in ^{48}Ca some while ago [18]. Such collective excitations are of interest in the case of exotic nuclei as they are sensitive to properties in the surface region and differences in the neutron and proton effective interactions.

There has been preliminary evidence for yet a third concentration of strength at around 10 MeV in ^{154}Sm [4]. In our calculations we find that some of the higher lying, but weaker, 1^+ QRPA solutions in the vicinity of 10 MeV are of isovector spin-flip character, which may give a clue to the nature of these new states. In this regard, recent observations obtained in photo-excitation experiments [5] as M1 strength located around 6.7 MeV, might also be identified with components of the same state, indicating it to be of highly fragmented character.

Partial support for this work by B.M.F.T. under contract number 06 TM 743 (6) is gratefully acknowledged.

References

1. K.-G. Dietrich et al; Phys. Lett. **B220** (1989) 351
2. D. Frekers et al; Phys. Lett. **B244** (1990) 178
3. D. Frekers et al; Notre Dame Workshop on "*Giant Resonances and Related Phenomena*", Oct. 1991
4. A. Richter; Pub. in "*Response of the Nuclear System to External Forces*", Lecture Notes in Physics, Springer Verlag, 1995, p13-59 Eds. J.M. Arias et al. and references cited therein
5. P. v Neumann-Cosel Prog. Part. Nucl. Phys. 38(1997)213
6. D. Zawischa and J. Speth; Phys. Lett. **B252** (1990) 4
7. C. de Coster, K. Heyde, and A. Richter; Nucl. Phys. **A542** (1992) 375 and references cited therein
8. A. Faessler, R. Nojarov, and F.G. Scholtz; Nucl. Phys. **A515** (1990) 237
9. P. Sarriguren et al J. Phys. G19 (1993) 291 and G20 (1994) 315
10. P. Sarriguren et al Phys. Rev. C54 (1996) 690
11. R.R. Hilton; Proc. Int. Conf. "*Perspectives of Nuclear Physics in the Late Nineties*", Hanoi (1994), Eds. Dinh Dang et al, World Scientific, 1995, p86
12. C.D. Goodman; Nucl. Phys. **A374** (1982) 241c
13. R.M. Laszewski et al; Phys. Lett. **54** (1985) 530; Phys. Rev. **C43** (1986) 2013; Phys. Rev. Lett. **59** (1987) 431
14. O. Civitarese, A. Faessler, and R. Nojarov; Phys. Rev. **C35** (1987) 2310
15. M. Beiner, H. Flocard, N. van Giai, and P. Quentin; Nucl. Phys. **A238** (1975) 29
16. P. Ring and P. Schuck; *The Nuclear Many-body Problem*, Springer Verlag, (1980)
17. C. de Coster and K. Heyde; Phys. Rev. Lett. **66** (1991) 2456
18. W. Steffan et al; Phys.Letts **B95** (1980) 23